

Determine The Mean And Variance Of The Random Variable With The Following Probability Mass Function.

Determine The Mean And Variance Of The Random Variable With The Following Probability Mass Function.

Understanding how to compute the mean and variance of a random variable is fundamental in probability theory and statistics. These measures provide crucial insights into the behavior and characteristics of stochastic processes, enabling statisticians and data analysts to interpret data accurately and make informed decisions. When dealing with discrete random variables, the probability mass function (PMF) plays a central role in these calculations. This article aims to guide you through the detailed process of determining the mean and variance of a discrete random variable given its PMF, emphasizing clarity, practical examples, and optimization for SEO to ensure comprehensive coverage of the topic.

What Is a Probability Mass Function (PMF)?

Before diving into the calculations, it's essential to understand what a probability mass function is.

Definition of PMF

A probability mass function (PMF) assigns probabilities to each possible value of a discrete random variable. Formally, for a discrete random variable X , the PMF $p(x)$ satisfies:

- $p(x) \geq 0$ for all x ,
- $\sum_x p(x) = 1$.

These properties ensure that the total probability across all possible outcomes sums to one, adhering to the axioms of probability.

Example of a PMF

Suppose a die is rolled, and X represents the outcome. The PMF for a fair six-sided die is:

- $p(1) = \frac{1}{6}$,
- $p(2) = \frac{1}{6}$,
- ...
- $p(6) = \frac{1}{6}$.

In this case, each outcome has an equal probability, but PMFs can vary to represent more complex distributions.

Calculating the Mean (Expected Value) of a Discrete Random Variable

The mean, or expected value $E[X]$, measures the central tendency of the distribution of X . It indicates the average value you would expect if the experiment were repeated many times.

Formula for the Mean

For a discrete random variable X with PMF $p(x)$, the mean is calculated as:

$$E[X] = \sum_{x} x \cdot p(x)$$

This formula involves summing over all possible values of X , multiplying each value by its probability.

Step-by-Step Example

Suppose X has the following PMF:

X	1	2	3	4
$p(x)$	0.1	0.3	0.4	0.2

To find $E[X]$:

$$E[X] = (1)(0.1) + (2)(0.3) + (3)(0.4) + (4)(0.2) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7$$

This value indicates the average outcome over many trials.

Calculating the Variance of a Discrete Random Variable

Variance quantifies the spread or dispersion of the distribution around the mean. A higher variance indicates outcomes are more spread out.

Formula for Variance

The variance $Var(X)$ is given by:

$$Var(X) = E[(X - E[X])^2] = \sum_{x} (x - E[X])^2 p(x)$$

Alternatively, it can be computed using the formula:

$$Var(X) = E[X^2] - (E[X])^2$$

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

where $E[X^2]$ is the second moment of the distribution.

Calculating $E[X^2]$

$$E[X^2] = \sum_x x^2 p(x)$$

Once $E[X]$ and $E[X^2]$ are known, the variance can be obtained straightforwardly.

Example Calculation

Using the previous PMF:

x	1	2	3	4
$p(x)$	0.1	0.3	0.4	0.2

Calculate $E[X^2]$:

$$E[X^2] = (1^2)(0.1) + (2^2)(0.3) + (3^2)(0.4) + (4^2)(0.2) = 1 \times 0.1 + 4 \times 0.3 + 9 \times 0.4 + 16 \times 0.2 = 0.1 + 1.2 + 3.6 + 3.2 = 8.1$$

Recall $E[X] = 2.7$:

$$\text{Var}(X) = E[X^2] - (E[X])^2 = 8.1 - (2.7)^2 = 8.1 - 7.29 = 0.81$$

Thus, the variance of X is 0.81, indicating the degree of spread around the mean.

General Steps to Find Mean and Variance from a PMF

To systematically compute the mean and variance of a discrete random variable given its PMF, follow these steps:

Step 1: List All Possible Values

Identify all outcomes $(x_1, x_2, \dots, x_n)$ for the random variable.

