Consider The System Of Equations -2x + 3y = -5 And 3x - 4y = 6.-Describe Two Different Ways To Solve

Consider The System Of Equations -2x + 3y = -5 And 3x - 4y = 6.-Describe Two Different Ways To Solve

Understanding how to solve systems of linear equations is fundamental in algebra and many applied mathematics fields. When faced with the system:

$-2x + 3y = -5$
$3x - 4y = 6$

it’s essential to explore different methods to find the values of x and y that satisfy both equations simultaneously. This article will delve into two effective techniques: the substitution method and the elimination method. These approaches not only help in solving this particular system but also build foundational skills applicable to a wide range of algebraic problems.

Introduction to Systems of Equations

A system of equations comprises two or more equations with the same variables. The goal is to find the values of these variables that satisfy all equations at once. In a two-variable system, solutions are typically points on the coordinate plane where the equations intersect.

For the given system:

$-2x + 3y = -5$ (Equation 1)
$3x - 4y = 6$ (Equation 2)

our task is to identify the pair (x, y) that makes both equations true.

Method 1: Solving by Substitution

The substitution method involves solving one of the equations for one variable and then substituting that expression into the other equation. This approach is especially useful when one of the equations is easily solvable for one variable.

Step 1: Solve one equation for one variable

Let's choose Equation 1 and solve for x:

-2x + 3y = -5

Rearranged for x:

-2x = -5 - 3y
x = (5 + 3y) / 2

Step 2: Substitute this expression into the other equation

Now, substitute x = (5 + 3y) / 2 into Equation 2:

3x - 4y = 6

Replace x:

3 [(5 + 3y) / 2] - 4y = 6

Step 3: Simplify and solve for y

Multiply through:

(3 (5 + 3y)) / 2 - 4y = 6

Calculate numerator:

(15 + 9y) / 2 - 4y = 6

To clear the denominator, multiply entire equation by 2:

15 + 9y - 8y = 12

Simplify:

15 + y = 12

Subtract 15 from both sides:

y = 12 - 15
y = -3

Step 4: Find x using the value of y

Recall x = (5 + 3y) / 2

Plug in y = -3:

x = (5 + 3 (-3)) / 2
x = (5 - 9) / 2
x = (-4) / 2
x = -2

Solution via Substitution:

The solution to the system is:

x = -2, y = -3

This point (-2, -3) is where both lines intersect.

Method 2: Solving by Elimination

The elimination method seeks to eliminate one variable by combining the equations, making it easier to solve for the remaining variable. This technique is particularly effective when the coefficients of a variable are opposites or can be made so by multiplication.

Step 1: Prepare the system for elimination

Our system:

-2x + 3y = -5 (Equation 1)
3x - 4y = 6 (Equation 2)

To eliminate either x or y, align coefficients. Let's eliminate x by making the coefficients opposites.

Multiply Equation 1 by 3:

(3) (-2x + 3y) = 3 -5
-6x + 9y = -15 (Equation 3)

Multiply Equation 2 by 2:

(2) (3x - 4y) = 2 6

6x - 8y = 12 (Equation 4)

Now, add Equations 3 and 4:

(-6x + 9y) + (6x - 8y) = -15 + 12

Simplify:

0x + (9y - 8y) = -3

Which reduces to:

y = -3

Step 2: Substitute y back into one of the original equations

Using Equation 1:

-2x + 3y = -5

Plug in y = -3:

-2x + 3 (-3) = -5
-2x - 9 = -5

Add 9 to both sides:

-2x = 4

Divide both sides by -2:

x = -2

Solution via Elimination:

Again, the solution is:

x = -2, y = -3

The elimination method efficiently led us to the solution by eliminating one variable entirely through multiplication and addition.

Comparison of the Two Methods

Both substitution and elimination methods are valid and effective for solving systems of equations. Here’s a quick comparison:

- **Substitution Method:** Best when one equation is already solved for a variable or can be easily rearranged. It provides a straightforward path to the solution but can become cumbersome if the equations are complex.
- **Elimination Method:** Ideal when the coefficients of a variable are opposites or can be easily made to be so through multiplication. It often involves fewer steps and is efficient for systems where variables can be eliminated directly.

Practical Tips for Solving Systems of Equations

- Always check if equations are in the same form; converting them to slope-intercept form (y = mx + b) can make substitution easier.
- When coefficients are already aligned or can be easily aligned, elimination is generally quicker.
- Verify your solutions by substituting back into original equations.
- Practice both methods to determine which works best for different types of systems.

Conclusion

Solving systems of equations like:

$-2x + 3y = -5$
$3x - 4y = 6$

can be approached effectively using the substitution or elimination methods. Each method has its advantages and is suitable for different scenarios. The substitution method involves solving one equation for one variable and substituting into the other, while the elimination method involves manipulating the equations to cancel out one variable directly. Mastering both techniques enhances your problem-solving toolbox, allowing you to handle a wide variety of algebraic systems with confidence.

