

Decide Which Values X, Y ∈ ℝ Give The Solution Of The Given Set Of Equations $\log X - \log Y = 0$ and $Y = 2^X$

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When solving systems of equations involving logarithmic expressions, it is essential to carefully analyze the equations, understand the domain restrictions, and methodically find the solutions that satisfy all conditions. In this article, we will explore the process of determining the values of (X) and (Y) in the set of equations:

$$\begin{cases} \log X - \log Y = 0 \\ Y = 2^X \end{cases}$$

with the stipulation that $(X, Y \in \mathbb{R})$. We will break down each equation, analyze their implications, and systematically arrive at the solution.

Understanding the Given Equations

Before diving into calculations, it's important to understand the meaning and properties of the equations involved.

Equation 1: $\log X - \log Y = 0$

This is a logarithmic equation involving two variables. Recall the logarithmic property:

$$\log a - \log b = \log \left(\frac{a}{b} \right)$$

Applying this, the first equation becomes:

$$\log \left(\frac{X}{Y} \right) = 0$$

The property of logarithms states that:

$$\log a = 0 \implies a = 10^0 = 1$$

Assuming the common logarithm base 10 (which is standard unless specified otherwise), the equation simplifies to:

$$\frac{X}{Y} = 1$$

which implies:

$$X = Y$$

Note: The domain restrictions for logarithmic functions require that both X and Y are positive real numbers, i.e.,

$$X > 0, \quad Y > 0$$

Equation 2: $Y - 2X = 0$

This is a linear equation relating Y and X :

$$Y = 2X$$

Solving the System of Equations

Having simplified the first equation to $X = Y$, and knowing the second is $Y = 2X$, we can find the common values of X and Y that satisfy both.

Step 1: Set the equations equal

Since from the first equation:

$$\begin{aligned} & \backslash[\\ & X = Y \\ & \backslash] \end{aligned}$$

and from the second:

$$\begin{aligned} & \backslash[\\ & Y = 2X \\ & \backslash] \end{aligned}$$

Equate these two expressions:

$$\begin{aligned} & \backslash[\\ & X = 2X \\ & \backslash] \end{aligned}$$

Subtract $\backslash(X)$ from both sides:

$$\begin{aligned} & \backslash[\\ & X - 2X = 0 \implies -X = 0 \\ & \backslash] \end{aligned}$$

which yields:

$$\begin{aligned} & \backslash[\\ & X = 0 \\ & \backslash] \end{aligned}$$

Step 2: Find $\backslash(Y)$

Recall that $\backslash(Y = 2X)$. Substituting $\backslash(X=0)$:

$$\begin{aligned} & \backslash[\\ & Y = 2 \times 0 = 0 \\ & \backslash] \end{aligned}$$

Step 3: Verify domain restrictions

Recall that for the original equations involving $\backslash(\log X)$ and $\backslash(\log Y)$, both $\backslash(X)$ and $\backslash(Y)$ must be positive real numbers.

However, the solution obtained is:

$$\begin{aligned} & \backslash[\\ & X = 0, \quad Y = 0 \\ & \backslash] \end{aligned}$$

which violates the domain restrictions because the logarithm of zero is undefined:

$\log 0$ is undefined

Therefore, the solution $(X=0, Y=0)$ is invalid in the context of the original equations.

Conclusion: No Valid Solution in Real Numbers

Since the only algebraic solution contradicts the domain restrictions, there are no real values of (X, Y) in $\mathbb{R}$ that satisfy both equations simultaneously.

Summary:

- The first equation simplifies to $(X = Y)$.
- The second equation gives $(Y = 2X)$.
- Equating these yields $(X = 2X \Rightarrow X=0)$.
- Correspondingly, $(Y=0)$.
- These values are invalid for the logarithmic functions involved.

Alternative Approach: Considering Domain Restrictions

Given the restrictions for the logarithmic functions $(X > 0, Y > 0)$, and the derived solution being invalid, we conclude:

- No solutions exist in the set of positive real numbers (X, Y) that satisfy both equations simultaneously.

Summary of Key Points for Solving Logarithmic Systems

To successfully solve equations involving logarithms:

1. Simplify using logarithmic properties:
 - $\log a - \log b = \log \frac{a}{b}$
 - $\log a + \log b = \log(ab)$
2. Identify domain restrictions:

- The arguments of logarithms must be positive.
3. Solve algebraic equations carefully considering these restrictions.
 4. Verify solutions by substituting back into the original equations and checking domain validity.

Final Notes and Tips

- Always analyze the domain restrictions first when dealing with logarithms.
- Remember that $\log 0$ is undefined; solutions leading to zero inside a logarithm are invalid.
- When solving systems, substitute and check whether solutions meet the original conditions.
- In cases where algebraic solutions violate domain conditions, conclude that no solutions exist in the real number set.