Step 2: Obtain or Define the PMF

Ensure the probability for each (x_i) is known, satisfying the properties of a PMF.

Step 3: Compute the Expected Value $(E[X])$

Use:

$$E[X] = \sum_{i=1}^n x_i p(x_i)$$

Step 4: Compute the Second Moment $(E[X^2])$

Calculate:

$$E[X^2] = \sum_{i=1}^n x_i^2 p(x_i)$$

Step 5: Calculate the Variance

Apply:

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

Special Cases and Additional Considerations

While the above steps cover most cases, certain scenarios require additional attention:

1. Infinite Support

If (X) can take infinitely many values, ensure the sums converge; otherwise, the mean or variance may be undefined or infinite.

2. Symmetric Distributions

Symmetry can simplify calculations, especially when the PMF is symmetric about the mean.

3. Discrete Uniform Distribution

For uniform distributions over (n) outcomes:

- $(p(x) = \frac{1}{n})$,
- $(E[X] = \frac{a + b}{2})$,
- $(\text{Var}(X) = \frac{(b - a + 1)^2 - 1}{12})$, where (a, b) are the minimum and maximum outcomes.

Practical Applications of Mean and Variance in Probability

Calculating the mean and variance of a discrete random variable with a known PMF is vital across various fields:

- **Quality Control:** Assessing the average defect rate and its variability in manufacturing processes.
- **Risk Management:** Quantifying expected losses and their volatility in finance.
- **Game Theory and Gambling:** Estimating average winnings and the risk involved.
- **Data Science:** Summarizing categorical data distributions for inference.

Understanding these statistical measures enhances decision-making and predictive modeling.

Conclusion

Determining the mean and variance of a random variable from its probability mass function is a foundational skill in probability and statistics. By methodically applying the formulas for expectation and variance, and carefully handling the summations over the distribution's support, analysts can extract meaningful insights about the distribution's behavior. Remember to verify that the PMF satisfies the necessary properties before performing calculations, especially for distributions with infinite support. Mastery of these techniques empowers professionals to analyze, interpret, and leverage discrete probability distributions effectively across diverse applications.

Keywords for SEO optimization:

- Probability mass function calculation
- How to compute mean and variance from PMF
- Discrete random variable analysis
- Expected value formula
- Variance of a discrete distribution
- Step-by-step PMF examples
- Applications of mean and variance in statistics
- Analyzing discrete probability distributions

Frequently Asked Questions

How do you calculate the mean (expected value) of a discrete random variable given its probability mass function (pmf)?

The mean is calculated by summing the product of each value of the random variable and its corresponding probability: $E[X] = \sum x P(X = x)$.

What is the formula for finding the variance of a discrete random variable once the mean is known?

The variance is given by $\text{Var}(X) = \sum (x - \mu)^2 P(X = x)$, where μ is the mean of the distribution.

In the context of a pmf, how do you handle calculations if the probabilities are given for only a subset of possible values?

You sum over all the specified values, multiplying each value by its probability for the mean, and similarly for variance. If some probabilities are missing, ensure they sum to 1 or adjust calculations accordingly.

Can you provide a step-by-step method to determine the mean and variance from a given pmf?

Yes. First, identify all possible values and their probabilities. Then, compute the mean using $E[X] = \sum x P(X = x)$. Next, find the variance by calculating $\sum (x - \mu)^2 P(X = x)$.

What are common pitfalls when calculating mean and variance from a pmf?

Common pitfalls include forgetting to verify that probabilities sum to 1, mixing up the values, or incorrectly calculating variance by not subtracting the mean before squaring.

How does the shape of the pmf influence the mean and variance of the distribution?

The shape indicates how values are distributed; for example, a skewed pmf shifts the mean and can increase variance, while symmetric pmfs tend to have means near the center with potentially lower variance.

