Convert The Rectangular Equation X^2 + Y^2 = 2y To Polar Form A) R= 4 Sin Theta B) R= 2 Cos Theta C)

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Understanding how to convert equations from rectangular (Cartesian) coordinates to polar coordinates is a fundamental skill in mathematics, especially in the study of conic sections, calculus, and analytical geometry. The equation $X^2 + Y^2 = 2y$ is a classic circle in rectangular form, and converting it into its polar form reveals its geometric nature more intuitively. In this guide, we will explore the step-by-step process of converting this equation into polar form, specifically aiming to derive the forms:

- A) $r = 4 \sin \theta$
- B) $r = 2 \cos \theta$

This process involves understanding the relationships between Cartesian and polar coordinates, applying substitutions, and simplifying equations to reach the desired forms. Whether you're a student preparing for exams, a teacher designing lessons, or an enthusiast exploring coordinate systems, mastering this conversion will enhance your grasp of the geometry involved.

Understanding Cartesian and Polar Coordinates

Before diving into the conversion process, it's essential to understand the fundamental differences and relationships between Cartesian and polar coordinate systems.

Cartesian Coordinates (x, y)

- Represent points in a plane using horizontal (x) and vertical (y) distances from the origin.
- The standard form of equations (like circles, lines, parabolas) is expressed in x and y.

Polar Coordinates (r, θ)

- Represent points using a distance r from the origin and an angle θ measured from the positive x-axis.

- The relationships between Cartesian and polar coordinates are:

- $x = r \cos \theta$
- $y = r \sin \theta$
- $r^2 = x^2 + y^2$

Understanding these relationships is crucial because they form the basis of converting any Cartesian equation into its polar form.

Given Equation: $X^2 + Y^2 = 2y$ in Cartesian Coordinates

This is a standard circle equation in Cartesian form. To better understand its geometric nature, let's analyze it:

- The equation resembles the general form of a circle:

$$x^2 + y^2 + Dx + Ey + F = 0$$

- Rewriting it:

$$x^2 + y^2 - 2y = 0$$

- Completing the square for the y-term:

$$x^2 + (y^2 - 2y + 1) = 1$$

$$x^2 + (y - 1)^2 = 1$$

This indicates a circle with center at $(0, 1)$ and radius 1.

Step-by-Step Conversion Process

The main goal is to replace x and y with their polar equivalents and manipulate the equation into the desired forms:

- $r = 4 \sin \theta$
- $r = 2 \cos \theta$

Let's proceed systematically.

Step 1: Express x and y in terms of r and θ

Recall:

$$
x = r \cos \theta
$$
$$
y = r \sin \theta
$$

Substitute these into the original equation:

$$
x^2 + y^2 = 2 y
$$
$$
(r \cos \theta)^2 + (r \sin \theta)^2 = 2 r \sin \theta
$$

Step 2: Simplify the equation

Using the identity:

$$
r^2 (\cos^2 \theta + \sin^2 \theta) = 2 r \sin \theta
$$
$$
r^2 (1) = 2 r \sin \theta
$$

Resulting in:

$$
r^2 = 2 r \sin \theta
$$

Step 3: Solve for r

Divide both sides by r, noting that $r \neq 0$ unless the point is at the origin:

$$
r = 2 \sin \theta
$$

Note: The division by r is valid for all points except at $r = 0$, which corresponds to the origin.

Now, observe that the equation simplifies to:

$$
r = 2 \sin \theta
$$

which is Option A.

Deriving the Polar Forms: $r = 4 \sin \theta$ and $r = 2 \cos \theta$

The previous derivation shows that the original circle can be expressed in the form:

$$
r = 2 \sin \theta
$$

which matches option B. However, the problem asks to relate the original equation to two specific forms:

- $r = 4 \sin \theta$
- $r = 2 \cos \theta$

Let's analyze how these forms relate to the original circle and how to derive them.

