D) Using Rankine. Tresca And Von-Mises Failure Criteria Determine The Safety Factor Against Yielding

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Yielding is a critical concern in the design and analysis of mechanical components and structures. Ensuring that a component can withstand operational stresses without yielding is essential for safety, reliability, and longevity. To evaluate the likelihood of yielding, engineers utilize various failure criteria that relate the applied stresses to the material's yield strength. Among the most prominent criteria are the Rankine, Tresca, and Von-Mises failure theories. These criteria help determine the safety factor against yielding, guiding engineers in designing safer and more efficient structures.

This article provides a comprehensive overview of these three failure criteria, explaining their principles, applications, and how they are used to calculate the safety factor against yielding.

Understanding the Failure Criteria

Failure criteria are mathematical models that predict the onset of plastic deformation (yielding) in materials subjected to complex stress states. They are essential tools in the field of solid mechanics and structural analysis, allowing engineers to interpret the stress state within a component and predict whether it will yield.

The three primary criteria discussed here are:

- Rankine’s (Maximum Principal Stress) Criterion
- Tresca (Maximum Shear Stress) Criterion
- Von-Mises (Distortion Energy) Criterion

Each of these theories offers a different perspective on failure, based on different assumptions about stress distribution and material behavior.

Rankine Failure Criterion

Principle of Rankine’s Criterion

Rankine’s criterion states that failure occurs when the maximum principal stress in the material reaches the material’s yield strength in uniaxial tension or compression. It is based on the idea that the most critical stress component causes the onset of yielding.

Mathematically:

$$
\sigma_{max} \geq \sigma_{yield}
$$

Where:

- σ_{max} is the maximum principal stress at a point.
- σ_{yield} is the yield strength of the material in uniaxial tension.

Application:

- Best suited for brittle materials or situations where failure is governed by the maximum tensile stress.
- Conservative, as it does not consider the effects of shear stresses or the combined effect of multiple stress components.

Calculating Safety Factor Using Rankine Criterion

The safety factor N based on Rankine's criterion is calculated as:

$$
N_{Rankine} = \frac{\sigma_{yield}}{\sigma_{max}}
$$

Where:

- σ_{max} is the maximum principal stress experienced by the component.

Interpretation:

- If $N_{Rankine} > 1$, the component is safe against yielding.
- If $N_{Rankine} \leq 1$, the component is at risk of failure.

Tresca Failure Criterion

Principle of Tresca's Criterion

The Tresca criterion states that yielding begins when the maximum shear stress in the material reaches a critical value, which is related to the uniaxial yield strength.

Mathematically:

$$
\tau_{max} = \frac{\sigma_{yield}}{2}
$$

Where:

- τ_{max} is the maximum shear stress within the material.

In terms of principal stresses, the Tresca criterion is expressed as:

$$
\max \left| \sigma_i - \sigma_j \right| = \sigma_{yield}
$$

for $i,j = 1,2,3$ (the principal stresses).

Application:

- Suitable for ductile materials.
- Often considered more conservative than Von-Mises but less so than Rankine.

Calculating Safety Factor Using Tresca Criterion

The safety factor N based on Tresca's criterion is:

$$
N_{Tresca} = \frac{\sigma_{yield}}{\max |\sigma_i - \sigma_j|/2}
$$

or equivalently,

$$
N_{Tresca} = \frac{\sigma_{yield}}{\tau_{max}}
$$

Interpretation:

- When the maximum shear stress reaches $\sigma_{yield}/2$, the material yields.
- A safety factor greater than 1 indicates safety.

Von-Mises Failure Criterion

Principle of Von-Mises Criterion

The Von-Mises criterion is based on the concept of distortional energy in the material. It states that yielding occurs when the distortion energy per unit volume reaches a critical value corresponding to the yield stress in uniaxial tension.

Mathematically:

$$
\sigma_{v} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}{2}}
$$

Yielding occurs when:

$$
\sigma_{v} = \sigma_{yield}
$$

Where:

- $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses.

The Von-Mises stress is often used because it correlates well with experimental observations for ductile materials.

Application:

- Widely used in finite element analysis.
- Suitable for ductile materials subjected to complex stress states.

