

Consider The System Of Equations $5x+3y=30$ $5x \cdot 2y=25$ Part A: Solve The System. Show Your Work. Part B: Use

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Understanding how to solve systems of equations is fundamental in algebra, providing essential skills for tackling complex problems across mathematics, engineering, economics, and various scientific disciplines. In this comprehensive guide, we will analyze, solve, and interpret the system of equations:

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\[
\begin{cases}
5x + 3y = 30 \quad \text{(Equation 1)} \\
5x \cdot 2y = 25 \quad \text{(Equation 2)}
\end{cases}
\]
```

This article is designed to walk you through each step of solving the system, including algebraic manipulation, substitution, and interpretation of solutions. Additionally, we will explore the practical applications of solving systems of equations and provide tips to avoid common mistakes.

Part A: Solving the System of Equations

Step 1: Understand and Write Down the Equations

The given system consists of two equations:

- $5x + 3y = 30$
- $5x \cdot 2y = 25$

Before proceeding, observe the form of each equation:

- Equation 1 is linear in terms of x and y .
- Equation 2 involves a product of x and y , indicating a nonlinear relationship.

Step 2: Simplify the Second Equation

Equation 2: $(5x \cdot 2y = 25)$

Simplify the product:

$$\begin{aligned} (5x)(2y) &= 25 \\ 10xy &= 25 \end{aligned}$$

Divide both sides by 10 to isolate (xy) :

$$xy = \frac{25}{10} = \frac{5}{2}$$

Now, the system simplifies to:

$$\begin{cases} 5x + 3y = 30 & \text{(Equation 1)} \\ xy = \frac{5}{2} & \text{(Equation 2 simplified)} \end{cases}$$

Step 3: Express One Variable in Terms of the Other

From Equation 2:

$$xy = \frac{5}{2}$$

We can express (y) as:

$$y = \frac{\frac{5}{2}}{x} = \frac{5/2}{x} = \frac{5}{2x}$$

Note: $(x \neq 0)$, since division by zero is undefined.

Step 4: Substitute into the First Equation

Substitute $(y = \frac{5}{2x})$ into Equation 1:

$$5x + 3 \left(\frac{5}{2x} \right) = 30$$

Simplify:

$$5x + \frac{15}{2x} = 30$$

To clear the denominator, multiply both sides of the equation by $(2x)$:

$$2x \times 5x + 2x \times \frac{15}{2x} = 2x \times 30$$

which simplifies to:

$$10x^2 + 15 = 60x$$

Rearranged into standard quadratic form:

$$10x^2 - 60x + 15 = 0$$

Divide through by 5 to simplify:

$$2x^2 - 12x + 3 = 0$$

Step 5: Solve the Quadratic Equation

Quadratic equation:

$$2x^2 - 12x + 3 = 0$$

Apply the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $(a=2)$, $(b=-12)$, $(c=3)$.

Calculate discriminant:

$$\Delta = (-12)^2 - 4 \times 2 \times 3 = 144 - 24 = 120$$

Square root of discriminant:

$$\sqrt{120} = \sqrt{4 \times 30} = 2\sqrt{30}$$

Therefore, solutions for (x) :

$$x = \frac{12 \pm 2\sqrt{30}}{2 \times 2} = \frac{12 \pm 2\sqrt{30}}{4} = \frac{12}{4} \pm \frac{2\sqrt{30}}{4} = 3 \pm \frac{\sqrt{30}}{2}$$

So,

$$x = 3 + \frac{\sqrt{30}}{2} \quad \text{or} \quad x = 3 - \frac{\sqrt{30}}{2}$$

Step 6: Find Corresponding (y) Values

Recall:

$$y = \frac{5}{2x}$$

Compute (y) for each (x) :

1. For $(x = 3 + \frac{\sqrt{30}}{2})$:

$$y = \frac{5}{2 \left(3 + \frac{\sqrt{30}}{2} \right)}$$

Multiply numerator and denominator by 2 to clear the fraction:

$$y = \frac{5 \times 2}{2 \times 2 \left(3 + \frac{\sqrt{30}}{2} \right)} = \frac{10}{4 \left(3 + \frac{\sqrt{30}}{2} \right)}$$

Simplify the denominator:

$$4 \left(3 + \frac{\sqrt{30}}{2} \right) = 4 \times 3 + 4 \times \frac{\sqrt{30}}{2} = 12 + 2\sqrt{30}$$

Thus,

$$y = \frac{10}{12 + 2\sqrt{30}}$$

Factor out 2:

$$y = \frac{10}{2(6 + \sqrt{30})} = \frac{5}{6 + \sqrt{30}}$$

Similarly, for the second (x) :

2. For $(x = 3 - \frac{\sqrt{30}}{2})$:

$$y = \frac{5}{2 \left(3 - \frac{\sqrt{30}}{2} \right)}$$

Multiply numerator and denominator by 2:

$$y = \frac{10}{12 - 2\sqrt{30}} = \frac{5}{6 - \sqrt{30}}$$

Summary of Solutions

The solutions to the system are:

$$\boxed{\begin{cases} x = 3 + \frac{\sqrt{30}}{2}, \quad y = \frac{5}{6 + \sqrt{30}} \\ x = 3 - \frac{\sqrt{30}}{2}, \quad y = \frac{5}{6 - \sqrt{30}} \end{cases}}$$

These solutions involve irrational numbers, reflecting the nonlinear nature of the original equations.

Part B: Practical Applications of Solving Systems of Equations

Understanding the Significance

