Determine The Maximin And Minimax Strategies For The Two-person, Zero-sum Matrix Game. 2. 5 1 1 -3 3

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Understanding strategic decision-making in two-person, zero-sum matrix games is crucial for optimizing outcomes in competitive environments. In particular, determining the maximin and minimax strategies provides valuable insights into the optimal choices for players involved in such games. This article explores these strategies in detail, using the specific game matrix:

	Player B: Column 1	Player B: Column 2	Player B: Column 3	Player B: Column 4	Player B: Column 5
Player A: Row 1	2	5	1	1	-3
Player A: Row 2	3				

(Note: The matrix provided appears to have some formatting issues; for clarity, assume the full matrix is as follows:)

	Column 1	Column 2	Column 3	Column 4	Column 5
Row 1	2	5	1	1	-3
Row 2	3	1	-3	3	?

Note: Since the original matrix seems incomplete or inconsistent, this analysis assumes the matrix is as given in the initial statement: "2. 5 1 1 -3 3". For the purpose of this discussion, we'll interpret the matrix as a 2x5 matrix:

	Col 1	Col 2	Col 3	Col 4	Col 5
Row 1	2	5	1	1	-3
Row 2	3	1	-3	3	?

Assuming the last element in row 2 is missing, and the matrix is intended to be a 2x5 matrix with the first four columns as above and the last as 3.

In this guide, we will walk through the process of analyzing such a game to determine the optimal strategies for both players, focusing on the concepts of maximin and minimax. These concepts are fundamental in game theory, particularly for zero-sum games, where one player's gain is precisely the other's loss.

Understanding Two-Person, Zero-Sum Matrix Games

What Is a Zero-Sum Game?

A zero-sum game is a strategic interaction where one player's gain is exactly balanced by the losses of the other player. The total payoff remains constant; thus, the interests of players are completely opposed.

- Examples include: Poker, chess, and certain economic or military scenarios.
- Key property: The sum of the payoffs for both players in any outcome equals zero.

Representation of the Game Matrix

In a matrix game:

- Rows represent Player A's strategies.
- Columns represent Player B's strategies.
- Entries represent the payoff to Player A (the maximizer), with Player B's payoff being the negative of that.

For the matrix:

	Col 1	Col 2	Col 3	Col 4	Col 5
Row 1	2	5	1	1	-3
Row 2	3	1	-3	3	?

Assuming the last entry in Row 2 is 3 for completeness.

Maximin and Minimax Strategies Explained

Definitions

- Maximin Strategy: The strategy that maximizes the minimum payoff a player can guarantee, regardless of the opponent's actions.
- Minimax Strategy: The strategy that minimizes the maximum payoff the opponent can achieve, effectively minimizing potential losses.

The Significance in Zero-Sum Games

- In zero-sum games, the maximin for Player A and the minimax for Player B are crucial for identifying the game's value and optimal strategies.
- The Minimax Theorem (von Neumann): In finite, zero-sum games, the maximin equals the minimax, ensuring the existence of a saddle point.

Step-by-Step Approach to Determine Strategies

1. Calculating Player A’s Maximin Strategy

Player A aims to select a strategy that maximizes their minimum guaranteed payoff.

- Process:
1. For each row, identify the minimum payoff.
2. Select the row with the highest of these minimum payoffs.

Example:

Row	Payoffs	Minimum in row
Row 1	2, 5, 1, 1, −3	−3
Row 2	3, 1, −3, 3, 3	−3

Note: Since the matrix entries for Row 2 are assumed to be 3, 1, −3, 3, 3.

- The minimum in Row 1: −3.
- The minimum in Row 2: −3.

Maximum of these minima: −3.

- Interpretation: Player A's maximin payoff is −3, achieved by either row.

2. Calculating Player B’s Minimax Strategy

Player B aims to select a strategy that minimizes Player A's maximum payoff.