In conclusion, the set of equations:

$$\begin{cases} \log X - \log Y = 0 \\ Y - 2X = 0 \end{cases}$$

has no solutions in $\mathbb{R}$ that satisfy the domain restrictions of the logarithmic functions. This highlights the importance of considering both algebraic manipulation and domain constraints when solving logarithmic equations.

If you need further clarification or assistance with similar equations, feel free to explore more examples or ask!

Frequently Asked Questions

What does the equation $\log X - \log Y = 0$ imply about the relationship between X and Y?

The equation $\log X - \log Y = 0$ implies that $\log(X) = \log(Y)$, which means $X = Y$, given that X and Y are positive real numbers.

How do you solve the set of equations: $\log X - \log Y = 0$ and $Y - 2X = 0$?

From $\log X - \log Y = 0$, we get $X = Y$. Substituting into $Y - 2X = 0$ gives $Y - 2Y = 0$, which simplifies to $-Y = 0$, so $Y = 0$, but since logs are undefined at zero, the solution must satisfy $X > 0$ and $Y > 0$, leading to no real solution in this case.

Are there any restrictions on the values of X and Y for the given equations to hold true?

Yes, since the equations involve logarithms, both X and Y must be positive real numbers ($X > 0$ and $Y > 0$).

What is the solution set for the equations $\log X - \log Y = 0$ and $Y - 2X = 0$?

Since $\log X = \log Y$, then $X = Y$. Substituting into $Y - 2X = 0$ gives $Y - 2Y = 0$, leading to $Y = 0$, which is invalid for logs. Therefore, the only solution is when the equations are consistent with positive values, which leads to the solution $X = Y > 0$ with $Y = 2X$, but this conflicts with the earlier derived relation. Hence, the only valid solutions are those where $X = Y > 0$ and satisfy $Y = 2X$, so $X = Y$ and $Y = 2X$ implies $X = 0$, $Y = 0$, which are invalid. Therefore, the equations have no solution in the domain of positive real numbers.

How can the equations be rewritten to find the values of X and Y?

Rewrite $\log X - \log Y = 0$ as $\log(X/Y) = 0$, which implies $X/Y = 1$, or $X = Y$. Then, using $Y - 2X = 0$, substitute $Y = X$ to get $X - 2X = 0$, leading to $-X = 0$, thus $X = 0$, which is invalid for logs. Therefore, no valid solution exists in the positive real domain.

What is the key takeaway when solving equations involving logarithms and linear relations like these?

The key is to remember the domain restrictions of logarithms ($X > 0$, $Y > 0$) and to carefully substitute and analyze the resulting equations to ensure solutions are valid within those domains.

Additional Resources

Deciphering the Solution of the Set of Equations: Logarithmic Equations Involving X, Y, and R

When exploring complex algebraic and logarithmic equations, it becomes essential to approach the problem systematically. The set of equations at hand, involving logarithms, variables X, Y, and R, presents a compelling challenge that combines properties of logarithms, algebraic manipulation, and logical reasoning. This detailed review aims to dissect the problem thoroughly, understand the underlying principles, and arrive at the solution with clarity and depth.

Understanding the Given Problem: A Breakdown of the

Equations

The initial step in solving any set of equations is to comprehend the structure and nature of the problem. The given set is:

- $\log X - \log Y = 0$
- $Y - 2X = 0$

Key Points:

- The equations involve logarithmic expressions ($\log X$) and ($\log Y$)
- The second equation is linear and straightforward: $Y = 2X$
- The goal is to find the values of X , Y , and R that satisfy both equations simultaneously

Clarification of Variables:

- X and Y are variables, typically real numbers
- R appears in the problem statement but is not explicitly present in the equations provided; its role could be contextual or possibly a parameter to be determined from the solution

Note: Since the problem states "which values X , Y , R give the solution of the given set of equations," and the equations involve only X and Y , it is reasonable to infer that R might be related to the logarithmic base or an additional parameter that could influence the solution.

Analyzing the Logarithmic Equation: $\log X - \log Y = 0$

This equation can be simplified based on properties of logarithms:

Property Used:

$$\log a - \log b = \log \left(\frac{a}{b} \right)$$

Simplification:

$$\log \left(\frac{X}{Y} \right) = 0$$

Implication:

Since $\log \left(\frac{X}{Y} \right) = 0$, it follows that:

$$\frac{X}{Y} = 10^0 = 1$$

Note: The base of the logarithm is assumed to be 10 unless otherwise specified. If the base (R) is variable, then:

$$\log_R \left(\frac{X}{Y} \right) = 0 \Rightarrow \frac{X}{Y} = R^0 = 1$$

which again implies:

$$X = Y$$

Conclusion:

$$X = Y$$

Solving the Linear Equation: $(Y - 2X = 0)$

This equation simplifies directly:

$$Y = 2X$$

Since from the logarithmic equation, we have $(X = Y)$, substituting into this gives:

$$X = 2X$$

which implies:

$$X - 2X = 0 \Rightarrow -X = 0 \Rightarrow X = 0$$

Similarly, since $(Y = X)$:

$$Y = 0$$

\]

Verifying the Solution: Are $(X=0)$ and $(Y=0)$ Valid?

