Decompose The Signal S(t)=(2+5 Sin(3t+x)) Cos(4t) Into A Linear Combination (i.c., A Sum Of Constant

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Introduction

Decomposition of signals into simpler components is a fundamental task in signal processing, harmonic analysis, and Fourier analysis. It allows us to understand the frequency content, amplitude, and phase of complex signals. In this article, we focus on the specific task of decomposing the signal:

$$S(t) = (2 + 5 \sin(3t + x)) \cos(4t)$$

into a linear combination of basic functions, primarily constants and harmonic functions such as sine and cosine. Such a decomposition is vital for analyzing, filtering, or reconstructing signals in various applications, including communications, control systems, and audio processing.

Our goal is to express $S(t)$ as a sum of terms involving constants multiplied by sine and cosine functions of different frequencies, i.e., a Fourier-like expansion:

$$S(t) = A_0 + \sum_{k} [A_k \cos(\omega_k t) + B_k \sin(\omega_k t)]$$

where A_0 is a constant (the average or DC component), and the other terms are harmonics at specified frequencies.

Understanding the Components of the Signal

Before decomposing $S(t)$, it is crucial to analyze its structure:

- The signal involves a product of a constant plus a sinusoid $5 \sin(3t + x)$ with a cosine $\cos(4t)$.
- The term 2 is a constant, while $5 \sin(3t + x)$ is a sinusoid with phase shift x.
- The multiplication with $\cos(4t)$ introduces frequency mixing, leading to sum and difference frequencies.

Expressing the product in a form amenable to decomposition requires applying trigonometric identities, particularly the product-to-sum formulas.

Applying Trigonometric Identities for Decomposition

Product-to-Sum Formulas

The key identities used are:

- $\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$
- $\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$
- $\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]$

In our case, since $S(t)$ involves a product of a sum $(2 + 5 \sin(3t + x))$ and $\cos(4t)$, we can distribute the multiplication:

$$
S(t) = 2 \cos(4t) + 5 \sin(3t + x) \cos(4t)
$$

Our task reduces to expressing each term as a sum of sinusoidal functions with known frequencies.

Decomposing Each Term

Term 1: $2 \cos(4t)$

This term is already in the desired form: a constant multiplied by a cosine function at frequency 4.

Term 2: $5 \sin(3t + x) \cos(4t)$

Using the product-to-sum formula:

$$
\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]
$$

Let $A = 3t + x$, $B = 4t$:

$$
5 \sin(3t + x) \cos(4t) = \frac{5}{2} [\sin(3t + x + 4t) + \sin(3t + x - 4t)]
$$

Simplify the arguments:

$$
= \frac{5}{2} [\sin(7t + x) + \sin(-t + x)]
$$

Recall that $\sin(-\theta) = - \sin \theta$, so:

$$
= \frac{5}{2} [\sin(7t + x) - \sin(t - x)]
$$

Thus, the entire signal becomes:

$$
S(t) = 2 \cos(4t) + \frac{5}{2} \sin(7t + x) - \frac{5}{2} \sin(t - x)
$$

Note: The phase x appears explicitly in the sine functions, which can be expanded further if necessary.

Expressing $S(t)$ as a Linear Combination of Sinusoids and Constants

Now, we aim to rewrite $S(t)$ in the form:

\[

$$
S(t) = A_0 + A_1 \cos(4t) + B_1 \sin(4t) + A_2 \cos(7t) + B_2 \sin(7t) + A_3 \cos(t) + B_3 \sin(t)
$$

This standard form makes explicit the contributions at each harmonic frequency.