Calculating Safety Factor Using Von-Mises Criterion

The safety factor N is:

$$
N_{Von-Mises} = \frac{\sigma_{yield}}{\sigma_{v}}
$$

Where:

σ_{v} is the Von-Mises equivalent stress calculated from the principal stresses.

Interpretation:

- When σ_{v} equals σ_{yield}, the material yields.
- A safety factor greater than 1 indicates the component is safe under the given stress state.

Comparison of the Failure Criteria

Criterion	Focus	Suitable Materials	Conservativeness	Application Examples
Rankine	Maximum principal stress	Brittle materials, concrete	Most conservative	Structural concrete, brittle ceramics
Tresca	Maximum shear stress	Ductile metals	Moderate	Steel shafts, pressure vessels
Von-Mises	Distortional energy	Ductile metals	Less conservative than Tresca	Automotive components, aircraft structures

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Step-by-Step Approach to Determine Safety Factors

1. Obtain the stress components: Identify all principal stresses ($\sigma_1, \sigma_2, \sigma_3$) acting on the component through analysis or experimentation.

2. Select the appropriate failure criterion: Based on material properties and application.

3. Calculate the equivalent stress or critical stress:
- For Rankine: σ_{max}
- For Tresca: $\max |\sigma_i - \sigma_j|$
- For Von-Mises: σ_{v}

4. Calculate the safety factor:
- $N = \frac{\text{Yield strength}}{\text{Equivalent stress}}$

5. Interpret the safety factor:
- $N > 1$: Safe
- $N \leq 1$: Failure predicted; redesign needed.

Practical Example

Suppose a steel component is subjected to principal stresses:

- $\sigma_1 = 150\, \text{MPa}$
- $\sigma_2 = 50\, \text{MPa}$
- $\sigma_3 = 0\, \text{MPa}$

The yield strength of the steel is 250 MPa.

Calculate safety factors:

- Rankine:

$$
\sigma_{max} = 150\, \text{MPa}
$$

$$
N_{Rankine} = \frac{250}{150} \approx 1.67
$$

- Tresca:

$$
\max |\sigma_i - \sigma_j| = \max |150 - 50|, |50 - 0|, |0 - 150| = 100, 50, 150\, \text{MPa}
$$

$$
\text{Maximum difference} = 150\, \text{MPa}
$$

$$
N_{Tresca} = \frac{250}{150/2} = \frac{250}{75} \approx 3.33
$$

- Von-Mises:

$$
\sigma_{v} = \sqrt{\frac{(150 - 50)^2 + (50 - 0)^2 + (0 - 150)^2}{2}} = \sqrt{\frac{100^2 + 50^2 + 150^2}{2}}
$$

$$
= \sqrt{\frac{10,000 + 2,500 + 22,500}{2}} = \sqrt{\frac{35,000}{2}} = \sqrt{17,500} \approx 132.29\, \text{MPa}
$$

$$
N_{Von-Mises} = \frac{250}{132.29} \approx 1.89
$$

Result:

- All three criteria suggest the component is safe, with safety factors well above 1.

Conclusion

Choosing the appropriate failure criterion depends on the material,

Frequently Asked Questions

What is the purpose of using Rankine, Tresca, and Von-Mises failure criteria in determining the safety factor against yielding?

These criteria are used to evaluate the likelihood of material yielding under complex stress states by comparing the applied stresses with the material's yield strength, thus helping to determine the safety factor against yielding.

How do the Rankine, Tresca, and Von-Mises criteria differ in predicting yielding?

Rankine's criterion considers the maximum principal stress, Tresca's criterion uses the maximum shear stress, and Von-Mises' criterion evaluates the distortional energy; each provides a different approach to predicting yielding based on stress state.

Which failure criterion is most conservative for ductile materials: Tresca or Von-Mises?

Tresca's criterion is generally more conservative for ductile materials as it predicts yielding at lower stress levels compared to Von-Mises' criterion.

How is the safety factor calculated using the Von-Mises criterion?

The safety factor is calculated by dividing the material's yield strength by the Von-Mises equivalent stress obtained from the applied stresses: Safety Factor = Yield Strength / Von-Mises stress.

Can the Tresca criterion be used for brittle materials? Why or why not?

No, Tresca is primarily used for ductile materials because it focuses on shear stresses related to ductile yielding; brittle materials tend to fail due to different mechanisms not accurately captured by Tresca.