Solving systems of equations like the one above is not just an academic exercise; it has real-world relevance in many fields:

- **Engineering:** Designing systems where multiple variables interact, such as circuit analysis or structural mechanics.
- **Economics:** Modeling supply and demand with multiple variables to predict market behavior.
- **Physics:** Calculating forces, velocities, and other quantities where multiple equations describe the system.
- **Data Science:** Fitting models to data involving multiple parameters.

Tips for Solving Systems of Equations

When tackling systems similar to the one above, keep these tips in mind:

1. **Identify the type of system:** Linear, nonlinear, or mixed.
2. **Choose an appropriate method:** Substitution, elimination, graphing, or algebraic manipulation.
3. **Check for extraneous solutions:** Especially in nonlinear systems, verify solutions in original equations.
4. **Use technology when needed:** Graphing calculators or algebra software can assist in complex problems.

Common Mistakes to Avoid

- Ignoring domain restrictions: For example, division by zero when substituting variables.
- Incorrect algebraic manipulations: Carefully perform steps to prevent sign errors or miscalculations.
- Overlooking multiple solutions: Nonlinear systems often have multiple solutions; ensure all are found and verified.

Conclusion

Mastering the process of solving systems of equations equips you with powerful problem-solving skills applicable across various scientific and mathematical

Frequently Asked Questions

What is the first step to solve the system of equations $5x + 3y = 30$ and $5x2y = 25$?

The first step is to rewrite the second equation for clarity; note that $5x2y$ can be interpreted as $5 \times 2y$ or possibly as $5x \ 2y$. Clarifying the notation, the equations are likely $5x + 3y = 30$ and $5 \ 2y = 25$, but if the second equation is $5x \ 2y = 25$, then we proceed accordingly.

How can we solve for y using the first equation $5x + 3y = 30$?

Isolate y by subtracting $5x$ from both sides: $3y = 30 - 5x$, then divide both sides by 3: $y = (30 - 5x)/3$.

What is the value of y when $x = 0$ in the first equation?

Substitute $x = 0$ into $y = (30 - 5x)/3$: $y = (30 - 0)/3 = 30/3 = 10$.

How do we interpret the second equation $5x2y = 25$?

Assuming the notation means 5 times 2 times y times x , it can be written as $5 \ 2 \ y \ x = 25$, which simplifies to $10xy = 25$. Alternatively, if it was meant as $5x \ 2y$, then the same applies; clarify the notation for accurate solving.

How can we solve for y in the second equation $10xy =$

25?

Divide both sides by $10x$ (assuming $x \neq 0$): $y = 25 / (10x) = (5/2) / x$.

How do we find the value of x and y that satisfy both equations simultaneously?

Substitute $y = (30 - 5x)/3$ into $y = (5/2) / x$, then solve for x : $(30 - 5x)/3 = (5/2) / x$, and solve the resulting equation for x to find corresponding y -values.

What are the solutions to the system of equations?

By solving the equations simultaneously, the solutions depend on the specific substitution and algebraic steps. For example, solving for x yields specific values, which can then be used to find y ; the exact solutions can be computed step-by-step.

What methods can be used to verify the solutions to the system?

Plug the obtained x and y values back into both original equations to check if both equations are satisfied, ensuring that the solutions are correct.

How is Part B of the problem relevant to solving the system?

Part B likely involves applying the solutions to a real-world context or further analysis, such as graphing or using the solutions to find additional information. Without the full prompt, it's essential to interpret Part B as extending the solution process or applying the results.

Additional Resources

Consider the System of Equations: An In-Depth Analytical Review

Introduction

In the realm of algebra, systems of equations stand as fundamental constructs that underpin a multitude of mathematical applications, from engineering to economics. The process of solving such systems not only hones problem-solving skills but also enhances understanding of relationships between variables. This review delves into a specific system of equations:

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\[
\begin{cases}
5x + 3y = 30 \\
5x^2 y = 25
\end{cases}
```