- Process:
1. For each column, identify the maximum payoff to Player A.
2. Select the column with the lowest of these maximums.

Example:

Column	Payoffs in columns	Max in column
Col 1	2, 3	3
Col 2	5, 1	5
Col 3	1, −3	1
Col 4	1, 3	3
Col 5	−3, 3	3

- The maximums per column: 3, 5, 1, 3, 3.

- The minimum of these maximums: 1 (Column 3).

- Interpretation: Player B’s minimax is 1, achieved by selecting Column 3.

Determining the Game’s Value and Optimal Strategies

Analysis

- The maximin value for Player A is −3.
- The minimax value for Player B is 1.

Since these are not equal, the game does not have a pure strategy saddle point; instead, players will adopt mixed strategies to optimize their expected payoff.

Implications

- Player A’s strategy: Focus on maximizing the minimum payoff, potentially mixed between rows.
- Player B’s strategy: Focus on minimizing the maximum payoff to Player A, potentially mixed between columns.

Calculating Optimal Mixed Strategies

1. Player A’s Mixed Strategy

- Assign probabilities p_1 and p_2 to Rows 1 and 2, respectively.
- Expected payoff for Player A when Player B chooses a pure column strategy is:

$$
E = p_1 \times \text{Row 1 payoff} + p_2 \times \text{Row 2 payoff}
$$

- Player A aims to maximize the minimum expected payoff across all Player B's strategies.

2. Player B’s Mixed Strategy

- Assign probabilities q_1, q_2, q_3, q_4, q_5 for selecting each column.
- Player B minimizes Player A’s expected payoff:

$$
E = \sum_{i=1}^2 \sum_{j=1}^5 p_i \times q_j \times \text{payoff}_{ij}
$$

To find the optimal strategies, proceed with linear programming techniques:

- For Player A:

Maximize \(v \) subject to:

```
\[
\begin{cases}
2p_1 + 3p_2 \geq v \quad \text{(Column 1)} \\
5p_1 + 1p_2 \geq v \quad \text{(Column 2)} \\
1p_1 - 3p_2 \geq v \quad \text{(Column 3)} \\
1p_1 + 3p_2 \geq v \quad \text{(Column 4)} \\
-3p_1 + 3p_2 \geq v \quad \text{(Column 5)} \\
p_1 + p_2 = 1 \\
p_
```

Frequently Asked Questions

What is the primary goal of the maximin strategy in a two-person, zero-sum matrix game?

The maximin strategy aims to maximize the minimum payoff a player can receive, ensuring the best worst-case outcome regardless of the opponent's actions.

How do you determine the minimax strategy in a matrix game?

The minimax strategy involves the player minimizing the maximum payoff that the opponent can force, effectively choosing actions that limit the potential gains of the adversary.

Given the matrix [[5, 1], [1, -3], [3, 0]], how can you find the maximin value for the row player?

Calculate the minimum payoff in each row and select the highest among these minima. For the matrix: row 1 min is 1, row 2 min is -3, row 3 min is 0. The maximin value is 1 from row 1.

What is the significance of the saddle point in a zero-sum matrix game?

A saddle point exists when the maximin equals the minimax, indicating an equilibrium where both players' strategies are optimal and the game is stable.

Does a saddle point always exist in a two-person zero-sum matrix game?

No, a saddle point exists only if the maximin equals the minimax. If they differ, the game requires mixed strategies for optimal play.

How can mixed strategies be used to determine the optimal strategies if no saddle point exists?

Mixed strategies involve assigning probabilities to actions to maximize expected payoffs or minimize potential losses, and are found using linear programming methods or solving for equilibrium conditions.

What are the maximin and minimax strategies for the given matrix [[5, 1], [1, -3], [3, 0]]?

Maximin strategy for the row player is to choose the first row (since its minimum is 1, higher than others). Minimax strategy for the column player involves analyzing the columns to minimize the maximum payoff, often using linear programming to find the optimal mixed strategies.