What is the significance of the principal stresses in applying these failure criteria?

Principal stresses simplify complex stress states by representing them as normal stresses on mutually perpendicular planes, making it easier to apply failure criteria like Rankine, Tresca, and Von-Mises.

How do you determine the equivalent stress for Tresca and Von-Mises criteria from principal stresses?

For Tresca, the maximum shear stress is half the difference between the maximum and minimum principal stresses. For Von-Mises, the equivalent stress is calculated using all principal stresses through a specific formula involving their squares and sums.

Why is it important to compare the calculated safety factor against the yield strength?

Comparing the safety factor against the yield strength helps assess whether the material will safely withstand the applied loads without yielding, ensuring structural integrity.

What assumptions are typically made when applying these failure criteria in safety factor calculations?

Assumptions include isotropic and homogeneous material behavior, linear elastic response until yield, and that the stress state is accurately known and steady under loading conditions.

How does the choice of failure criterion affect the design safety factor in engineering applications?

Different criteria may predict yielding at different stress levels; choosing a more conservative criterion (like Tresca) results in a higher safety factor, potentially leading to safer but more conservative designs.

Additional Resources

Using Rankine, Tresca, and Von-Mises Failure Criteria to Determine the Safety Factor Against Yielding

Understanding the safety and reliability of structural components is a fundamental aspect of mechanical and civil engineering. When designing components subjected to complex stress states, engineers rely on failure theories or criteria to predict the onset of yielding or failure. Among the most widely used are the Rankine, Tresca, and Von-Mises failure criteria. Each provides a different perspective on when a material will yield under combined stresses, enabling engineers to determine the safety factor against yielding with greater confidence.

This comprehensive review delves into these three failure theories, elucidates their principles, mathematical formulations, applications, and how they are employed to determine the safety factor. Understanding these theories not only enhances safety margins but also optimizes material utilization, leading to cost-effective and reliable designs.

Understanding the Need for Failure Criteria

Before exploring specific failure theories, it's essential to grasp why failure criteria are necessary:

- Complex Stress States: Real-world components often experience multi-axial stresses rather than simple uniaxial tension or compression.

- Material Behavior: Materials may behave differently under complex stress combinations, and pure stress or strain theories might not suffice.

- Predictive Capability: Failure criteria enable prediction of yielding or failure based on the stress state, facilitating the design of safe structures.

- Safety Factors: Quantifying how close a component operates to its failure point allows for the calculation of safety factors, ensuring the structure's integrity under various load conditions.

Overview of the Failure Theories

The three primary failure criteria discussed are:

- Rankine Criterion (Maximum Principal Stress Theory)
- Tresca Criterion (Maximum Shear Stress Theory)
- Von-Mises Criterion (Distortion Energy Theory)

Each has its assumptions, advantages, and limitations, making them suitable for different materials and conditions.

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1. Rankine (Maximum Principal Stress) Criterion

Principle and Assumptions

- Based on the idea that failure occurs when the maximum principal stress reaches the ultimate tensile strength (UTS) or compressive strength of the material.
- It is most applicable to brittle materials like ceramics, concrete, or glass, where failure is governed predominantly by maximum normal stresses rather than shear stresses.
- Assumes failure occurs along a plane where the principal stress is maximum, ignoring shear effects in the failure process.

Mathematical Formulation

For a given state of stress with principal stresses $\sigma_1, \sigma_2, \sigma_3$:

$$
\text{Failure occurs if} \quad \max |\sigma_i| \geq \sigma_{allow}
$$

where σ_{allow} is the allowable stress, often derived from material strength divided by safety factor.

To determine the safety factor (SF) against yielding:

$$
SF_{Rankine} = \frac{\text{Material strength (e.g., } \sigma_{t} \text{)}}{\max |\sigma_i|}
$$

In the case of combined stresses:

$$
SF_{Rankine} = \frac{\sigma_{allow}}{\max |\sigma_1|, |\sigma_2|, |\sigma_3|}
$$

Note: For ductile materials, Rankine criterion tends to be conservative, as it does not consider shear stresses or distortional energy.

Application and Limitations

- Best suited for brittle materials where failure is dominated by maximum normal stress.

- Tends to be conservative for ductile materials.

- Does not account for shear stresses or the combined effect of multiple stress components.