Is it necessary to normalize the pmf before calculating the mean and variance?

Yes, the probabilities must sum to 1. If they do not, normalize them by dividing each probability by the total sum before performing calculations.

How can the mean and variance help in understanding the

behavior of a random variable with a given pmf?

The mean indicates the expected value or center of the distribution, while the variance measures the spread or variability. Together, they summarize the distribution's characteristics and are useful for predictions and decision-making.

Additional Resources

Determine The Mean And Variance Of The Random Variable With The Following Probability Mass Function

In the realm of probability theory and statistics, understanding the behavior of random variables is fundamental. These variables, which can take on different values based on chance, are characterized by functions that describe their likelihoods. Among the essential tools for summarizing the distribution of a discrete random variable are the mean (or expected value) and the variance. The mean provides a measure of the central tendency, while the variance quantifies the spread or dispersion around that mean. This article aims to guide readers through the process of calculating the mean and variance for a discrete random variable when its probability mass function (pmf) is known, highlighting the steps involved, common pitfalls, and practical applications.

Understanding the Foundations: What Is a Probability Mass Function?

Before diving into calculations, it is crucial to understand what a probability mass function is. In probability theory, a pmf assigns a probability to each possible value that a discrete random variable can take. Formally, for a discrete random variable X , the pmf $p(x)$ satisfies:

- $p(x) \geq 0$ for all x
- $\sum_{x} p(x) = 1$

This function provides a complete description of the distribution of X . Common examples include the Bernoulli distribution, the Binomial distribution, and the Poisson distribution.

Step 1: Identifying the Distribution and Its Support

The first step in calculating the mean and variance is to clearly identify the pmf and its support—the set of all values where the pmf is non-zero.

Support: The set $\{x_1, x_2, \dots, x_n\}$ over which the pmf is defined.

For example, if the pmf is:

$$p(x) = \frac{1}{6} \quad \text{for} \quad x = 1, 2, 3, 4, 5, 6$$

then the support is $\{1, 2, 3, 4, 5, 6\}$.

Understanding the support is vital because the calculations for the mean and variance involve summing over all possible values of x within this support.

Step 2: Calculating the Mean (Expected Value)

The mean (or expected value) of a discrete random variable X with pmf $p(x)$ is defined as:

$$\boxed{\mu = E[X] = \sum_{x} x \cdot p(x)}$$

This formula computes the average value of X weighted by the probability of each x .

Practical steps:

1. List all possible values x_i in the support.
2. Multiply each x_i by its probability $p(x_i)$.
3. Sum all these products to obtain the mean.

Example:

Suppose X is the roll of a fair six-sided die. The pmf:

$$p(x) = \frac{1}{6} \quad \text{for} \quad x=1,2,3,4,5,6$$

The mean is:

$$E[X] = \sum_{x=1}^6 x \cdot \frac{1}{6} = \frac{1}{6}(1 + 2 + 3 + 4 + 5 + 6) = \frac{1}{6} \times 21 = 3.5$$

This indicates that the average outcome over many rolls approaches 3.5.

Step 3: Calculating the Variance

Variance measures how much the values of X spread out from the mean. It is given by:

$$\boxed{\sigma^2 = \text{Var}(X) = E[(X - \mu)^2]}$$

Alternatively, variance can be computed using the formula:

$$\text{Var}(X) = E[X^2] - (E[X])^2$$

where $E[X^2]$ is the second moment about zero, calculated as:

$$E[X^2] = \sum_x x^2 \cdot p(x)$$

Steps for calculating variance:

1. Compute $E[X]$ as above.
2. Calculate $E[X^2]$ by summing $x^2 \cdot p(x)$ over all x .
3. Use the relation $\text{Var}(X) = E[X^2] - (E[X])^2$.