Here, we encounter a fundamental property of logarithms:

- Logarithm Domain Restriction: $(\log X)$ and $(\log Y)$ are defined only for positive real numbers $(X > 0), (Y > 0)$
- Problem with Zero: The solution $(X=0), (Y=0)$ violates the domain restrictions

Implication:

- The derived solution is mathematically consistent with the algebraic manipulation but invalid within the domain of the logarithm
- Therefore, no valid real solution exists where both (X) and (Y) are positive, satisfying the original equations

Re-evaluating the Problem: Are There Other Solutions?

Given the domain restrictions, the previous solution is invalid. We need to explore alternative approaches or interpret the equations differently.

Alternative interpretations:

- The use of $(\log)$ might involve a different base (R) , which could influence solutions
- The problem might be seeking solutions in complex numbers, where $(\log)$ can take on multiple values
- There may be an error or omission in the original problem statement, such as missing constraints or additional equations

Assumption:

- To proceed, assume the logarithm is base 10, and variables are positive real numbers, as is standard.

Exploring Valid Solutions within Logarithmic Domain

Constraints

Given that:

- $(\log X - \log Y = 0 \Rightarrow X = Y)$
- $(Y - 2X = 0 \Rightarrow Y = 2X)$

Substituting $(Y = X)$ into $(Y = 2X)$:

$$X = 2X \Rightarrow X = 0$$

which is invalid for logarithms. Therefore, no solutions exist where both (X) and (Y) are positive real numbers.

Conclusion:

- The equations are incompatible within the domain of positive real numbers
- The only algebraic solution suggests $(X=Y=0)$, which is invalid

Involving the Parameter R: Could It Provide a Different Solution?

The problem mentions "Values (X, Y, R) ." If (R) is the base of the logarithm, then:

$$\log_R X - \log_R Y = 0$$

which simplifies to:

$$\frac{X}{Y} = R^0 = 1 \Rightarrow X = Y$$

Similarly, the second equation is unaffected:

$$Y = 2X$$

Again, substituting $(Y = X)$:

$$X = 2X$$

$$X = 2X \rightarrow X = 0$$

\]

which still contradicts the domain restrictions.

Alternatively, if (R) is a parameter unrelated to the base of the logarithm, perhaps an additional equation or constraint involving (R) is missing.

Summary of the Analytical Process

- The logarithmic equation simplifies to $(X = Y)$
- The linear equation simplifies to $(Y = 2X)$
- Combining both yields $(X = 2X \rightarrow X=0)$
- Since $(X=0)$ violates the domain of the logarithm, no valid real solutions exist under standard assumptions
- The apparent solution set is invalid within the constraints of real positive numbers, indicating the equations are inconsistent or have no solution in the real domain

Deeper Insights and Potential Extensions

Given the incompatibility within the real domain, potential areas for further exploration include:

1. Complex Number Domain:

- The logarithm can be extended to complex numbers, where multiple branches exist
- Solutions could involve complex logarithms, leading to an infinite set of solutions
- However, this complicates the problem significantly and might be beyond the intended scope

2. Different Logarithm Bases:

- If the base (R) is variable, the relationship remains the same, as the ratio remains 1, leading to $(X=Y)$
- Without additional constraints, this does not resolve the core inconsistency

3. Additional Equations or Constraints:

- The problem might be incomplete, missing other equations involving (R)
- Additional constraints could potentially lead to viable solutions

Practical Implications and Educational Takeaways

This problem serves as an excellent teaching example of:

- The importance of considering the domain of functions involved (logarithms require positive arguments)
- The necessity of verifying solutions after algebraic manipulation
- Recognizing when a set of equations is inherently inconsistent within a given domain
- Understanding how assumptions about variables and parameters influence solutions

Conclusion: Final Assessment of the Set of Equations

In essence, the set of equations:

- Simplifies to $(X = Y)$ from the logarithmic equation
- Leads to $(X = 2X)$ from substituting into the second equation
- The only algebraic solution is $(X=0)$, which is invalid for positive real arguments in logs

Therefore:

- No valid solution exists in the realm of positive real numbers for the variables (X) and (Y)
- The problem highlights fundamental properties of logarithmic functions and their constraints
- If considering complex numbers or alternative bases, solutions might exist, but they extend beyond the typical scope of real-valued algebra

Final note: When approaching similar problems, always scrutinize the domain restrictions imposed by functions like logarithms and verify that solutions satisfy these constraints. Recognizing such limitations is crucial in mathematical problem-solving and ensures that solutions are both mathematically consistent and practically meaningful.

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