Can you explain the process of solving for the maximin and minimax strategies in this specific matrix?

Yes. For the row player, identify the minimum payoff in each row: row 1 is 1, row 2 is -3, row 3 is 0. The maximin is 1. For the column player, analyze the columns to find strategies that minimize the maximum payoff to the row player, often through linear programming or equilibrium analysis.

What is the value of the game based on the matrix [[5, 1], [1, -3], [3, 0]]?

Since the maximin is 1 and the minimax can be determined through optimization, the value of the game is approximately 1, indicating the expected payoff when both players use their optimal strategies.

Additional Resources

Determine The Maximin And Minimax Strategies For The Two-Person, Zero-Sum Matrix Game. 2. 5 1 1 -3 3

Introduction

In the realm of game theory, two-person zero-sum games have long served as fundamental models for understanding competitive decision-making scenarios. They encapsulate situations where one player's gain is precisely another's loss, emphasizing the importance of strategic planning to optimize outcomes against an opponent. A central focus in such games is identifying the strategies that players can adopt to secure the best possible outcome under worst-case assumptions—these are known as the maximin and minimax strategies.

This article aims to thoroughly analyze these strategies for a specific matrix game characterized by the data: 2. 5 1 1 -3 3. Although this notation may appear cryptic, it can be interpreted as a payoff matrix representing the game structure. The goal is to determine the optimal strategies for both players, understand the significance of the maximin and minimax principles, and explore the broader implications within game theory.

Understanding the Matrix Game Structure

Before delving into the strategies, it is essential to interpret the given data accurately.

Decoding the Data: "2. 5 1 1 -3 3"

The notation suggests a matrix with the following characteristics:

- The first number might indicate the number of rows or columns; assuming it indicates the number of rows or strategies for Player A.
- The subsequent numbers are likely the payoff entries, which could be arranged in a matrix form.

Given the sequence "5 1 1 -3 3," one plausible interpretation is that these represent the entries of a 2x3 payoff matrix, with Player A's strategies as rows and Player B's strategies as columns:

	B1	B2	B3
A1	5	1	1
A2	-3	3	?

However, the last value "3" suggests a third column in the second row, making the matrix:

	B1	B2	B3
A1	5	1	1
A2	-3	3	3

This 2x3 matrix would be consistent with the sequence of numbers.

Therefore, the payoff matrix for Player A (the row player) can be represented as:

```
B1 B2 B3
A1 5 1 1
A2 -3 3 3
```

Fundamental Concepts: Maximin and Minimax Strategies

Maximin Strategy

The maximin strategy involves Player A choosing a strategy that maximizes their minimum guaranteed payoff. In other words, Player A looks at each of their strategies' worst-case outcomes and selects the strategy with the highest payoff among these minimum payoffs.

Mathematically, for each strategy i:

$$
\text{Min payoff for } i = \min_{j} a_{ij}
$$

\]

The maximin value $V_{maximin}$ is:

$$
V_{maximin} = \max_{i} \left(\min_{j} a_{ij} \right)
$$

Minimax Strategy

Conversely, the minimax strategy for Player B (the column player) involves choosing a strategy that minimizes Player A's maximum possible payoff. Player B considers the maximum payoff Player A can secure in response to each of their strategies and chooses the one that minimizes this maximum.

For each column j:

$$
\text{Max payoff for Player A} \text{ in column } j = \max_{i} a_{ij}
$$

The minimax value $V_{minimax}$ is:

$$
V_{minimax} = \min_{j} \left(\max_{i} a_{ij} \right)
$$

In equilibrium, these two values should coincide if the game has a saddle point.

Step-by-Step Calculation of Strategies

1. Determine Player A's Maximin Strategy

Identify the minimum payoff in each row:

- For A1: $\min(5, 1, 1) = 1$
- For A2: $\min(-3, 3, 3) = -3$

Maximin value:

$$
V_{maximin} = \max(1, -3) = 1
$$

Optimal maximin strategy:

- Player A should choose the strategy corresponding to row 1 (A1), which guarantees at least 1 regardless of Player B's actions.

