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Understanding the graph of a quadratic function is fundamental in algebra and helps in visualizing the parabola's shape, position, and key features. In this article, we will analyze the quadratic function $G(x) = 2x^2 + 8x + 10$ by identifying its key components: the y-intercept, the vertex, and the zeros. These elements provide insight into the graph's behavior and are essential in solving quadratic equations, optimizing functions, and graphing.

Understanding the Quadratic Function $G(x) = 2x^2 + 8x + 10$

Before diving into the specific features, it's important to understand the general form of a quadratic function. The standard quadratic form is:

$$ y = ax^2 + bx + c $$

where:
- a determines the opening direction and width of the parabola,
- b influences the position of the vertex along the x-axis,
- c is the y-intercept.

In our case, $G(x) = 2x^2 + 8x + 10$, the coefficients are:
- $a = 2$,
- $b = 8$,
- $c = 10$.

Since $a > 0$, the parabola opens upward, meaning it has a minimum point at its vertex.

Finding the Y-Intercept of $G(x) = 2x^2 + 8x + 10$

The y-intercept of a quadratic function occurs where the graph crosses the y-axis, which is at $x=0$. To find the y-intercept, substitute $x=0$ into the function:

$$
G(0) = 2(0)^2 + 8(0) + 10 = 0 + 0 + 10 = 10
$$

Therefore, the y-intercept is at the point (0, 10).

This point is significant because it provides a starting reference for graphing and understanding the parabola's position relative to the y-axis.

Determining the Vertex of G(x) = 2x^2 + 8x + 10

The vertex of a parabola is its highest or lowest point, depending on the direction it opens. Since the parabola opens upward (because $a=2 > 0$), the vertex represents its minimum point.

The vertex's coordinates (x_v, y_v) can be found using the vertex formula:

$$
x_v = -\frac{b}{2a}
$$

$$
y_v = G(x_v)
$$

Calculating the x-coordinate of the vertex:

$$
x_v = -\frac{8}{2 \times 2} = -\frac{8}{4} = -2
$$

Calculating the y-coordinate:

$$
G(-2) = 2(-2)^2 + 8(-2) + 10
$$

$$
= 2(4) + (-16) + 10
$$

$$
= 8 - 16 + 10 = 2
$$

Thus, the vertex is at the point $(-2, 2)$.

This point is crucial because it indicates the minimum value of the function and helps in sketching the graph accurately.

Finding the Zeros of G(x) = 2x^2 + 8x + 10

Zeros, or roots, of a quadratic function are the points where the graph crosses the x-axis, i.e., where $G(x) = 0$. To find these zeros, we solve the quadratic equation:

$$
2x^2 + 8x + 10 = 0
$$

Using the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

where:
- $a = 2$,
- $b = 8$,
- $c = 10$.

Calculating the discriminant (Δ):

$$
\Delta = b^2 - 4ac = 8^2 - 4 \times 2 \times 10 = 64 - 80 = -16
$$

Since the discriminant is negative ($\Delta = -16$), the quadratic has no real zeros. Instead, it has two complex conjugate zeros.

Calculating the complex roots:

$$
x = \frac{-8 \pm \sqrt{-16}}{2 \times 2} = \frac{-8 \pm 4i}{4}
$$

$$
x = -2 \pm i
$$

Therefore, the zeros are at $-2 + i$ and $-2 - i$.

This implies the parabola does not cross the x-axis and is entirely above or below it, depending on the vertex and the leading coefficient.

Summary of Key Features of G(x) = 2x^2 + 8x + 10

Feature	Coordinates / Description	Explanation
Y-intercept	(0, 10)	The point where the graph crosses the y-axis.
Vertex	(-2, 2)	The minimum point of the parabola, located at $(-2, 2)$.
Zeros	$-2 \pm i$	No real zeros; complex conjugate roots indicating the graph does not cross the x-axis.

Graphing the Function G(x) = 2x^2 + 8x + 10

To graph the quadratic function accurately, follow these steps:

1. Plot the y-intercept: Mark the point (0, 10).
2. Plot the vertex: Mark the point (-2, 2). Since the parabola opens upward, this is the lowest point.
3. Identify the zeros: Although the zeros are complex, understanding their nature helps in visualizing that the parabola does not intersect the x-axis.
4. Determine additional points: Choose other x-values (e.g., x = -1, -3) and compute their corresponding y-values to refine the shape.

For example:

- At $x = -1$:

$$
G(-1) = 2(1) + 8(-1) + 10 = 2 - 8 + 10 = 4
$$

- At $x = -3$:

$$
G(-3) = 2(9) + 8(-3) + 10 = 18 - 24 + 10 = 4
$$

5. Draw the parabola passing through these points, ensuring symmetry about the vertical line $x = -2$.

Applications and Significance of Identifying Key Features

Understanding the y-intercept, vertex, and zeros of a quadratic function is essential in various mathematical and real-world contexts.

