Cube B Is The Image Of Cube A After Dilation By A Scale Factor Of 4. If The Volume Of Cube B Is 7872

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Understanding the relationship between a cube and its dilated image involves exploring concepts of geometric transformations, specifically dilation, and how these transformations affect a solid's dimensions and volume. In this article, we will delve into the principles of dilation, the mathematical relationships involved, and how to determine the original cube's dimensions given the volume of the dilated cube. We will also examine practical applications of these concepts in real-world scenarios.

Introduction to Dilation and Its Effect on Cubes

What Is Dilation?

Dilation is a transformation that produces an image that is a scaled version of the original figure. It involves resizing a figure proportionally from a fixed point called the center of dilation, using a specific scale factor.

Key points about dilation:
- The shape of the figure remains the same.
- Distances from the center of dilation are multiplied by the scale factor.
- The transformation is characterized by a scale factor, which determines how much larger or smaller the image will be.

Dilation Applied to Cubes

When a cube undergoes dilation:
- All edges are scaled uniformly.
- The volume and surface area change according to the scale factor.
- The center of dilation can be any point in the plane or space, but in many problems, it is assumed to be the cube's center for simplicity.

Mathematical Relationships in Dilation of Cubes

Scale Factor and Edge Lengths

Let:
- a be the edge length of the original cube (Cube A).
- a' be the edge length of the dilated cube (Cube B).
- k be the scale factor of dilation.

Because dilation is uniform:
$$
a' = k \times a
$$

Given:
$$
k = 4
$$

Thus:
$$
a' = 4a
$$

Volume and Surface Area Scaling

The volume of a cube is given by:
$$
V = a^3
$$

After dilation:
$$
V' = (a')^3 = (k \times a)^3 = k^3 \times a^3
$$

Similarly, the surface area S of a cube is:
$$
S = 6a^2
$$

And after dilation:
$$
S' = 6(a')^2 = 6(k \times a)^2 = 6k^2 \times a^2
$$

Key Point:
- The volume scales by k^3.
- The surface area scales by k^2.

Calculating the Original Cube's Dimensions

Given Data

- Volume of Cube B, $V' = 7872$
- Scale factor, $k = 4$

Finding the Volume of Cube A

Since:

$$
V' = k^3 \times V
$$

then:

$$
V = \frac{V'}{k^3}
$$

Calculating:

$$
V = \frac{7872}{4^3} = \frac{7872}{64} = 123
$$

So, the volume of Cube A:

$$
V = 123
$$

Determining the Edge Length of Cube A

Recall:

$$
V = a^3
$$

Therefore:

$$
a = \sqrt[3]{V} = \sqrt[3]{123}
$$

Using a calculator:

$$
a \approx \sqrt[3]{123} \approx 4.97
$$

Thus, the original cube (Cube A) has an edge length approximately 4.97 units.

Edge Length of Cube B

Since:

$$
a' = k \times a = 4 \times 4.97 \approx 19.88
$$

The dilated cube (Cube B) has an edge length approximately 19.88 units.

Implications and Applications of Dilation in Geometry

Understanding Scale Factors in Real-World Contexts

The principles of dilation are fundamental in various fields such as architecture, engineering, and computer graphics. For instance:
- Scaling models in architecture.
- Enlarging or reducing images in graphic design.
- Analyzing stress and strain in materials by scaling prototypes.

Volume and Surface Area Changes

The cubic and quadratic relationships highlight how small changes in scale factor dramatically affect volumes and surface areas:
- Doubling the size of a cube increases volume by a factor of 8.
- Tripling the size increases volume by a factor of 27.

This understanding aids in designing structures, packaging, and other engineering applications where volume and surface area are critical.

Practical Example: Building a Scale Model

Suppose an engineer wants to create a scale model of a cubic container:
- The actual container has a volume of 7872 units, as in Cube B.
- The engineer chooses a scale factor of 4 to make a manageable model.

Using the calculations above:
- The original container (Cube A) has a volume of approximately 123 units.
- The model's edge length would be approximately 4.97 units.

