Determine The Equation Of The Tangent To The Graph Of Y=(x^2-3)^2 At The Point (-2, 1). A) Y = 8x+15

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Understanding how to find the tangent line to a curve at a specific point is a fundamental skill in calculus and analytical geometry. In this article, we will walk through the detailed process of determining the equation of the tangent to the graph of the function $y = (x^2 - 3)^2$ at the point (-2, 1). Specifically, we will verify whether the tangent line matches the given option $y = 8x + 15$ and explain the steps involved in deriving this tangent line.

Understanding the Problem

Before diving into calculations, it is essential to understand what is being asked.

What is a Tangent Line?

A tangent line to a curve at a specific point is a straight line that just touches the curve at that point, sharing the same slope as the curve at that point. It provides a local linear approximation of the function near that point.

The Given Function

The function under consideration is:

$$y = (x^2 - 3)^2$$

This is a quadratic function squared, which creates a parabola-like curve with specific symmetry and turning points.

The Point of Interest

The point provided is (-2, 1). Our goal is to:

- Verify that this point lies on the curve.

- Find the slope of the tangent line at this point.
- Write the equation of the tangent line using the point-slope form.
- Confirm whether the tangent line matches the given option $y = 8x + 15$.

Step 1: Verify the Point Lies on the Curve

First, ensure that the point (-2, 1) is on the graph of $y = (x^2 - 3)^2$.

Calculating the y-value at x = -2

Substitute $x = -2$:

$$y = ((-2)^2 - 3)^2 = (4 - 3)^2 = (1)^2 = 1$$

Since the y-value is 1, which matches the point's y-coordinate, the point (-2, 1) indeed lies on the curve.

Step 2: Find the Derivative of the Function

To find the slope of the tangent line at $x = -2$, we need to compute the derivative $\frac{dy}{dx}$.

Applying Chain Rule

Rewrite the function for clarity:

$$y = (u)^2 \quad \text{where} \quad u = x^2 - 3$$

Using the chain rule:

$$\frac{dy}{dx} = 2u \cdot \frac{du}{dx}$$

Calculate $\frac{du}{dx}$:

$$\frac{du}{dx} = 2x$$

Therefore:

$$\frac{dy}{dx} = 2(x^2 - 3) \cdot 2x = 4x(x^2 - 3)$$

This is the general expression for the slope of the tangent at any x.

Step 3: Find the Slope at the Point $x = -2$

Substitute $x = -2$:

$$\frac{dy}{dx} = 4 \times (-2) \times ((-2)^2 - 3)$$

Calculate step-by-step:

- $(-2)^2 = 4$
- $4 - 3 = 1$
- $4 \times (-2) = -8$

Thus:

$$\frac{dy}{dx} = -8 \times 1 = -8$$

The slope of the tangent line at $x = -2$ is -8.

Step 4: Write the Equation of the Tangent Line

Using the point-slope form:

$$y - y_1 = m (x - x_1)$$

where:

- $(x_1, y_1) = (-2, 1)$
- $m = -8$

Substitute:

$$y - 1 = -8 (x + 2)$$

Simplify:

$$y - 1 = -8x - 16$$

$$y = -8x - 16 + 1$$

$$y = -8x - 15$$

Therefore, the equation of the tangent line is:

$$ y = -8x - 15 $$

Step 5: Comparing with the Given Option

The problem states an option: $ y = 8x + 15 $. Our derived tangent line is:

$$ y = -8x - 15 $$

which is the negative of the given option and has a different slope.

Conclusion: The tangent line at the point (-2, 1) is $ y = -8x - 15 $, not $ y = 8x + 15 $.

Summary:
- The point (-2, 1) lies on the curve $ y = (x^2 - 3)^2 $.
- The slope of the tangent at this point is -8.
- The equation of the tangent line is $ y = -8x - 15 $.
- The given option $ y = 8x + 15 $ does not match the tangent line at that point.

Additional Insights and Tips

Understanding how to determine the tangent line involves several key steps that can be generalized for different functions and points.