- In physics: Quadratic functions model projectile motion, where the vertex represents the highest or lowest point.
- In economics: Parabolas can model profit functions, with the vertex indicating maximum profit.
- In engineering: Analyzing zeros helps in stability and control systems.
- In mathematics education: These features are fundamental in understanding the properties of quadratic functions and their graphs.

Conclusion

Analyzing the quadratic function G(x) = 2x^2 + 8x + 10 reveals important features that help in graphing and understanding its behavior. The y-intercept at (0, 10) indicates where the graph crosses the y-axis, while the vertex at (-2, 2) shows the parabola's lowest point, confirming the upward opening due to a positive leading coefficient. The zeros, being complex conjugates \(-2 \pm i\), indicate the parabola does not intersect the x-axis.

By mastering how to find these key features, students and professionals can better interpret quadratic functions, solve equations, and apply these concepts across various disciplines. Accurate graphing and interpretation of quadratic functions are vital skills in mathematics, physics, engineering, economics, and beyond.

For further understanding, consider practicing with different quadratic functions and exploring how changing coefficients affect the parabola's shape and position.

Frequently Asked Questions

What is the y-intercept of the graph of G(x) = 2x^2 + 8x + 10?

The y-intercept is the point where the graph crosses the y-axis, which occurs when x=0. Substituting x=0 into G(x): G(0) = 2(0)^2 + 8(0) + 10 = 10. So, the y-intercept is at (0, 10).

How do you find the vertex of the quadratic function G(x) = 2x^2 + 8x + 10?

The vertex of a quadratic ax^2 + bx + c can be found using the formula x = -b/(2a). Here, a=2 and b=8, so x = -8/(22) = -8/4 = -2. Plugging x = -2 back into $G(x)$: $G(-2) = 2(-2)^2 + 8(-2) + 10 = 2(4) - 16 + 10 = 8 - 16 + 10 = 2$. Therefore, the vertex is at (-2, 2).

What are the zeros of the quadratic function $G(x) = 2x^2 + 8x + 10$?

Zeros are the x-values where $G(x) = 0$. Solve $2x^2 + 8x + 10 = 0$. Divide through by 2: $x^2 + 4x + 5 = 0$. Use the quadratic formula: $x = [-4 \pm \sqrt{(4^2 - 415)}] / 2 = [-4 \pm \sqrt{(16 - 20)}] / 2 = [-4 \pm \sqrt{(-4)}] / 2$. Since the discriminant is negative, the zeros are complex: $x = [-4 \pm 2i] / 2 = -2 \pm i$. So, the function has no real zeros, only complex zeros at $-2 + i$ and $-2 - i$.

Is the parabola of $G(x) = 2x^2 + 8x + 10$ opening upwards or downwards?

Since the coefficient of x^2 is positive (a=2), the parabola opens upwards.

What is the axis of symmetry for the graph of $G(x) = 2x^2 + 8x + 10$?

The axis of symmetry is the vertical line that passes through the vertex. It can be found using $x = -b/(2a)$. Here, $x = -8/(22) = -2$. So, the axis of symmetry is $x = -2$.

How does the discriminant help in determining the zeros of $G(x)$?

The discriminant, given by $b^2 - 4ac$, indicates the nature of the roots. If it's positive, there are two real zeros; if zero, one real zero; if negative, zeros are complex. For $G(x)$, the discriminant is $16 - 20 = -4$, which is negative, indicating complex zeros.

Can the graph of $G(x) = 2x^2 + 8x + 10$ intersect the x-axis?

No, because the zeros are complex (not real), meaning the parabola does not intersect the x-axis.

Additional Resources

Consider The Graph Of $G(x) = 2x^2 + 8x + 10$. Identify The Y-intercept, The Vertex, And The Zeros Of The

Understanding the characteristics of a quadratic function like $G(x) = 2x^2 + 8x + 10$ is fundamental in algebra and important for various applications across science, engineering, and economics. This article delves into the key features of this quadratic function, including its y-intercept, vertex, and zeros, providing a comprehensive yet accessible guide to interpreting its graph.

Breaking Down the Quadratic Function $G(x) = 2x^2 + 8x + 10$

Before exploring specific features, it’s essential to understand the general form of a quadratic function. The standard form is:

$G(x) = ax^2 + bx + c$

where:

- a, b, and c are constants
- $a \neq 0$

In our case, a = 2, b = 8, and c = 10. The quadratic graph is a parabola that opens upwards because the coefficient a is positive. The shape and position of this parabola can be analyzed through its y-intercept, vertex, and zeros.