1. Deriving $r = 4 \sin \theta$

Suppose we want to adjust the original equation to match:

$$
r = 4 \sin \theta
$$

This represents a circle with a different radius or position.

- Recall that in polar coordinates:

$$
r = a \sin \theta
$$

represents a circle with diameter a, centered at $(0, a/2)$. Specifically:

- $r = 4 \sin \theta$ corresponds to a circle centered at $(0, 2)$ with radius 2.

Derivation:

Starting from the general form:

$$
r = 4 \sin \theta
$$

- The circle's center is at $(0, 2)$, radius 2.
- Its Cartesian form:

$$
(x)^2 + (y - 2)^2 = 4
$$

which expands to:

$$
x^2 + y^2 - 4 y + 4 = 4
$$
$$
x^2 + y^2 - 4 y = 0
$$

Compare this with the original equation $x^2 + y^2 = 2 y$. Rewriting:

$$
x^2 + y^2 - 2 y = 0

\]

which is a circle centered at $(0, 1)$ with radius 1, as previously identified.

Thus, $r = 4 \sin \theta$ describes a larger circle, displaced upwards.

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2. Deriving $r = 2 \cos \theta$

Similarly, for:

$$
r = 2 \cos \theta
$$

- It represents a circle centered at $(a/2, 0)$, with radius $a/2$.

- Cartesian form:

$$
(x - a/2)^2 + y^2 = (a/2)^2
$$

- For $a = 2$:

$$
(x - 1)^2 + y^2 = 1
$$

which is a circle centered at $(1, 0)$ with radius 1.

Summary and Geometric Interpretations

Understanding the geometric implications helps in visualizing these equations:

- Equation $r = 2 \sin \theta$: circle centered at $(0, 1)$ with radius 1.
- Equation $r = 4 \sin \theta$: larger circle centered at $(0, 2)$ with radius 2.
- Equation $r = 2 \cos \theta$: circle centered at $(1, 0)$ with radius 1.

These forms are useful because they make it easy to identify the position and size of the circles directly from the polar equations.

Conclusion and Practical Applications

Converting the given Cartesian circle $x^2 + y^2 = 2 y$ into polar form reveals various equivalent representations, each highlighting different geometric features.

- The core conversion process involves substituting $x = r \cos \theta$ and $y = r \sin \theta$, simplifying, and solving for r.
- Recognizing the forms $r = a \sin \theta$ and $r = a \cos \theta$ helps quickly identify the circle's center and radius.
- These conversions are essential in fields like physics, engineering, and computer graphics, where polar coordinates simplify calculations involving rotations, wave functions, and circular motion.

Mastering these conversions enhances spatial reasoning and mathematical fluency, making it easier to analyze and interpret complex geometric shapes in different coordinate systems.

Additional Tips for Converting Equations to Polar Form

- Always recall the fundamental relationships between Cartesian and polar coordinates.
- Complete the square when needed to identify the geometric nature of the Cartesian equation.
-

Frequently Asked Questions

How do you convert the rectangular equation $x^2 + y^2 = 2y$ into polar form?

To convert $x^2 + y^2 = 2y$ into polar form, express x and y in terms of r and θ: $x = r \cos \theta$, $y = r \sin \theta$. Substitute into the equation: $(r \cos \theta)^2 + (r \sin \theta)^2 = 2(r \sin \theta)$. Simplify: $r^2 (\cos^2 \theta + \sin^2 \theta) = 2r \sin \theta$. Since $\cos^2 \theta + \sin^2 \theta = 1$, the equation becomes $r^2 = 2r \sin \theta$. Divide both sides by r (assuming $r \neq 0$): $r = 2 \sin \theta$. So, the polar form is $r = 2 \sin \theta$.

What is the correct polar form of the equation x^2 + y^2 = 2y?

The correct polar form is $r = 2 \sin \theta$.

Is the polar form r = 4 sin θ correct for the given equation?

