Determine The General Solution Of Sin X Cos X + Sin X = 3 Cos X + 3 Cos X 5.3 Given The Identity Sin

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Understanding how to solve trigonometric equations is fundamental in mathematics, especially in fields such as engineering, physics, and applied mathematics. The problem titled "Determine The General Solution Of Sin X Cos X + Sin X = 3 Cos X + 3 Cos X 5.3 Given The Identity Sin" presents an intriguing challenge that involves manipulating core trigonometric identities to find the comprehensive set of solutions for the variable X. This article explores this problem in depth, providing step-by-step solutions, explanations of key concepts, and tips for solving similar equations efficiently.

Introduction to Trigonometric Equations and Identities

Before delving into the specific problem, it's essential to review some fundamental concepts related to trigonometric equations and identities.

What Are Trigonometric Equations?

Trigonometric equations involve unknown angles within trigonometric functions such as sine ($\sin$), cosine ($\cos$), tangent ($\tan$), and their reciprocals. These equations often require techniques like applying identities, algebraic manipulation, and inverse functions to find solutions.

Key Trigonometric Identities

Several identities are pivotal in solving complex trigonometric equations:

- Pythagorean Identity: $\sin^2 X + \cos^2 X = 1$
- Double-Angle Formulas:
- $\sin 2X = 2 \sin X \cos X$
- $\cos 2X = \cos^2 X - \sin^2 X$
- Sum and Difference Formulas:
- $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

Understanding and applying these identities can simplify complex equations and reveal their solutions.

Analyzing the Given Equation

The core problem is:

Determine the general solution of

$$
\sin X \cos X + \sin X = 3 \cos X + 3 \cos X \times 5.3
$$

Given the phrase "Given The Identity Sin," which suggests the problem involves utilizing the sine identity in the process.

Clarifying the Equation

First, observe the expression:

$$
\sin X \cos X + \sin X = 3 \cos X + 3 \cos X \times 5.3
$$

Note that:

- The right side contains $3 \cos X$ and $3 \cos X \times 5.3$.
- The multiplication $3 \cos X \times 5.3 = 15.9 \cos X$.

Thus, the right side simplifies to:

$$
3 \cos X + 15.9 \cos X = (3 + 15.9) \cos X = 18.9 \cos X
$$

So, the equation reduces to:

$$
\sin X \cos X + \sin X = 18.9 \cos X
$$

Rearranged, it becomes:

$$
\sin X \cos X + \sin X - 18.9 \cos X = 0
$$

Step-by-Step Solution Process

Now, we'll systematically solve this simplified equation.

Step 1: Factor the Equation

Observe common factors:

$$
\sin X \cos X + \sin X - 18.9 \cos X = 0
$$

Group terms:

$$
\sin X (\cos X + 1) - 18.9 \cos X = 0
$$

Rewrite as:

$$
\sin X (\cos X + 1) = 18.9 \cos X
$$

Step 2: Express in Terms of Known Identities

The equation is:

$$
\sin X (\cos X + 1) = 18.9 \cos X
$$

To solve for X, consider two cases:

Case 1: $\cos X + 1 \neq 0$

Case 2: $\cos X + 1 = 0$

Case 1: $\cos X + 1 \neq 0$

Divide both sides by $\cos X + 1$:

$$
\sin X = \frac{18.9 \cos X}{\cos X + 1}
$$

This expression relates $\sin X$ and $\cos X$. Since $\sin^2 X + \cos^2 X = 1$, substitute $\sin X$:

$$
\left(\frac{18.9 \cos X}{\cos X + 1}\right)^2 + \cos^2 X = 1
$$

Simplify:

$$
\frac{(18.9)^2 \cos^2 X}{(\cos X + 1)^2} + \cos^2 X = 1
$$

Multiply through by $(\cos X + 1)^2$:

$$
(18.9)^2 \cos^2 X + \cos^2 X (\cos X + 1)^2 = (\cos X + 1)^2
$$

Expressed explicitly:

$$
(18.9)^2 \cos^2 X + \cos^2 X (\cos^2 X + 2 \cos X + 1) = \cos^2 X + 2 \cos X + 1
$$

Calculate $\cos^2 X (\cos^2 X + 2 \cos X + 1)$:

$$
\cos^4 X + 2 \cos^3 X + \cos^2 X
$$

Now, the entire equation becomes:

$$
(18.9)^2 \cos^2 X + \cos^4 X + 2 \cos^3 X + \cos^2 X = \cos^2 X + 2 \cos X + 1
$$

Bring all to one side:

$$
(18.9)^2 \cos^2 X + \cos^4 X + 2 \cos^3 X + \cos^2 X - \cos^2 X - 2 \cos X - 1 = 0
$$

Simplify:

$$
(18.9)^2 \cos^2 X + \cos^4 X + 2 \cos^3 X - 2 \cos X - 1 = 0
$$

Note that $(18.9)^2 \approx 357.21$. So:

$$
357.21 \cos^2 X + \cos^4 X + 2 \cos^3 X - 2 \cos X - 1 = 0
$$

This is a quartic equation in $\cos X$:

$$
\cos^4 X + 2 \cos^3 X + 357.21 \cos^2 X - 2 \cos X - 1 = 0
$$

Let $t = \cos X$, then:

$$
t^4 + 2 t^3 + 357.21 t^2 - 2 t - 1 = 0
$$

This polynomial can be solved numerically or graphically for real solutions within $[-1,1]$, since $\cos X$ lies within this interval.