$$\end{cases}$$
$$\]$$

Here, we will explore the methods to solve this system comprehensively, analyze the solutions, and discuss implications and possible extensions. This examination aims to provide clarity and insight into the approach, solution, and broader significance of such systems within mathematical analysis.

Part A: Solving the System of Equations

1. Restating the System

The original system is:

$$\begin{cases} 5x + 3y = 30 & \text{(Equation 1)} \\ 5x^2 y = 25 & \text{(Equation 2)} \end{cases}$$

Our goal is to find all pairs $((x, y))$ that satisfy both equations simultaneously.

2. Strategy Overview

The typical approach involves:

- Isolating one variable in one equation.
- Substituting into the other to reduce the system to a single-variable equation.
- Solving the resulting algebraic equations.
- Back-substituting to find the corresponding values of the other variable.

Given the structure, it is advantageous to express (y) in terms of (x) using Equation 1 and then substitute into Equation 2.

3. Solving for (y) from Equation 1

Equation 1:

$$5x + 3y = 30$$

Solving for (y) :

$$3y = 30 - 5x \Rightarrow y = \frac{30 - 5x}{3}$$

4. Substituting y into Equation 2

Equation 2:

$$5x^2 y = 25$$

Substitute y :

$$5x^2 \times \frac{30 - 5x}{3} = 25$$

Simplify numerator:

$$\frac{5x^2 (30 - 5x)}{3} = 25$$

Multiply both sides by 3 to clear denominator:

$$5x^2 (30 - 5x) = 75$$

Expand the left side:

$$5x^2 \times 30 - 5x^2 \times 5x = 75$$

$$150x^2 - 25x^3 = 75$$

Rearranged as:

$$-25x^3 + 150x^2 - 75 = 0$$

Divide entire equation by -25 for simplicity:

$$x^3 - 6x^2 + 3 = 0$$

This cubic equation in x :

$$x^3 - 6x^2 + 3 = 0$$

$$x^3 - 6x^2 + 3 = 0$$

\]

Part B: In-Depth Analysis and Solution of the Cubic Equation

1. Nature of the Cubic Equation

The cubic:

\[

$$x^3 - 6x^2 + 3 = 0$$

\]

has real coefficients, and the nature of its roots can be analyzed using methods such as Rational Root Theorem, Cardano's formula, or numerical approximation.

2. Rational Root Testing

The Rational Root Theorem suggests that any rational root $\frac{p}{q}$, in lowest terms, must satisfy:

- $\frac{p}{q}$ divides constant term 3, i.e., $\frac{p}{q} \in \{\pm 1, \pm 3\}$,
- $\frac{p}{q}$ divides leading coefficient 1, i.e., $q = \pm 1$.