This example demonstrates how understanding the relationships between scale factors and volume can facilitate accurate and proportionate model creation.

Summary and Key Takeaways

- Dilation is a transformation that scales all dimensions of a figure by a common factor.
- For cubes, the volume scales by the cube of the scale factor, and the surface area scales by the square of the scale factor.
- Given the volume of the dilated cube and the scale factor, the original cube's dimensions can be calculated directly.
- In this problem, the original cube (Cube A) has an approximate edge length of 4.97 units, and the dilated cube (Cube B) has an edge length of about 19.88 units.
- These principles are widely applicable in practical situations involving scaling, modeling, and design.

Conclusion

Understanding the relationship between a cube and its dilated image provides vital insights into how geometric transformations influence dimensions and volume. The mathematical framework involving scale factors allows for precise calculations and predictions, essential for both theoretical studies and practical applications. Whether designing scaled models or analyzing physical structures, mastery of these concepts is fundamental to spatial reasoning and geometric problem-solving.

Frequently Asked Questions

What is the scale factor used in the dilation from Cube A to Cube B?

The scale factor used is 4.

How is the volume of the dilated cube (Cube B) related to the original cube (Cube A)?

The volume of the dilated cube is scaled by the cube of the scale factor, so Volume of B = (Scale factor)^3 × Volume of A.

Given the volume of Cube B is 7872, how can we find the volume of Cube A?

Divide the volume of Cube B by the cube of the scale factor: Volume of A = 7872 / (4^3) = 7872 / 64 = 123.

What is the volume of Cube A based on the given information?

The volume of Cube A is 123 cubic units.

How do you find the side length of Cube A if its volume is 123?

Calculate the cube root of 123: side length = $\sqrt[3]{123} \approx 4.97$ units.

What is the side length of Cube B after dilation?

Since the side length scales by a factor of 4, side length of B = 4 × side length of A ≈ 4 × 4.97 ≈ 19.88 units.

Why does the volume of Cube B increase by a factor of 64 compared to Cube A?

Because volume scales with the cube of the scale factor, and 4^3 = 64, so the volume increases by a factor of 64.

If the volume of Cube B is 7872, what is the scale factor used in the dilation?

The scale factor is 4, as given in the problem, which aligns with the volume scaling by 64 times.

Additional Resources

Cube B is the image of Cube A after dilation by a scale factor of 4. If the volume of Cube B is 7,872, then understanding the geometric transformations involved, especially dilation and volume scaling, becomes essential for grasping the relationship between the two cubes. This analysis offers a comprehensive breakdown of this transformation, shedding light on how scale factors influence volume and providing a step-by-step approach to solving related problems.

Understanding the Basics of Dilation and Scale Factors

Dilation is a type of geometric transformation that enlarges or reduces a figure proportionally from a fixed point, called the center of dilation. When applied to three-dimensional figures like cubes, dilation scales all dimensions uniformly.

What is a Scale Factor?

The scale factor determines how much a figure is enlarged or reduced during dilation:

- Scale factor > 1: The figure is enlarged.
- Scale factor < 1: The figure is reduced.
- Scale factor = 1: The figure remains unchanged.

In our specific problem, the scale factor is 4, meaning each linear dimension of Cube A is scaled by a factor of 4 to produce Cube B.

Effect of Dilation on Volumes

While linear dimensions are scaled by the scale factor, the volume scales by the cube of the scale factor because volume depends on three dimensions:

- Volume of a scaled figure = $(\text{Scale factor})^3 \times$ Original volume

This principle is essential for understanding how the volume of Cube B relates to Cube A.

Step-by-Step Breakdown of the Problem

Given:
- Cube B is the image of Cube A after dilation by a scale factor of 4.
- The volume of Cube B is 7,872.

Our goal:
- To find the volume of Cube A.
- To understand the relationship between the two cubes.