Y-Intercept: The Point Where the Graph Crosses the Y-Axis

The y-intercept of a graph is the point where the curve crosses the y-axis, corresponding to the value of the function when x = 0. It provides a starting reference point for sketching the graph and understanding the function's basic position.

Calculating the Y-Intercept

To find the y-intercept, substitute x = 0 into the function:

$G(0) = 2(0)^2 + 8(0) + 10 = 0 + 0 + 10 = 10$

Result:

- The y-intercept is at (0, 10).

Significance of the Y-Intercept

This point indicates that when x is zero, the output of the function is 10. It's a fixed point on the graph and a useful anchor for plotting the parabola.

The Vertex: The Parabola's Highest or Lowest Point

In a parabola, the vertex is the point at which the parabola turns; it represents either the maximum or minimum value of the quadratic function. Since our coefficient $a = 2$ is positive, the parabola opens upward, making the vertex its lowest point.

Finding the Vertex Coordinates

The vertex can be found using the vertex form or the vertex formula derived from the standard form.

The x-coordinate of the vertex is given by:

$$x_v = -b / (2a)$$

Substituting the known values:

$$x_v = -8 / (2\ 2) = -8 / 4 = -2$$

Next, find the y-coordinate by plugging x_v back into the original function:

$$G(-2) = 2(-2)^2 + 8(-2) + 10$$

Calculations:

- $(-2)^2 = 4$
- $2\ 4 = 8$
- $8\ (-2) = -16$

Adding them up:

$$G(-2) = 8 - 16 + 10 = 2$$

Vertex Point:

- The vertex is at (-2, 2).

Interpreting the Vertex

This point signifies the minimum value of the function $G(x)$. The parabola reaches its lowest point at $x = -2$, with a y-value of 2. This is crucial in optimization problems where finding the minimum point of a quadratic is necessary.

Zeros of the Function: The Roots Where G(x) = 0

Zeros, or roots, of a quadratic are the x-values where the graph intersects the x-axis, i.e., where $G(x) = 0$. These points are essential for solving equations and understanding the domain where the function is positive or negative.

Finding the Zeros

We need to solve:

$2x^2 + 8x + 10 = 0$

This is a quadratic equation. The most straightforward method is to use the quadratic formula:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Calculate the discriminant D:

$D = b^2 - 4ac$

$D = 8^2 - 4\ 2\ 10 = 64 - 80 = -16$

Since D is negative, the zeros are complex (non-real). Let's proceed to find them explicitly.

Calculating the Roots

$x = [-8 \pm \sqrt{(-16)}] / (2\ 2)$

$\sqrt{(-16)} = 4i$ (since $\sqrt{(-1)} = i$)

Thus,

$x = [-8 \pm 4i] / 4$

Simplify numerator and denominator:

$x = (-8 / 4) \pm (4i / 4) = -2 \pm i$

Zeros:

- The quadratic has complex conjugate roots at $x = -2 \pm i$.

Implications

Because the zeros are complex, the parabola does not cross the x-axis. This indicates the entire graph remains above the x-axis, consistent with the minimum point at $y = 2$. The negative discriminant reveals that the quadratic has no real solutions, and the parabola does not intersect the x-axis.

Additional Insights: Graph Behavior and Applications

Understanding the y-intercept, vertex, and zeros allows for a comprehensive visualization of G(x). Since the parabola opens upward with a vertex at (-2, 2) and a y-intercept at (0, 10), it is situated above the x-axis across its entire domain.

Graph Characteristics Summary:

- The parabola is symmetric about the vertical line x = -2.
- The minimum value of the function is 2 at x = -2.
- The parabola does not cross the x-axis, remaining entirely positive.
- The y-intercept at (0, 10) is to the right of the vertex, indicating the parabola rises on either side of the vertex.

Real-World Applications

Quadratic functions like G(x) are prevalent in physics (projectile motion), economics (profit maximization), and engineering (parabolic reflectors). Knowing the vertex helps optimize outcomes, while the y-intercept and zeros clarify the range and domain of interest.

Conclusion

Analyzing $G(x) = 2x^2 + 8x + 10$ reveals a parabola opening upward, with a y-intercept at (0, 10), a vertex at (-2, 2), and no real roots. These features collectively enable mathematicians and practitioners to interpret the graph's shape, position, and key points accurately. Whether plotting by hand or using graphing technology, understanding these properties is essential for applying quadratic functions to solve real-world problems efficiently and effectively.

By mastering how to identify the y-intercept, vertex, and zeros, students and professionals gain critical tools for analyzing quadratic behaviors, predicting outcomes, and making informed decisions based on mathematical models.

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