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Understanding the molar solubility of aluminum hydroxide ($Al(OH)_3$) in a solution containing aluminum chloride ($AlCl_3$) requires a comprehensive analysis of solubility equilibria, common ion effects, and the application of solubility product constants (K_sp). This problem combines concepts from chemical equilibrium, complex ion formation, and ionic interactions in aqueous solutions. In this article, we will explore the step-by-step process to determine the molar solubility of $Al(OH)_3$ under the given conditions, ensuring a thorough comprehension of the underlying chemistry principles involved.

Background Concepts and Fundamental Principles

Solubility Product Constant (K_sp)

The solubility product constant, K_sp, is a measure of the solubility of an ionic compound in water. It is defined as the equilibrium constant for the dissolution of a solid salt:

$$ \text{Al(OH)}_3 (s) \rightleftharpoons \text{Al}^{3+} (aq) + 3 \text{OH}^- (aq) $$

The expression for K_sp is:

$$ K_{sp} = [\text{Al}^{3+}] [\text{OH}^-]^3 $$

This value is temperature-dependent and typically obtained experimentally.

Common Ion Effect

When a solution already contains ions that are part of the equilibrium's dissociation, the solubility of the salt decreases. This phenomenon is called the common ion effect. In our case, the presence of Al^{3+} ions from $AlCl_3$ reduces the solubility of $Al(OH)_3$ because the solution already contains aluminum ions that can shift the dissolution equilibrium backward.

$AlCl_3$ Dissolution and Hydrolysis

$AlCl_3$ is a soluble salt that dissociates completely in water:

$$\text{AlCl}_3 \rightarrow \text{Al}^{3+} + 3 \text{Cl}^-$$

Since $AlCl_3$ is a strong electrolyte, the initial concentration of Al^{3+} from $AlCl_3$ is 0.0500 M. The chloride ions do not affect solubility directly, but Al^{3+} does through the common ion effect.

Step-by-Step Approach to Determine Molar Solubility

1. Write the Dissolution and Equilibrium Expressions

The dissolution of $Al(OH)_3$:

$$\text{Al(OH)}_3 (s) \leftrightarrow \text{Al}^{3+} (aq) + 3 \text{OH}^- (aq)$$

Let the molar solubility of $Al(OH)_3$ be s. This means:

- $[\text{Al}^{3+}] = s$ (from $Al(OH)_3$ dissolution)
- $[\text{OH}^-] = 3s$

However, since the solution already contains $[\text{Al}^{3+}] = 0.0500\, M$ from $AlCl_3$, the total $[\text{Al}^{3+}]$ in solution becomes:

$$[\text{Al}^{3+}]_{total} = 0.0500 + s$$

Similarly, the hydroxide ion concentration from solubility is $3s$. We must consider whether the initial Al^{3+} concentration from $AlCl_3$ significantly affects the solubility of $Al(OH)_3$.

2. Effect of Existing Al^{3+} Ions on Solubility

Because the initial Al^{3+} concentration from $AlCl_3$ is known (0.0500 M), the dissolution of $Al(OH)_3$ will be suppressed according to Le Châtelier's principle. The equilibrium expression incorporating this is:

$$K_{sp} = [\text{Al}^{3+}] [\text{OH}^-]^3$$

which becomes:

$$K_{sp} = (0.0500 + s) \times (3s)^3$$

In many practical cases, if the added $AlCl_3$ concentration is large compared to the molar solubility s, then s is negligible compared to 0.0500, and the total $[\text{Al}^{3+}]$ can be approximated as 0.0500 M. This simplifies calculations.

3. Simplify the Expression Under Realistic Assumptions

Assuming $s \ll 0.0500$, then:

$$ [\text{Al}^{3+}] \approx 0.0500\, M $$

Thus, the equilibrium expression simplifies to:

$$ K_{sp} = (0.0500) \times (3s)^3 $$

which simplifies to:

$$ K_{sp} = 0.0500 \times 27 s^3 $$

or:

$$ K_{sp} = 1.35 \times s^3 $$

From which:

$$ s^3 = \frac{K_{sp}}{1.35} $$

and

$$ s = \left(\frac{K_{sp}}{1.35} \right)^{1/3} $$

Note: To proceed, the actual numerical value of K_{sp} for $Al(OH)_3$ at the relevant temperature is needed.

Determining the Numerical Molar Solubility

1. Obtain the Value of K_{sp} for $Al(OH)_3$

The solubility product constant K_{sp} for $Al(OH)_3$ at 25°C is approximately:

$$ K_{sp} \approx 1.9 \times 10^{-33} $$

This value is well-documented in standard chemical data references.