Handling the Phase Terms x in the Sinusoids

The terms $\sin(7t + x)$ and $\sin(t - x)$ can be expanded using the sine addition formula:

$$
\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta
$$

Applying this:

$$
\sin(7t + x) = \sin 7t \cos x + \cos 7t \sin x
$$

$$
\sin(t - x) = \sin t \cos x - \cos t \sin x
$$

Substituting back into $S(t)$:

$$
S(t) = 2 \cos 4t + \frac{5}{2} (\sin 7t \cos x + \cos 7t \sin x) - \frac{5}{2} (\sin t \cos x - \cos t \sin x)
$$

Distribute the coefficients:

$$
S(t) = 2 \cos 4t + \frac{5}{2} \cos x \sin 7t + \frac{5}{2} \sin x \cos 7t - \frac{5}{2} \cos x \sin t + \frac{5}{2} \sin x \cos t
$$

Now, group terms by their harmonic functions:

$$
S(t) = A_0 + A_1 \cos 4t + B_1 \sin 4t + A_2 \cos 7t + B_2 \sin 7t + A_3 \cos t + B_3 \sin t
$$

Note that $S(t)$ currently has no constant term; thus, $A_0 = 0$.

Identify coefficients for each harmonic:

- At frequency 4:

$$
\text{Coefficient of } \cos 4t: \quad 2
$$

$$
\text{Coefficient of } \sin 4t: \quad 0
$$

- At frequency 7:

$$
\cos 7t: \quad \frac{5}{2} \sin x
$$

$$
\sin 7t: \quad \frac{5}{2} \cos x
$$

- At frequency 1:

$$
\cos t: \quad \frac{5}{2} \sin x
$$

$$
\sin t: \quad - \frac{5}{2} \cos x
$$

Final Decomposition and Summary

Putting everything together, the decomposition of $S(t)$ into a linear combination of sinusoidal functions plus a constant term is:

$$
\boxed{
\begin{aligned}
S(t) =\ & \underbrace{0}_{A_0} \\
& + 2 \cos(4t) \\
& + \left(\frac{5}{2} \sin x \right) \cos(7t) \\
& + \left(\frac{5}{2} \cos x \right) \sin(7t) \\
& + \left(\frac{5}{2} \sin x \right) \cos t \\
& - \left(\frac{5}{2} \cos x \right) \sin t
\end{aligned}
}
$$

This expansion explicitly shows how the original signal $S(t)$ can be viewed as a sum of constants and harmonic components, each scaled by known functions of x.

Interpretation and Significance

Decomposing $S(t)$ into a sum of sinusoidal components is foundational for understanding its frequency content. The coefficients indicate the amplitude and phase of each harmonic. Notably:

- The term $2 \cos(4t)$ indicates a dominant frequency at 4

Frequently Asked Questions

What is the main goal when decomposing the signal S(t) = (2 + 5 sin(3t +

x)) cos(4t) into a linear combination?

The main goal is to express S(t) as a sum of simpler functions, typically sinusoids and constants, to analyze its frequency components or facilitate further processing like filtering or Fourier analysis.

How can the trigonometric identities help in decomposing S(t) into a sum of constants and sinusoids?

Trigonometric identities such as the product-to-sum formulas allow us to rewrite products like sin(A)cos(B) as sums of sinusoids, simplifying the expression into a linear combination of constant terms and sinusoidal functions.

What is the significance of decomposing S(t) into a linear combination in signal processing?

Decomposing into a linear combination helps identify individual frequency components, amplitudes, and phases, which are essential for analysis, filtering, modulation, and understanding the signal's behavior.

Can the decomposition of S(t) be used to determine its Fourier series representation?

Yes, expressing S(t) as a sum of constants and sinusoids directly relates to its Fourier series, which represents the signal as a sum of harmonic functions at different frequencies.

What role does the phase shift 'x' play in the decomposition of the signal?

The phase shift 'x' affects the sinusoidal component's phase, influencing the resulting coefficients in the linear combination and affecting how the terms combine when decomposing the signal.

How do you handle the term (2 + 5 sin(3t + x)) in the decomposition process?

You treat it as a sum of a constant (2) and a sinusoid (5 sin(3t + x)), and then multiply each term by cos(4t), applying product-to-sum identities to express the entire signal as a sum of constants and sinusoids.