- Typically results in lower safety factors compared to other theories for ductile materials.

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2. Tresca (Maximum Shear Stress) Criterion

Principle and Assumptions

- Based on the maximum shear stress theory: failure occurs when the maximum shear stress in the material reaches the shear stress at failure in a simple tension test.
- It is particularly useful for ductile materials like steels, aluminum, and other metals.
- Assumes that the yield occurs when the maximum shear stress reaches the shear yield strength of the material.

Mathematical Formulation

The maximum shear stress (τ_{max}) in a state of stress is given by:

$$
\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2}
$$

where:

- σ_{max} and σ_{min} are the maximum and minimum principal stresses, respectively.

Failure occurs when:

$$
\tau_{max} \geq \tau_{yield}
$$

or in terms of principal stresses:

$$
\frac{\sigma_1 - \sigma_3}{2} \geq \sigma_{yield}
$$

The safety factor (SF) can be determined as:

$$
SF_{Tresca} = \frac{\sigma_{yield}}{\max \left(\frac{\sigma_1 - \sigma_3}{2} \right)}
$$

Alternatively, for a combined stress state:

$$
SF_{Tresca} = \frac{\sigma_{allow}}{\max \left(\frac{\sigma_1 - \sigma_3}{2} \right)}
$$

where σ_{allow} is derived from the material's yield strength.

Application and Limitations

- Suitable for ductile materials where shear plays a dominant role in yielding.
- Tends to be more conservative than Von-Mises for the same stress state.

- Simplifies the failure analysis by focusing on shear stress, which is often easier to interpret.

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3. Von-Mises (Distortion Energy) Criterion

Principle and Assumptions

- Based on the concept that yielding occurs when the distortional (or shear) energy per unit volume reaches a critical value.
- It is considered more accurate for ductile materials, providing a good correlation with experimental data.
- Assumes the material yields when the second invariant of the deviatoric stress tensor reaches a critical value.

Mathematical Formulation

The Von-Mises criterion states that yielding occurs when:

$$
\sigma_{vm} = \sqrt{\frac{1}{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]} \geq \sigma_{yield}
$$

where:

- σ_{vm} is the Von-Mises equivalent stress.

- σ_{yield} is the yield strength of the material in uniaxial tension.

The safety factor (SF) is then:

$$
SF_{Von-Mises} = \frac{\sigma_{yield}}{\sigma_{vm}}
$$

or, considering the applied stresses:

$$
SF_{Von-Mises} = \frac{\sigma_{allow}}{\sqrt{\frac{1}{2} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]}}
$$

Comparative Analysis of the Three Failure Criteria

Criterion	Dominant Material Type	Key Assumption	Mathematical Focus	Typical Use Cases	Conservativeness
Rankine	Brittle materials	Failure when maximum principal stress reaches strength	Maximum principal stress	Concrete, ceramics, brittle materials	Highly conservative for ductile materials
Tresca	Ductile materials	Failure when maximum shear stress reaches shear strength	Maximum shear stress	Metals, ductile alloys	Conservative but less so than Rankine
Von-Mises	Ductile materials	Yield occurs at critical distortional energy	Equivalent von-Mises stress	Metals, ductile materials	Less conservative, aligns well with experimental data

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Calculating Safety Factor Using These Criteria

The process of determining the safety factor against yielding involves:

1. Identify the Stress State:

- Obtain principal stresses ($\sigma_1, \sigma_2, \sigma_3$) through stress analysis.

2. Select Appropriate Failure Criterion:

- Choose based on material behavior (brittle vs. ductile).

3. Compute the Critical Stress or Equivalent Stress:

- For Rankine: maximum principal stress.

- For Tresca: maximum shear stress.

- For Von-Mises: equivalent stress.

4. Determine Allowable or Material Strength:

- Use yield strength (σ_y) or ultimate stress (σ_{u}) as relevant.

5. Calculate Safety Factor:

- Divide material strength by the critical stress:

$$
SF = \frac{\text{Material strength}}{\text{Critical stress}}
$$

6. Assess Design:

- Ensure $SF \geq 1$ (or a specified minimum safety factor, e.g., 1.5).

Practical Examples of Calculation

Example 1: Ductile Material Under Biaxial Tension

- Given stresses: \(\sigma_1=150\, MPa\), \(\sigma_2=50\, MPa

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