Practice with Derivatives

- Always verify the point lies on the curve before proceeding.
- Use the chain rule carefully when dealing with composite functions like $ (x^2 - 3)^2 $.

Equation of the Tangent Line

- Remember to calculate the slope at the specific point.
- Use the point-slope form to write the tangent line equation.
- Simplify the equation for clarity.

Common Mistakes to Avoid

- Confusing the sign of the slope; ensure to evaluate the derivative correctly.
- Using the wrong point or derivative value.
- Misreading the options; verify the derived equation against multiple choices.

Conclusion

Determining the equation of the tangent to a graph at a specific point is a fundamental skill in calculus and analytical geometry. By following a structured approach—verifying the point's presence on the curve, calculating the derivative for the slope, and using the point-slope form—you can accurately find the tangent line. In this case, the tangent to $y = (x^2 - 3)^2$ at (-2, 1) is $y = -8x - 15$, which differs from the provided option $y = 8x + 15$, highlighting the importance of thorough calculations and understanding.

Remember, practice makes perfect when mastering the art of finding tangent lines and understanding their properties on various functions.

Frequently Asked Questions

What is the first step to find the equation of the tangent line to the graph of y = (x^2 - 3)^2 at the point (-2, 1)?

The first step is to find the derivative y' to determine the slope of the tangent at the given point.

How do you differentiate y = (x^2 - 3)^2 with respect to x?

Use the chain rule: $y' = 2(x^2 - 3) 2x = 4x(x^2 - 3)$.

What is the slope of the tangent line at the point (-2, 1)?

Substitute $x = -2$ into $y' = 4x(x^2 - 3)$: $y' = 4(-2)((-2)^2 - 3) = -8(4 - 3) = -8(1) = -8$.

How do you verify that the point (-2, 1) lies on the graph y = (x^2 - 3)^2?

Calculate y at $x = -2$: $y = ((-2)^2 - 3)^2 = (4 - 3)^2 = 1^2 = 1$, confirming the point is on the graph.

What is the equation of the tangent line at the point (-2, 1)?

Using point-slope form: $y - 1 = -8(x + 2)$. Simplifying gives $y = -8x - 15$.

Does the given answer 'Y = 8x + 15' correctly represent the tangent line? Why or why not?

No, because the slope from calculation is -8, so the correct tangent line is $y = -8x - 15$, not $y = 8x + 15$.

What correction should be made to the provided answer 'Y = 8x + 15'?

The slope should be negative, so the correct equation is y = -8x - 15.

Why is it important to verify the point and the slope before finalizing the tangent line equation?

To ensure the tangent line accurately touches the curve at the given point with the correct slope, avoiding errors in the equation.

Can the equation of the tangent line be written in slope-intercept form? If yes, what is it?

Yes, the equation is y = -8x - 15.

What concept does this problem illustrate about tangent lines to curves?

It illustrates how to find the tangent line at a specific point by calculating the derivative (slope) and applying point-slope form.

Additional Resources

Determine The Equation Of The Tangent To The Graph Of Y = $(x^2 - 3)^2$ At The Point (-2, 1)

Introduction

In the realm of calculus and analytical geometry, understanding the behavior of curves through their tangents is fundamental. The tangent line at a particular point on a curve provides a linear approximation of the function near that point, revealing the instantaneous rate of change—an essential concept in both theoretical mathematics and practical applications such as physics, engineering, and computer graphics.

This article delves deeply into the problem: Determine the equation of the tangent to the graph of \(y = (x^2 - 3)^2 \) at the point \((-2, 1)\). Notably, this specific problem is a classic example of applying differential calculus to find tangent lines, and it underscores the importance of derivatives in analyzing curves.

Understanding the Given Function and Point

Before proceeding with calculations, it is crucial to understand the nature of the function and the specified point:

- Function:
$$
y = (x^2 - 3)^2
$$

- Point of interest:
$(-2, 1)$

- Verification of the point:
Substituting $x = -2$ into the function:
$$
y = ((-2)^2 - 3)^2 = (4 - 3)^2 = (1)^2 = 1
$$
Since $y = 1$ when $x = -2$, the point $(-2, 1)$ indeed lies on the graph.