Whether you’re a student learning algebra, a teacher preparing lessons, or someone working on applied mathematics problems, understanding these methods is essential. Practice with different systems to become proficient, and you’ll find solving equations becomes more intuitive and efficient.

Frequently Asked Questions

What are the two methods to solve the system of equations -2x + 3y = -5 and 3x - 4y = 6?

The two common methods are substitution and elimination. Substitution involves solving one equation for a variable and substituting into the other, while elimination involves adding or subtracting equations to eliminate a variable.

How can substitution be used to solve the system -2x + 3y = -5 and 3x - 4y = 6?

Solve one equation for one variable, for example, solve -2x + 3y = -5 for x: x = (3y + 5)/2. Then substitute this expression into the second equation to find y, and back-substitute to find x.

How does the elimination method work for solving the given system?

Multiply equations to align coefficients of one variable, then add or subtract the equations to eliminate that variable, allowing you to solve for the remaining variable directly.

Can you demonstrate solving the system using substitution?

Yes. From the first equation: -2x + 3y = -5, solve for x: x = (3y + 5)/2. Substitute into the second: 3(3y + 5)/2 - 4y = 6. Simplify and solve for y, then find x.

What are the steps to solve the system using elimination?

First, multiply equations to align coefficients of a variable, for example, multiply the first by 4 and the second by 3. Then add or subtract to eliminate a variable and solve for the remaining variable.

Which method is more straightforward for solving this system: substitution or elimination?

Both methods are effective. The choice depends on the specific equations, but elimination can be quicker when the coefficients are easily aligned, while substitution is helpful if one variable is already isolated.

What is the solution to the system of equations -2x + 3y = -5 and 3x - 4y = 6?

Solving the system yields x = 2 and y = -1, which can be verified by substituting back into the original equations.

How do I verify the solution once I find x and y?

Substitute the values of x and y into both original equations to check if both sides are equal. If they are, the solution is correct.

Are there any advantages to using substitution over elimination for this system?

Substitution is advantageous when one equation is already solved for a variable or when substitution simplifies calculations. It also helps in understanding the relationship between variables.

What should I do if the two methods give different solutions?

If different solutions are obtained, double-check calculations for errors. If the system is inconsistent, it may have no solution; if dependent, it may have infinitely many solutions.

Additional Resources

Solving Systems of Equations: An In-Depth Analysis of Two Effective Methods

When tackling systems of equations, understanding the various approaches to find solutions is fundamental in algebra. These systems often appear in real-world applications—from engineering to economics—making mastery over their solution techniques invaluable. Here, we will explore the system of equations:

-2x + 3y = -5
3x - 4y = 6

We will delve into two distinct methods: Substitution Method and Elimination Method. Both approaches have their own advantages and ideal scenarios for use. This comprehensive review aims to not only explain each method step-by-step but also to provide insights into when and why they are effective.

Understanding the System of Equations

Before diving into solution techniques, it's crucial to interpret the problem correctly.

- The system is composed of two linear equations in two variables, x and y.
- The goal is to find the values of x and y that satisfy both equations simultaneously.
- Graphically, these equations represent straight lines; the solution is the point(s) where these lines intersect.

Equations:

1. -2x + 3y = -5
2. 3x - 4y = 6

Method 1: Substitution Method

Fundamentals of the Substitution Method

The substitution method involves solving one of the equations for one variable in terms of the other, then substituting this expression into the other equation. This reduces the system to a single-variable equation, which can then be solved straightforwardly.

Advantages:

- Useful when one of the equations is already solved for a variable.
- Simplifies complex systems into manageable steps.
- Particularly effective when one variable has a coefficient of 1 or -1.

Step-by-Step Process:

1. Choose an Equation and Solve for a Variable

For the given system, both equations are relatively manageable. Let's solve the first equation for y:

$$
-2x + 3y = -5 \implies 3y = 2x - 5 \implies y = \frac{2x - 5}{3}
$$

2. Substitute into the Second Equation

Replace y in the second equation with the expression from step 1:

$$
3x - 4\left(\frac{2x - 5}{3}\right) = 6
$$

3. Simplify and Solve for x

Multiply through to clear fractions:

$$
3x - \frac{4(2x - 5)}{3} = 6
$$

Multiply entire equation by 3 to eliminate denominator:

$$
3 \times 3x - 4(2x - 5) = 6 \times 3
$$

$$
9x - 8x + 20 = 18
$$

Simplify:

$$
(9x - 8x) + 20 = 18 \implies x + 20 = 18
$$

$$
x = 18 - 20 = -2
$$

4. Find y Using the Expression

Substitute x back into the expression for y:

$$
y = \frac{2(-2) - 5}{3} = \frac{-4 - 5}{3} = \frac{-9}{3} = -3
$$

Solution:

$$
\boxed{
x = -2, \quad y = -3
}
$$

Verification:

Plugging these values back into the original equations to verify:

- First equation:

$$
-2(-2) + 3(-3) = 4 - 9 = -5 \quad \checkmark
$$

- Second equation:

$$
3(-2) - 4(-3) = -6 + 12 = 6 \quad \checkmark
$$

Both are satisfied; thus, the solution is correct.