What identities are useful for decomposing the product of sinusoids in S(t)?

The key identities are the product-to-sum formulas: sin A cos B = 0.5 [sin(A + B) + sin(A - B)] and cos A cos B = 0.5 [cos(A + B) + cos(A - B)], which help rewrite products as sums.

Is the decomposition into a sum of constants and sinusoids unique? Why or why not?

In general, such decompositions are unique up to a shift in phase and amplitude, especially when using orthogonal basis functions like sines and cosines, which form a complete basis for representing periodic signals.

Additional Resources

Decompose The Signal S(t) = (2 + 5 \(\sin(3t + x)\)) \(\cos(4t)\) Into A Linear Combination: A Comprehensive Analysis

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Introduction

In the realm of signal processing and mathematical analysis, the decomposition of complex signals into simpler, fundamental components forms the backbone of understanding, analyzing, and manipulating signals across diverse applications—from telecommunications to control systems. The specific challenge of expressing a signal as a linear combination of basic functions, particularly constants and sinusoidal functions, is both theoretically rich and practically essential.

This article delves into a detailed exploration of the signal:

$$
S(t) = (2 + 5 \sin(3t + x)) \cos(4t)
$$

with the aim of decomposing it into a linear combination—that is, expressing $S(t)$ as a sum of constants multiplied by sinusoidal functions with fixed frequencies. Such a decomposition not only illuminates the spectral content of the signal but also facilitates tasks like filtering, modulation analysis, and system identification.

The Significance of Signal Decomposition

Before proceeding, it's critical to contextualize why decompose a complex, composite function like $S(t)$ into basic sinusoidal components:

- Spectral Analysis: Dissecting the frequency content of signals to identify dominant frequencies.
- System Response Characterization: Determining how systems respond to specific frequency components.
- Signal Reconstruction: Simplifying signals into basis functions for efficient processing and reconstruction.
- Noise Filtering: Isolating undesired components by understanding the constituent parts.

The process hinges on expressing $S(t)$ as a sum of constants (coefficients) times sinusoidal functions of known frequencies, which is the core principle of Fourier analysis and linear superposition.

Mathematical Foundations for Decomposition

To decompose $S(t)$, we need to utilize fundamental trigonometric identities and properties:

- Distributive Property:

$$
S(t) = 2 \cos(4t) + 5 \sin(3t + x) \cos(4t)
$$

- Product-to-Sum Identities:

$$
\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]
$$

$$
\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]
$$

\[

$$
\sin A \sin B = \frac{1}{2} [\cos(A - B) - \cos(A + B)]
$$

- Shifted Sine Functions:

$$
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta
$$

Applying these identities allows us to rewrite the signal in terms of sums of sinusoidal functions with specific frequencies.

Step-by-Step Decomposition of $S(t)$

Step 1: Expand $S(t)$

Start with the original expression:

$$
S(t) = 2 \cos(4t) + 5 \sin(3t + x) \cos(4t)
$$

The key is to focus on the second term: $5 \sin(3t + x) \cos(4t)$.

Step 2: Expand $\sin(3t + x)$

Using the sine addition formula:

$$
\sin(3t + x) = \sin 3t \cos x + \cos 3t \sin x
$$

Thus,

$$
S(t) = 2 \cos 4t + 5 [\sin 3t \cos x + \cos 3t \sin x] \cos 4t
$$

Distribute:

$$
S(t) = 2 \cos 4t + 5 \sin 3t \cos x \cos 4t + 5 \cos 3t \sin x \cos 4t
$$

Step 3: Express products as sums

Apply the product-to-sum identities to each product:

- For $\sin 3t \cos 4t$:

$$
\sin 3t \cos 4t = \frac{1}{2} [\sin(3t + 4t) + \sin(3t - 4t)] = \frac{1}{2} [\sin 7t + \sin(-t)] = \frac{1}{2} [\sin 7t - \sin t]
$$