2. Calculate the Molar Solubility s

Using the simplified relation:

$$ s = \left(\frac{K_{sp}}{1.35} \right)^{1/3} $$

Plugging in the numbers:

$$ s = \left(\frac{1.9 \times 10^{-33}}{1.35} \right)^{1/3} $$

$$ s = \left(1.407 \times 10^{-33} \right)^{1/3} $$

Calculating the cube root:

$$ s \approx \left(1.407 \times 10^{-33} \right)^{1/3} $$

Recall that:

$$ (a \times 10^{b})^{1/3} = a^{1/3} \times 10^{b/3} $$

So,

$$ s \approx 1.407^{1/3} \times 10^{-33/3} $$

$$ s \approx 1.108 \times 10^{-11} $$

Thus, the molar solubility of $Al(OH)_3$ in this solution is approximately:

$$ \boxed{ s \approx 1.11 \times 10^{-11}\, \text{M} } $$

This extremely low solubility indicates that $Al(OH)_3$ is nearly insoluble in water, and the presence of Al^{3+} from $AlCl_3$ further suppresses its solubility.

Refinement and Considerations for Accurate Calculations

1. Inclusion of the Initial Al^{3+} Ion Concentration

If the molar solubility s is comparable to 0.0500 M, the approximation that $ [\text{Al}^{3+}] \approx 0.0500\, M $ would be invalid. To account for this, the quadratic (or higher degree) equations derived from the full equilibrium expression should be solved explicitly:

$$ K_{sp} = (0.0500 + s) \times (3s)^3 $$

which leads to a cubic in s:

$$ K_{sp} = (0.0500 + s) \times 27 s^3 $$

Rearranged:

$$ 27 s^3 (0.0500 + s) = K_{sp} $$

This cubic can be solved numerically for s, but given the extremely small value of K_{sp}, the approximation is generally valid.

2. Effect of Hydrolysis and Complex Formation

In real systems, aluminum ions can undergo hydrolysis, forming species such as Al(OH)^{2+}, Al(OH)_2^+, and others, depending on pH. These reactions can influence the effective solubility. Additionally, complexation with hydroxide ions can increase solubility if complexes such as Al(OH)_4^- form. These phenomena complicate the analysis but are beyond the scope of this simplified calculation.

Summary of Key Results and Conclusions

- The molar solubility of $Al(OH)_3$ in pure water

Frequently Asked Questions

What is the Ksp expression for $Al(OH)_3$?

The solubility product expression for $Al(OH)_3$ is $K_{sp} = [Al^{3+}][OH^-]^3$.

How does the presence of 0.0500 M $AlCl_3$ affect the solubility of $Al(OH)_3$?

Since $AlCl_3$ dissociates completely into Al^{3+} ions, the common Al^{3+} ions from $AlCl_3$ suppress the solubility of $Al(OH)_3$ via the common ion effect.

What is the initial concentration of Al^{3+} ions in the solution due to $AlCl_3$?

The initial concentration of Al^{3+} ions from $AlCl_3$ is 0.0500 M.

How do you set up the solubility equilibrium expression for $Al(OH)_3$ in this solution?

Let s be the molar solubility of $Al(OH)_3$. Then, $[Al^{3+}] = 0.0500 + s \approx 0.0500$ (since s is small), and $[OH^-] = 3s$. The Ksp expression becomes $K_{sp} = (0.0500 + s)(3s)^3$.

Given that Ksp is typically very small, how can we approximate the molar solubility in this scenario?

Because s is small compared to 0.0500 M, we approximate $[Al^{3+}]$ as 0.0500 M in the Ksp expression to simplify calculations.

What is the formula to determine the molar solubility of $Al(OH)_3$ in the presence of a common ion?

The molar solubility s can be calculated using the simplified relation: $K_{sp} = [Al^{3+}][OH^-]^3$, where $[Al^{3+}] \approx 0.0500$ M + s, and $[OH^-] = 3s$.

How can we solve for s, the molar solubility, using the given data?

Assuming $[Al^{3+}] \approx 0.0500$ M, the equation simplifies to $K_{sp} = (0.0500)(3s)^3$. Rearranged, s = [(Ksp) / (0.0500)]^(1/3) / 3.

Why is it important to consider the common ion effect when calculating solubility in this scenario?

The common ion effect reduces the solubility of $Al(OH)_3$ because the presence of Al^{3+} ions from $AlCl_3$ shifts the equilibrium toward the solid form, decreasing the amount that dissolves.

Additional Resources

Determine the Molar Solubility of $Al(OH)_3$ in a Solution Containing 0.0500 M $AlCl_3$ and Its Ksp

Understanding the solubility behavior of aluminum hydroxide ($Al(OH)_3$) in aqueous solutions is vital in fields ranging from environmental chemistry to industrial processes. The solubility product constant (Ksp) provides a quantitative measure of the solubility equilibrium, and when combined with the presence of common ions like Al^{3+} from $AlCl_3$, it influences the molar solubility significantly. This article offers a comprehensive analysis of how to determine the molar solubility of $Al(OH)_3$ in a solution containing 0.0500 M $AlCl_3$, considering the effects of the common ion and the associated equilibrium principles.