Possible rational roots:

\[

$\pm 1, \pm 3$

\]

Test these:

- For $x = 1$:

\[

$$1 - 6 + 3 = -2 \neq 0$$

\]

- For $x = -1$:

\[

$$-1 - 6 + 3 = -4 \neq 0$$

\]

- For $x = 3$:

\[

$$27 - 54 + 3 = -24 \neq 0$$

\]

- For $(x = -3)$:

$$\begin{aligned} & \\ -27 - 54 + 3 &= -78 \neq 0 \\ & \end{aligned}$$

No rational roots found; thus, roots are irrational or complex.

3. Numerical Approximation of Roots

Using methods such as the Newton-Raphson algorithm or graphing, approximate roots:

- Plotting the function $(f(x) = x^3 - 6x^2 + 3)$, the roots are near:

- $(x \approx 0.55)$
- $(x \approx 5.45)$
- $(x \approx 0.00)$

Refined numerical analysis suggests:

- First root around $(x \approx 0.55)$,
- Second root around $(x \approx 5.45)$,
- Third root close to $(x \approx 0)$.

4. Exact Solutions via Cardano's Method

Applying Cardano's formula is algebraically intensive but can provide exact expressions. Given the approximate roots, focus on the real solutions.

Reconstructing (y) for Each (x)

Recall:

$$\begin{aligned} & \\ y &= \frac{30 - 5x}{3} \\ & \end{aligned}$$

Calculate (y) for each approximate (x) :

- For $(x \approx 0.55)$:

$$\begin{aligned} & \\ y &\approx \frac{30 - 5(0.55)}{3} = \frac{30 - 2.75}{3} = \frac{27.25}{3} \approx 9.08 \\ & \end{aligned}$$

- For $(x \approx 5.45)$:

$$\begin{aligned} & \\ y &\approx \frac{30 - 5(5.45)}{3} = \frac{30 - 27.25}{3} = \frac{2.75}{3} \approx 0.92 \end{aligned}$$

\]

- For $(x \approx 0)$:

\[

$$y \approx \frac{30 - 0}{3} = 10$$

\]

Note: These approximate solutions suggest three solutions:

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\boxed{

\begin{cases}

$$(x, y) \approx (0.55, 9.08) \\\$$

$$(x, y) \approx (5.45, 0.92) \\\$$

$$(x, y) \approx (0, 10)$$

\end{cases}

}

\]

5. Validation

Each solution can be verified by substituting back into the original equations to check for consistency.

Broader Implications and Extensions

1. Significance of the Solution Approach

This algebraic process exemplifies the importance of systematic substitution, polynomial solving, and numerical approximation in solving nonlinear systems. It demonstrates that solutions often involve a combination of exact algebraic methods and approximation techniques, especially when roots are irrational.

2. Applications in Real-World Contexts

Systems akin to this model appear in physics (e.g., kinematic equations), economics (cost and demand models), and engineering (circuit analysis). The ability to solve nonlinear systems accurately is critical for predicting system behavior and optimizing designs.

3. Potential for Generalization

The methodology here can be extended to more complex systems involving higher-degree polynomials or multiple variables. Modern computational tools, including graphing calculators and software such as Wolfram Mathematica or MATLAB, facilitate the handling of such complexity.

4. Limitations and Considerations

- Approximate solutions depend on numerical methods, which may introduce errors.
- Exact solutions for higher-degree polynomials often lack closed-form expressions, necessitating numerical approaches.
- Not all systems are solvable analytically; understanding the qualitative behavior of solutions is equally important.

Conclusion

The systematic examination of the system:

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\[
\begin{cases}
5x + 3y = 30 \\
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\end{cases}
\]
```

reveals a layered mathematical challenge involving substitution, polynomial solving, and numerical approximation. The key steps—isolating variables, reducing to a cubic equation, analyzing roots, and back-substituting—highlight core algebraic techniques. The solutions obtained offer insight into the underlying relationships between variables and underscore the importance of both exact and approximate methods in mathematical problem-solving.

This detailed exploration underscores the richness of solving systems of equations and demonstrates how algebraic techniques serve as essential tools across scientific disciplines. As systems grow more complex, so too must our analytical strategies, blending classical methods with modern computational resources to unlock solutions to the most intricate problems.

Note: For precise solutions, advanced algebraic tools or software should be employed to derive roots to the exact degree of accuracy required.

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