No, the correct polar form is $r = 2 \sin \theta$, not $r = 4 \sin \theta$.

Which of the options A) r = 4 sin θ or B) r = 2 cos θ correctly represents the equation x^2 + y^2 = 2y?

Option A) $r = 4 \sin \theta$ is incorrect; the correct representation is $r = 2 \sin \theta$, which is not listed among the options.

Why is r = 2 sin θ the appropriate polar form for the equation?

Because substituting $x = r \cos \theta$ and $y = r \sin \theta$ into the original equation simplifies to $r = 2 \sin \theta$, accurately representing the circle in polar coordinates.

Can the given options be used to represent the original equation?

Only option B) $r = 2 \cos \theta$ corresponds to a different circle; the correct form for $x^2 + y^2 = 2y$ is $r = 2 \sin \theta$, which is not listed among the provided options.

What is the geometric significance of the polar equation r = 2 sin θ?

It represents a circle with a diameter of 2 units centered at (0, 1) in Cartesian coordinates.

Additional Resources

Conversion of the Rectangular Equation \(x^2 + y^2 = 2y \) to Polar Form: An In-Depth Analysis

Understanding the conversion of equations from Cartesian (rectangular) coordinates to polar coordinates is a fundamental skill in mathematics, particularly in the study of curves, calculus, and analytical geometry. This article offers a comprehensive exploration of transforming the rectangular equation $x^2 + y^2 = 2y$ into its polar equivalents, specifically focusing on the forms:

- $R = 4 \sin \theta$
- $R = 2 \cos \theta$

We will delve into each step with clarity, detail, and expert insight, ensuring that readers not only grasp the process but also appreciate the underlying concepts that govern these transformations.

Understanding the Foundations: Cartesian to Polar Coordinates

Before diving into the specific transformation of the given equation, it's essential to establish a solid understanding of the coordinate systems involved.

Rectangular Coordinates (Cartesian System)

- Defined by the pair (x, y).
- Equations are written in terms of x and y.
- Suitable for many geometric problems but sometimes less convenient for curves with symmetry around the origin or involving angular components.

Polar Coordinates

- Defined by the pair (r, θ):
- r: the distance from the origin to the point.
- θ: the angle between the positive x-axis and the line connecting the origin to the point.
- Transformation relations:

$$
x = r \cos \theta,\quad y = r \sin \theta
$$

The goal is to express the given rectangular equation solely in terms of r and θ. This often simplifies the understanding of the curve’s symmetry and shape.

Analyzing the Given Equation: $x^2 + y^2 = 2y$

This equation represents a geometric shape in the xy-plane. Let's analyze it step-by-step.

Step 1: Recognize the Equation's Type

- The general form resembles a circle:
$$
x^2 + y^2 = 2y
$$
- To better understand its geometric nature, complete the square for the y terms.

Step 2: Complete the Square

- Rewrite the equation:
$$
x^2 + y^2 - 2y = 0
$$
- Complete the square for y:
$$
y^2 - 2y = (y - 1)^2 - 1
$$
- Substitute back:
$$
x^2 + (y - 1)^2 - 1 = 0
$$
- Rearrange:
$$
x^2 + (y - 1)^2 = 1
$$

This is the equation of a circle with:

- Center at $(0, 1)$
- Radius 1

Key insight: The original rectangular equation describes a circle centered above the origin at $(0, 1)$ with radius 1.

Converting $x^2 + y^2 = 2 y$ to Polar Form

Now, with the geometric understanding, we proceed to express the circle in polar coordinates.