Case 2: $\cos X + 1 = 0$

This implies:

$$
\cos X = -1
$$

When $\cos X = -1$, the original simplified equation becomes:

$$
\sin X (\cos X + 1) = 18.9 \cos X
$$

But $\cos X + 1 = 0$, so left side:

$$
\sin X \times 0 = 0
$$

Right side:

$$
18.9 \times (-1) = -18.9
$$

So:

$$
0 = -18.9
$$

which is not true. Therefore, $\cos X = -1$ is not a solution in this context.

Summary of Solutions

- The solutions are derived from solving the quartic in $t = \cos X$:

$$
t^4 + 2 t^3 + 357.21 t^2 - 2 t - 1 = 0
$$

- Only roots t within $[-1, 1]$ are physically valid for $\cos X$.

- For each valid root t, compute:

$$
\sin X = \frac{18.9 t}{t + 1}
$$

- Verify that $\sin^2 X + \cos^2 X = 1$ holds for each solution.

- Finally, find the general solutions for X:

$$
X = \arccos(t) + 2 \pi k \quad \text{or} \quad X = 2 \pi - \arccos(t
$$

Frequently Asked Questions

How do I start solving the equation sin x cos x + sin x = 3 cos x + 3 cos x 5.3 using identities?

Begin by simplifying the given equation and using identities such as sin 2x = 2 sin x cos x to rewrite the terms, which can help in combining like terms and isolating the variable.

What is the first step to find the general solution for the equation involving sin x and cos x given in the problem?

Rewrite the equation in terms of a single trigonometric function or use identities to combine terms, then solve for the resulting expression to find all possible solutions.

Can the given equation be simplified using the double angle identity for sine?

Yes, since the term sin x cos x can be expressed as $\frac{1}{2}$ sin 2x, which simplifies the equation and makes solving for x more straightforward.

How do I interpret the part of the problem mentioning 'Given The Identity Sin'?

This indicates that the problem involves using the sine identity, likely to transform the equation into a solvable form, such as expressing products of sine and cosine in terms of double angles.

What is the general solution for an equation involving sin 2x and cos x after simplification?

After expressing the equation in terms of sin 2x and cos x, you can set each expression equal to specific values and solve for x, obtaining the general solutions that include all angles satisfying the original equation.

Additional Resources

Determine The General Solution Of Sin X Cos X + Sin X = 3 Cos X + 3 Cos X 5.3 Given The Identity Sin

Understanding and solving trigonometric equations is fundamental in mathematics, especially in fields such as engineering, physics, and computer science. The problem titled "Determine The General Solution Of Sin X Cos X + Sin X = 3 Cos X + 3 Cos X 5.3 Given The Identity Sin" is a complex yet interesting exercise that combines multiple trigonometric concepts, identities, and algebraic manipulations. This comprehensive review aims to dissect the problem, explore the underlying identities, solve step-by-step, and discuss the broader implications and learning outcomes associated with such problems.

Understanding the Given Equation

Before diving into the solution process, it is essential to understand the structure and components of the given equation:

$$ \sin X \cos X + \sin X = 3 \cos X + 3 \cos X \times 5.3 $$

At first glance, the equation appears to involve multiple trigonometric functions—sine and cosine—and possibly some constants or coefficients. The presence of "5.3" suggests a constant multiplier, and the phrase "Given The Identity Sin" indicates that certain identities related to sine (and possibly cosine) are to be applied.

The key steps in understanding the problem include:

- Clarifying the equation's exact form, including any potential typographical errors or ambiguities.
- Recognizing the standard identities that can simplify or transform the equation.
- Analyzing whether the equation is meant to be solved for a specific angle X or as a general solution.

Note: The wording "3 Cos X 5.3" is somewhat ambiguous. It could mean $3 \cos X \times 5.3$ or $3 \cos X + 5.3$. For clarity, we will interpret it as $3 \cos X \times 5.3$, i.e., $3 \times 5.3 \times \cos X$.

Breaking Down the Equation

Let's rewrite the equation clearly:

$$
\sin X \cos X + \sin X = 3 \cos X \times 5.3
$$

which simplifies to:

$$
\sin X \cos X + \sin X = 15.9 \cos X
$$

This clarification helps us understand the structure and makes the algebraic manipulation more straightforward.