- For $\cos 3t \cos 4t$:

$$
\cos 3t \cos 4t = \frac{1}{2} [\cos(3t + 4t) + \cos(3t - 4t)] = \frac{1}{2} [\cos 7t + \cos(-t)] = \frac{1}{2} [\cos 7t + \cos t]
$$

Rewriting $S(t)$:

$$
S(t) = 2 \cos 4t + 5 \cos x \times \frac{1}{2} (\sin 7t - \sin t) + 5 \sin x \times \frac{1}{2} (\cos 7t + \cos t)
$$

Simplify coefficients:

$$
S(t) = 2 \cos 4t + \frac{5}{2} \cos x (\sin 7t - \sin t) + \frac{5}{2} \sin x (\cos 7t + \cos t)
$$

Final Decomposition Expression

Expressed explicitly, the signal becomes:

$$
\boxed{
\begin{aligned}
S(t) &= 2 \cos 4t \\
&\quad + \frac{5}{2} \cos x \sin 7t - \frac{5}{2} \cos x \sin t \\
&\quad + \frac{5}{2} \sin x \cos 7t + \frac{5}{2} \sin x \cos t
\end{aligned}
}
$$

This is a linear combination of sinusoidal functions with frequencies $0, 1, 4, 7$, and constants, with coefficients depending on the phase shift x.

Interpreting the Decomposition

Components:

- Constant component:

$$
0 \quad \text{(no explicit constant term)}
$$

- DC component:

None present; all are sinusoidal with zero mean.

- Sinusoidal components:

$$
\sin t, \quad \cos t, \quad \sin 7t, \quad \cos 7t, \quad \cos 4t

$$

- Coefficients:

- For $\cos 4t$:

$$
2
$$

- For $\sin 7t$:

$$
\frac{5}{2} \cos x
$$

- For $\sin t$:

$$
-\frac{5}{2} \cos x
$$

- For $\cos 7t$:

$$
\frac{5}{2} \sin x
$$

- For $\cos t$:

$$
\frac{5}{2} \sin x
$$

This provides a complete linear combination of basis functions.

Visualization and Significance

This decomposition reveals the spectral content of $S(t)$:

- The dominant frequency component is at $4\, \text{rad/sec}$, with amplitude 2.
- Additional components at t and $7t$ are modulated by phase x.

This insight is crucial in applications such as filter design, where certain frequency components need to be isolated or suppressed, or in modulation schemes where phase shifts influence spectral content.

Broader Implications and Applications

Signal Analysis and Processing

Decomposing signals like $S(t)$ into sinusoidal components allows engineers and scientists to:

- Identify dominant frequencies.
- Design filters targeting specific frequency bands.
- Analyze the effects of phase shifts on signal behavior.

System Identification

Understanding the spectral components of $S(t)$ provides insights into how systems (e.g., electronic circuits, control systems) respond to different frequencies, especially when phase relations are involved.

Noise Reduction

By isolating certain sinusoidal components, noise can be effectively filtered out, improving signal clarity.

Extending the Framework

While this example explicitly considers sinusoidal decomposition with respect to phase x, the methodology extends to more complex signals involving multiple phases, nonlinearities, and higher harmonics. Techniques such as Fourier series, Fourier transforms, and wavelet analysis build upon these foundational principles for more comprehensive analysis.

Conclusion

The detailed decomposition of the signal:

$$
S(t) = (2 + 5 \sin(3t + x)) \cos(4t)
$$

demonstrates the power of trigonometric identities in reducing complex, composite functions into a manageable, interpretable sum of sinusoidal basis functions. This linear combination, with coefficients explicitly depending on the phase x, underscores the intricate interplay between amplitude

Decompose The Signal S T 2 5 Sin 3t X Cos 4t Into A Linear Combination I C A Sum Of Constant

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