Method 2: Elimination Method

Fundamentals of the Elimination Method

The elimination method aims to eliminate one variable by combining the equations through addition or subtraction after appropriate multiplication. This method is particularly efficient when the coefficients of one variable are opposites or can be made so with simple multiplication.

Advantages:

- Efficient when coefficients are readily manipulated to cancel out a variable.
- Avoids substitution, which can sometimes lead to fractions.
- Often faster for systems with coefficients that are multiples or close in value.

Step-by-Step Process:

1. Align the Equations

Write the system:

$$
-2x + 3y = -5 \quad (1)
$$
$$
3x - 4y = 6 \quad (2)
$$

2. Multiply Equations to Align Coefficients

To eliminate either x or y, multiply equations to match coefficients:

- To eliminate x, find LCM of -2 and 3, which is 6.

Multiply equation (1) by 3:

$$
3 \times (-2x + 3y) = 3 \times -5
$$
$$
-6x + 9y = -15
$$

Multiply equation (2) by 2:

$$
2 \times (3x - 4y) = 2 \times 6
$$
$$
6x - 8y = 12
$$

3. Add or Subtract to Eliminate a Variable

Add the two new equations:

$$
(-6x + 9y) + (6x - 8y) = -15 + 12
$$

$$
(-6x + 6x) + (9y - 8y) = -3
$$

$$
0x + y = -3
$$

$$
\boxed{
y = -3
}
$$

4. Back-Substitute to Find the Other Variable

Use y = -3 in one of the original equations, say the first:

\[

$$
-2x + 3(-3) = -5
$$

$$
-2x - 9 = -5
$$

$$
-2x = -5 + 9 = 4
$$

$$
x = \frac{4}{-2} = -2
$$

Solution:

$$
\boxed{
x = -2, \quad y = -3
}
$$

Verification:

Substitute into the second original:

$$
3(-2) - 4(-3) = -6 + 12 = 6 \quad \checkmark
$$

The solution aligns with the previous method, confirming its accuracy.

Comparative Analysis of Both Methods

Efficiency and Suitability

Aspect	Substitution Method	Elimination Method
When to Use	When one variable is easy to isolate or equations are straightforward	When coefficients are convenient for elimination or when equations are already aligned for elimination
Complexity	Slightly more algebraic manipulation with substitution	Slightly more straightforward when coefficients are compatible
Risk of Fractions	Higher if substitution involves dividing by variables with small coefficients	Usually less prone to fractions if coefficients are multiples or can be easily aligned

Practical Considerations

- Choice of Method: The choice often depends on the form of the equations. If an equation is already solved for a variable, substitution is quicker. If coefficients align favorably, elimination is efficient.
- Speed: For systems where coefficients are multiples or can be easily manipulated, elimination tends to be faster.
- Errors: Substitution may introduce fractions, increasing the chance of arithmetic mistakes, while elimination reduces fractions but involves multiplication steps.

Additional Insights and Advanced Considerations

While the current system yields a unique solution, understanding the broader landscape of systems can deepen your mastery:

- Consistency and Types of Solutions:
- Unique solution: The system has a single point of intersection (as in our case).
- Infinite solutions: The equations are dependent (the same line), leading to infinitely many intersection points.
- No solution: The equations are inconsistent, representing parallel lines that never intersect.

- Graphical Interpretation:
- The solution point (-2, -3) corresponds to the intersection of the two lines.
- Visualizing the lines can often provide intuition about the nature of solutions.

- Extension to Larger Systems:
- Techniques such as Gaussian elimination extend these methods to systems with more variables.
- Matrix methods, including Cramer's rule, can also be employed for systems with multiple equations and variables.

- Numerical Methods and Approximate Solutions:
- For non-linear or more complex systems, numerical methods like graphing calculators, iterative algorithms, or software (e.g., MATLAB, WolframAlpha) are used.

Conclusion: A Holistic Approach to Solving Systems

Mastering the substitution and elimination methods provides a robust toolkit for solving linear systems of equations. Each method has its strengths:

- Substitution is ideal when an equation is already solved for one variable or involves fractions that are easy to manipulate.
- Elimination is efficient when coefficients are compatible for straightforward

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