Step 1: Factor common terms

On the left-hand side, both terms contain $\sin X$:

\[

\sin X (\cos X + 1) = 15.9 \cos X
\]

This factoring is crucial as it sets the stage for applying identities and solving for \(\sin X\) or \(\cos X\).

Applying Trigonometric Identities

The key to solving this equation lies in utilizing well-known identities, particularly the double-angle identity:

\[
\sin 2X = 2 \sin X \cos X
\]

This identity relates \(\sin X \cos X\) to \(\sin 2X\), which can simplify the equation further.

Rearranging the previous step:

\[
\sin X (\cos X + 1) = 15.9 \cos X
\]

We can write \(\sin X \cos X\) as \(\frac{1}{2} \sin 2X\), leading to:

\[
\frac{1}{2} \sin 2X + \sin X = 15.9 \cos X
\]

But this involves both \(\sin 2X\) and \(\sin X\), and the presence of \(\cos X\) on the right suggests that expressing everything in terms of a single trigonometric function might be more efficient.

Step 2: Express \(\sin X\) in terms of \(\cos X\) or vice versa

From the factored form:

\[

\sin X (\cos X + 1) = 15.9 \cos X
\]

If \(\cos X + 1 \neq 0\):

\[
\sin X = \frac{15.9 \cos X}{\cos X + 1}
\]

Alternatively, if \(\cos X + 1 = 0\):

\[
\cos X = -1
\]

which implies:

\[
X = \pi + 2k\pi, \quad k \in \mathbb{Z}
\]

Now, for the case when \(\cos X \neq -1\), the above expression allows us to relate \(\sin X\) and \(\cos X\).

Expressing the Equation in Terms of a Single Variable

Using the Pythagorean identity:

\[
\sin^2 X + \cos^2 X = 1
\]

and substituting \(\sin X\) from earlier:

\[
\left(\frac{15.9 \cos X}{\cos X + 1}\right)^2 + \cos^2 X = 1
\]

This leads to a rational equation in \(\cos X\), which can be solved for \(\cos X\).

Step 3: Solve the Rational Equation

Let $y = \cos X$. Then:

$$
\left(\frac{15.9 y}{y + 1}\right)^2 + y^2 = 1
$$

Simplify:

$$
\frac{(15.9 y)^2}{(y + 1)^2} + y^2 = 1
$$

$$
\frac{(15.9)^2 y^2}{(y + 1)^2} + y^2 = 1
$$

Multiply through by $(y + 1)^2$:

$$
(15.9)^2 y^2 + y^2 (y + 1)^2 = (y + 1)^2
$$

Expand $(y + 1)^2 = y^2 + 2y + 1$:

$$
(15.9)^2 y^2 + y^2 (y^2 + 2 y + 1) = y^2 + 2 y + 1
$$

Distribute:

$$
(15.9)^2 y^2 + y^2 y^2 + 2 y^3 + y^2 = y^2 + 2 y + 1
$$

Simplify:

$$
(15.9)^2 y^2 + y^4 + 2 y^3 + y^2 = y^2 + 2 y + 1

\]

Bring all to one side:

$$
y^4 + 2 y^3 + \left[(15.9)^2 y^2 + y^2 - y^2\right] - 2 y - 1 = 0
$$

Note that $(15.9)^2 y^2 + y^2 - y^2 = (15.9)^2 y^2$. So,

$$
y^4 + 2 y^3 + (15.9)^2 y^2 - 2 y - 1 = 0
$$

This quartic in y can be approached with numerical methods or factorization techniques, but it is complex to solve analytically without computational tools.

Special Cases and Solutions

Case 1: $\cos X = -1$

As previously identified, when:

$$
\cos X = -1
$$

$$
X = \pi + 2k \pi, \quad k \in \mathbb{Z}
$$

Check if it satisfies the original equation:

$$
\sin X \cos X + \sin X = 15.9 \cos X
$$

$$
\sin (\pi + 2k\pi) \times (-1) + \sin (\pi + 2k\pi) = 15.9 \times (-1)
$$

$$
0 \times (-1) + 0 = -15.9
$$

which simplifies to:

$$
0 = -15.9
$$

Contradiction, so $\cos X = -1$ is not a solution.

Case 2: $\cos X \neq -1$

We need to solve the quartic for y within the interval $[-1, 1]$, since $\cos X$ must lie within this range.

Final General Solution

From the earlier substitution:

$$
\sin X = \frac{15.9 \cos X}{\cos X + 1}
$$

and the identity:

$$
\sin^2 X + \cos^2 X = 1
$$

we find the valid solutions for $\cos X$ by solving the quartic equation numerically or graphically. Once $\cos X$ (denoted y) is found, the corresponding $\sin X$ can be computed using the relation above.

The general solution for X then follows from:

$$
X = \arccos y + 2k \pi
$$

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