Continuing the die example:

Calculate $E[X^2]$:

$$E[X^2] = \frac{1}{6}(1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2) = \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) = \frac{1}{6} \times 91 = 15.1667$$

Variance:

$$\text{Var}(X) = 15.1667 - (3.5)^2 = 15.1667 - 12.25 = 2.9167$$

This indicates the level of dispersion around the mean for a fair die roll.

Practical Examples with Different Distributions

While the die example is straightforward, in real-world applications, pmfs are often more complex, involving parameters that define distributions such as Binomial, Poisson, or Geometric.

Example 1: Binomial Distribution

Suppose $X \sim \text{Binomial}(n=10, p=0.5)$. The pmf:

$$p(x) = \binom{10}{x} p^x (1-p)^{10-x}$$

for $x = 0, 1, \dots, 10$.

Mean:

$$\begin{aligned} E[X] &= n p = 10 \times 0.5 = 5 \end{aligned}$$

Variance:

$$\begin{aligned} \text{Var}(X) &= n p (1 - p) = 10 \times 0.5 \times 0.5 = 2.5 \end{aligned}$$

These formulas are standard for binomial distributions, simplifying calculations.

Example 2: Poisson Distribution

Suppose $X \sim \text{Poisson}(\lambda=4)$. The pmf:

$$\begin{aligned} p(x) &= \frac{e^{-\lambda} \lambda^x}{x!} \end{aligned}$$

for $x=0, 1, 2, \dots$.

Mean:

$$\begin{aligned} E[X] &= \lambda = 4 \end{aligned}$$

Variance:

$$\begin{aligned} \text{Var}(X) &= \lambda = 4 \end{aligned}$$

The Poisson distribution's mean and variance are equal, a unique property that simplifies many analyses.

Common Challenges and Mistakes

Calculating mean and variance is often straightforward, but some pitfalls can trip up students and practitioners:

- Incorrect support identification: Missing some values where the pmf is non-zero leads to inaccurate calculations.
- Ignoring dependencies or parameters: In distributions with parameters, ensure they are correctly incorporated.
- Computational errors: Especially with large supports, manual calculations can be tedious; using

software or calculator functions helps reduce errors.

- Misapplication of formulas: Remember to use $E[X^2]$ for variance, not just $(E[X])^2$.

Practical Applications of Mean and Variance Calculations

Understanding how to determine the mean and variance of a discrete random variable has numerous applications:

- Quality Control: Estimating average defect rates and their variability.
- Finance: Calculating expected returns and risks of discrete investments.
- Operations Research: Modeling the number of customer arrivals or failures.
- Epidemiology: Estimating the average number of cases in a population and the variability.

In each case, knowing the distribution's mean and variance informs decision-making and risk assessment.

Final Remarks

Mastering the calculation of the mean and variance of a discrete random variable from its pmf is a cornerstone of statistical literacy. It enables analysts to summarize complex distributions succinctly and make informed predictions. Whether dealing with simple discrete distributions like the die roll or complex parameterized distributions like the binomial or Poisson, the fundamental principles remain consistent: identify the support, compute the necessary moments, and interpret the results within the context of the problem.

With practice, these calculations become intuitive tools, empowering data-driven decisions across countless fields. As probability theory continues to underpin advancements in technology, economics, and science, the ability to determine the mean and variance from a pmf remains an essential skill for statisticians, researchers, and analysts alike.

Determine The Mean And Variance Of The Random Variable With The Following Probability Mass Function

Determine The Mean And Variance Of The Random Variable With The Following Probability Mass Function

Related Articles

- [G 600 Million Shares Of Common Stock And 3 Million Shares Of 6%, \\$100 Par Value Cumulative Preferred](#)
- [George Clausen \(age 48\) Is Employed By Kline Company And Is Paid An Annual Salary Of \\$42,640. He Has](#)
- [For Many Years, There Have Been Limits Set On The Amount Of Sugar That Foreign](#)

[Producers Can Sell In](#)

[Back to Home](#)