Step 1: Recall the fundamental relations

$$
x = r \cos \theta,\quad y = r \sin \theta
$$

Step 2: Substitute into the original equation

$$
x^2 + y^2 = 2 y
$$
becomes
$$
(r \cos \theta)^2 + (r \sin \theta)^2 = 2 r \sin \theta
$$

Step 3: Simplify using identities

- Left side:
$$
r^2 \cos^2 \theta + r^2 \sin^2 \theta = r^2 (\cos^2 \theta + \sin^2 \theta) = r^2
$$
- Right side:
$$
2 r \sin \theta
$$

Thus, the equation simplifies to:
$$
r^2 = 2 r \sin \theta
$$

Step 4: Solve for r

- For $r \neq 0$, divide both sides by r:
$$
r = 2 \sin \theta
$$

- Note that if $r = 0$, the point is at the origin, which also satisfies the original equation.

Result: The polar form of the circle is:

$$
\boxed{r = 2 \sin \theta}
$$

This is the standard form of a circle in polar coordinates, representing a circle with a specific position and size.

Interpreting the Polar Equation $r = 2 \sin \theta$

This form is significant for several reasons:

- It clearly indicates the locus of points forming a circle.
- The maximum value of r occurs when $\sin \theta = 1$, i.e., at $\theta = 90^\circ$ or $\pi/2$ radians:

$$
r_{\max} = 2
$$

- The circle is symmetric about the vertical axis because the sine function is symmetric about $\theta = 90^\circ$.

Geometric interpretation:
- The circle has a radius of 1 (since the maximum r is 2, which is twice the radius), and its center is located at $(0, 1)$ in Cartesian coordinates, confirming earlier analysis.

Transforming to the Given Options: Which Form is Correct?

The problem asks to relate the original equation to two specific forms:

- A) $R = 4 \sin \theta$
- B) $R = 2 \cos \theta$

Let's analyze each.

Option A: $R = 4 \sin \theta$

- This would describe a circle with a maximum radius of 4 when $\sin \theta = 1$.
- The corresponding circle's radius and position would be different from our derived circle.
- Since our original circle's polar form is $r = 2 \sin \theta$, this is not the correct transformation for our specific equation.

Option B: $R = 2 \cos \theta$

- This form describes a circle as well, but shifted along the x-axis.
- The maximum occurs at $\theta = 0^\circ$ (or 0 radians), where:
$$
r = 2 \cos 0 = 2
$$
- The circle represented by $r = 2 \cos \theta$ has:
- Center at $(1, 0)$
- Radius 1

Comparison with our original circle:
- The original circle's center is at $(0, 1)$, not $(1, 0)$.
- Therefore, $r = 2 \cos \theta$ describes a different circle, not the same as the original.

Conclusion: Identifying the Correct Polar Form

Given the analysis above, the original rectangular equation:

$$
x^2 + y^2 = 2 y
$$

transforms into:

$$
\boxed{r = 2 \sin \theta}
$$

which is not explicitly listed among the options provided. However, understanding the forms:

- $r = 2 \sin \theta$: Circle centered at $(0, 1)$,
- $r = 2 \cos \theta$: Circle centered at $(1, 0)$,
- $r = 4 \sin \theta$: Larger circle, centered at $(0, 2)$,
- $r = 4 \cos \theta$: Larger circle, centered at $(2, 0)$,

we see that the form \(r = 2 \sin \theta \) precisely matches the original equation.

Therefore, the correct polar form corresponding to the original equation is:

\[
\boxed{\textbf{None of the options exactly match } r = 2 \sin \theta, \text{ but the closest is } R= 2 \sin \theta.}
\]

If the question explicitly asks to select from options A or B, the correct answer based on the given options is:

A) \(R = 4 \sin \theta \) is not correct, and B) \(R= 2 \cos \theta \) is also not correct for the original circle.

Final Summary and Key Takeaways

- The original equation \(x^2 + y^2 = 2 y \) represents a circle centered at \((0, 1)\) with radius 1.
- Converting to polar coordinates involves substituting \(x = r \cos \theta\) and \(y = r \sin \theta\).
- The transformation yields \(r = 2 \sin \theta \).
- The options provided correspond to different circles:
- \(

Convert The Rectangular Equation X 2 Y 2 2y To Polar Form A R 4 Sin Theta B R 2 Cos Theta C

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