Deep Dive into the Mathematical Process

1. Derivative of the Function

To find the tangent line, we need to determine the slope of the tangent at the given point. The slope is given by the derivative of the function at that point:

$$
\frac{dy}{dx} = \frac{d}{dx} \left((x^2 - 3)^2 \right)
$$

Applying the chain rule:

$$
\frac{dy}{dx} = 2(x^2 - 3) \times \frac{d}{dx}(x^2 - 3)
$$

Since $\frac{d}{dx}(x^2 - 3) = 2x$, the derivative simplifies to:

$$
\frac{dy}{dx} = 2(x^2 - 3) \times 2x = 4x(x^2 - 3)
$$

This derivative function gives the slope of the tangent at any point x.

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2. Calculating the Slope at $x = -2$

Plugging $x = -2$ into the derivative:

$$
\frac{dy}{dx} \bigg|_{x = -2} = 4 \times (-2) \times ((-2)^2 - 3)
$$

Calculate step-by-step:

- $(-2)^2 = 4$
- $4 - 3 = 1$
- $4 \times (-2) = -8$

Thus:

$$
\frac{dy}{dx} \bigg|_{x = -2} = -8 \times 1 = -8
$$

Interpretation: The slope of the tangent line at the point $(-2, 1)$ is -8.

---

3. Equation of the Tangent Line

The general form of a line is:

$$
y = m x + c
$$

where m is the slope, and c is the y-intercept.

Using the point-slope form:

$$
y - y_1 = m (x - x_1)
$$

where $(x_1, y_1) = (-2, 1)$, and $m = -8$:

$$
y - 1 = -8 (x + 2)
$$

Expanding:

$$
y - 1 = -8x - 16
$$

Adding 1 to both sides:

$$
y = -8x - 15
$$

This is the tangent line's equation.

The Final Equation and Its Significance

The derived tangent line:

$$
\boxed{
\textbf{Y = -8x - 15}
}
$$

This line touches the curve $y = (x^2 - 3)^2$ precisely at $(-2, 1)$, with a slope indicating a steep decline at that point.

Validation and Additional Insights

1. Confirming the Tangent Line Passes Through the Point

Substitute $x = -2$:

$$
Y = -8(-2) - 15 = 16 - 15 = 1
$$

which matches the point's y-coordinate, confirming the correctness.

2. Graphical Interpretation

Visually, the tangent line has a negative slope, indicating the graph is decreasing at $x = -2$. The steepness (-8) suggests rapid decline near that point, consistent with the quadratic nature of the original function.

Broader Context and Applications

The ability to determine tangent lines to complex functions extends well beyond academic exercises. For instance:

- Physics:
Calculating velocity and acceleration involves derivatives, which directly relate to tangent lines.

- Engineering:
Stress analysis often involves tangent lines to stress-strain curves.

- Economics:
Marginal cost and revenue are derivatives, representing tangent slopes at specific points.

- Computer Graphics:
Rendering smooth curves relies on tangent calculations to animate or model objects precisely.

Conclusion

The systematic approach to determining the tangent line to the curve $y = (x^2 - 3)^2$ at the point $(-2, 1)$ involves differentiating the function, evaluating the derivative at the point to find the slope, and then applying the point-slope form of a line. The resulting equation:

$$
\boxed{
Y = -8x - 15
}
$$

\]

encapsulates the local linear approximation of the curve at the specified point, offering both theoretical insight and practical utility. This process exemplifies core calculus principles and demonstrates how derivative-based techniques facilitate a deeper understanding of curve behavior.

References

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About the Author

Jane Doe is a mathematician and educator specializing in calculus and mathematical analysis. She has over a decade of experience in teaching advanced mathematics concepts and applying them to real-world problems across various scientific disciplines.

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