Step 1: Recall the Volume Scaling Law

Since the dilation involves a scale factor of 4, the volume of Cube B relates to the volume of Cube A as follows:

$$V_B = (k)^3 \times V_A$$

Where:
- V_B is the volume of Cube B.
- V_A is the volume of Cube A.
- k is the scale factor (here, 4).

Step 2: Set Up the Equation

Plugging in the known values:

$$7,872 = (4)^3 \times V_A$$

Calculating 4^3:

$$4^3 = 4 \times 4 \times 4 = 64$$

So,

$$7,872 = 64 \times V_A$$

Step 3: Solve for V_A

Divide both sides by 64:

$$ V_A = \frac{7,872}{64} $$

Calculating:

$$ V_A = 123 $$

Thus, the volume of Cube A is 123 cubic units.

Interpreting the Results: From Volume to Dimensions

Knowing the volume of Cube A allows us to determine its side length, which is helpful for visualizing the transformation.

Step 4: Find the Side Length of Cube A

The volume of a cube is given by:

$$ V = s^3 $$

Where:
- s is the side length of the cube.

Rearranged:

$$ s = \sqrt[3]{V} $$

Applying this to Cube A:

$$ s_A = \sqrt[3]{123} $$

Calculating:

$$ s_A \approx \sqrt[3]{123} \approx 4.97 $$

So, the side length of Cube A is approximately 4.97 units.

Step 5: Find the Side Length of Cube B

Since the dilation scales all dimensions by 4:

$$ s_B = 4 \times s_A \approx 4 \times 4.97 \approx 19.88 $$

And, to verify the volume of Cube B:

$$ V_B = s_B^3 \approx 19.88^3 \approx 7,872 $$

which aligns with the given data, reaffirming our calculations.

Visualizing the Transformation

Understanding the geometric intuition behind the dilation helps appreciate the scaling process:

- Original Cube (Cube A): Side length ≈ 4.97 units, volume = 123.
- Dilated Cube (Cube B): Side length ≈ 19.88 units, volume = 7,872.

This significant increase in volume illustrates how cubic scaling impacts size:

Parameter	Cube A	Cube B
Side length	≈ 4.97 units	≈ 19.88 units
Volume	123 cubic units	7,872 cubic units

The linear dimensions multiply by 4, but the volume increases by $4^3 = 64$ times.

Practical Applications and Related Problems

Understanding the principles behind dilation and volume scaling has practical applications across various fields:

- Architecture: Scaling models of buildings.
- Manufacturing: Adjusting component sizes proportionally.
- Physics: Understanding scale models of objects for experiments.
- Mathematics Education: Teaching concepts of proportionality and geometric transformations.

Example Problems for Practice

1. If Cube B has a volume of 7,872 and is obtained from Cube A by dilation with a scale factor of 4, what is the volume of Cube A? (Answer: 123 cubic units)

2. If Cube A has a side length of approximately 4.97 units, what is the side length of Cube B after dilation? (Answer: approximately 19.88 units)

3. How would the volume change if the scale factor were doubled to 8? (Answer: Volume increases by $8^3 = 512$ times)

4. Given the volume of Cube A, calculate the volume of a cube obtained by dilation with a scale factor of 3.

Summary and Key Takeaways

- Dilation scales all linear dimensions of a figure proportionally from a fixed point.

- The volume of a figure after dilation scales by the cube of the scale factor.
- In this problem, a scale factor of 4 results in the volume of Cube B being 64 times that of Cube A.
- Calculating the original volume involves dividing the dilated volume by k^3.

Key formula:

$$V_B = (k)^3 \times V_A$$

Rearranged:

$$V_A = \frac{V_B}{k^3}$$

Applying this formula allows for quick determination of original volumes and dimensions when given scaled figures.

Final Thoughts

Understanding how scale factors influence volume is fundamental in geometry and real-world applications. This problem exemplifies the direct relationship between linear scaling and volumetric change, emphasizing the importance of the cube law in three-dimensional transformations. Whether designing models, solving geometry problems, or analyzing physical systems, mastering these concepts provides a strong foundation for spatial reasoning and mathematical proficiency.

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