2. Determine Player B's Minimax Strategy

Identify the maximum payoff in each column:

- For B1: $\max(5, -3) = 5$
- For B2: $\max(1, 3) = 3$
- For B3: $\max(1, 3) = 3$

Minimax value:

$$
V_{minimax} = \min(5, 3, 3) = 3
$$

Optimal minimax strategy:

- Player B should choose column 2 or 3, which both limit Player A's maximum payoff to 3.

Analyzing Equilibrium and Saddle Point Existence

The key question: do the maximin and minimax values match?

- Player A's maximin value: 1
- Player B's minimax value: 3

Since $1 \neq 3$, no saddle point exists in pure strategies. This indicates that the game does not have a pure strategy equilibrium, and players should consider mixed strategies to optimize their outcomes.

Mixed Strategies and Equilibrium Analysis

Given the absence of a pure strategy saddle point, we move towards mixed strategies.

1. Player A's Mixed Strategy

Let:

- p = probability of choosing A1
- $1 - p$ = probability of choosing A2

Player A aims to maximize their expected payoff, considering Player B's mixed strategy.

2. Player B's Mixed Strategy

Let:

- q_1 = probability of choosing B1
- q_2 = probability of choosing B2
- q_3 = probability of choosing B3

with $q_1 + q_2 + q_3 = 1$.

Formulating the Expected Payoff

The expected payoff to Player A, given the strategies, is:

$$
E(a) = p \times \text{Expected payoff with A1} + (1 - p) \times
$$

\text{Expected payoff with A2}
\]

which depends on Player B's mixed strategy:

$$
E(a) = p [q_1 \times 5 + q_2 \times 1 + q_3 \times 1] + (1 - p) [q_1 \times (-3) + q_2 \times 3 + q_3 \times 3]
$$

The goal for Player A is to choose p to maximize the minimum expected payoff over all possible strategies of Player B.

Similarly, Player B chooses q_1, q_2, q_3 to minimize Player A's maximum expected payoff.

Solving for the Equilibrium Strategies

Given the complexity, the standard approach is to set up a linear programming formulation or use the method of indifference to find equilibrium strategies.

Player A's Indifference Conditions:

- Player A wants to choose p so that Player B's strategies yield the same expected payoff, making Player B indifferent.

Player B's Indifference Conditions:

- Player B wants to choose q_j so that Player A's strategies yield the same expected payoff, making Player A indifferent.

Practical Computation

Let's analyze Player A's expected payoff when Player B chooses specific strategies:

- If Player B chooses B1: Expected payoff:

$$
E_{A}(A1) = 5, \quad E_{A}(A2) = -3
$$

- If Player B chooses B2:

$$
E_{A}(A1) = 1, \quad E_{A}(A2) = 3
$$

- If Player B chooses B3:

$$
E_{A}(A1) = 1, \quad E_{A}(A2) = 3
$$

Player A will choose p to make Player B indifferent between strategies.

For Player B to be indifferent between B1 and B2, the expected payoff to Player A must be equal when Player B mixes strategies:

\[
\text{Expected payoff with B1} = \text{Expected payoff with B2}
\]

\[
p \times 5 + (1-p) \times (-3) = p \times 1 + (1-p) \times 3
\]

Simplify:

\[
5p - 3 + 3p = p + 3 - 3p
\]
\[
8p - 3 = (p + 3) - 3p
\]
\[

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- F(x, Y, Z) = Y I (z Y) J X K S Is The Surface Of The Tetrahedron With Vertices (0, 0, 0), (4, 0, 0),
- Consider The Video You Just Watched. The Two Pucks Of Equal Mass Did Not Move Linearly (they Came To

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