Understanding the Fundamentals: Solubility Equilibria and the Role of Ksp

What is Solubility?

Solubility refers to the maximum amount of a substance that can dissolve in a solvent at a specific temperature to form a saturated solution. For sparingly soluble salts like $Al(OH)_3$, this is typically very low, and their solubility is governed by an equilibrium process.

Solubility Product Constant (Ksp)

Ksp is the equilibrium constant specific to the dissolution of a salt:

$$\text{Al(OH)}_3 (s) \rightleftharpoons \text{Al}^{3+} (aq) + 3\,\text{OH}^- (aq)$$

$$
K_{sp} = [\text{Al}^{3+}][\text{OH}^-]^3
$$

This expression relates the concentrations of ions in a saturated solution at equilibrium and is temperature-dependent.

Impact of the Common Ion Effect

When a solution contains ions already common to the dissolution equilibrium—here, Al^{3+} from $AlCl_3$—the solubility of $Al(OH)_3$ decreases. This phenomenon, known as the "common ion effect," suppresses the dissolution of the sparingly soluble salt because the system shifts to favor the solid form to reduce ion concentrations.

Experimental Context: The Presence of 0.0500 M $AlCl_3$

What Does 0.0500 M $AlCl_3$ Imply?

$AlCl_3$ is a soluble salt that dissociates completely in aqueous solution:

$$\text{AlCl}_3 \rightarrow \text{Al}^{3+} + 3\,\text{Cl}^-$$

Given its high solubility, the solution contains 0.0500 M Al^{3+} ions initially, which directly influences the solubility of $Al(OH)_3$.

Implications for Solubility

The pre-existing Al^{3+} ions from $AlCl_3$ mean that the total Al^{3+} concentration in the solution is the sum of the ions from the dissociation of $AlCl_3$ and the Al^{3+} ions produced by the dissolution of $Al(OH)_3$. Since the initial Al^{3+} concentration is significant, the molar solubility of $Al(OH)_3$ will be less than its solubility in pure water.

Determining the Molar Solubility: Step-by-Step Approach

Step 1: Define the Variables

Let:

- s = molar solubility of $Al(OH)_3$ in mol/L (the quantity to be determined)

- $[\text{Al}^{3+}]_{total}$ = total aluminum ion concentration in solution at equilibrium
- $[\text{OH}^-]$ = hydroxide ion concentration at equilibrium

Step 2: Account for Initial and Equilibrium Concentrations

- Initial Al^{3+} from $AlCl_3$: 0.0500 M
- Additional Al^{3+} from $Al(OH)_3$ dissolution: s

Total Al^{3+} concentration at equilibrium:

$$[\text{Al}^{3+}] = 0.0500 + s$$

Similarly, the hydroxide ion concentration resulting from the dissolution:

$$[\text{OH}^-] = 3s$$

because each mole of $Al(OH)_3$ dissolves to produce three moles of OH^-.

Step 3: Write the Solubility Product Expression

The Ksp expression for $Al(OH)_3$:

$$K_{sp} = [\text{Al}^{3+}][\text{OH}^-]^3$$

Substituting the concentrations:

$$K_{sp} = (0.0500 + s) \times (3s)^3$$

Simplify:

$$K_{sp} = (0.0500 + s) \times 27s^3$$

Step 4: Approximate and Solve for s

Because $Al(OH)_3$ is sparingly soluble and the initial Al^{3+} concentration is relatively high (0.0500 M), the additional dissolution s will be small relative to 0.0500 M. Therefore, an approximation can be made:

$$0.0500 + s \approx 0.0500$$

which simplifies the equation to:

$$K_{sp} \approx 0.0500 \times 27s^3$$

Rearranged:

$$s^3 = \frac{K_{sp}}{0.0500 \times 27}$$

Once s is calculated, it provides the molar solubility of $Al(OH)_3$ in the presence of 0.0500 M $AlCl_3$.

Numerical Calculation: Applying the Known Ksp Value

Typical Ksp Value for $Al(OH)_3$

At 25°C, the solubility product constant for aluminum hydroxide is approximately:

$$ K_{sp} = 1.9 \times 10^{-33} $$

Calculating s

Plugging into the approximate formula:

$$ s^3 = \frac{1.9 \times 10^{-33}}{0.0500 \times 27} $$

Compute the denominator:

$$ 0.0500 \times 27 = 1.35 $$

Then:

$$ s^3 = \frac{1.9 \times 10^{-33}}{1.35} \approx 1.41 \times 10^{-33} $$

Taking cube root:

$$ s = \sqrt[3]{1.41 \times 10^{-33}} $$

Using calculator:

$$ s \approx 1.12 \times 10^{-11} \, \text{M} $$

Interpretation

The molar solubility of $Al(OH)_3$ in this solution is approximately 1.12×10^{-11} mol/L, indicating extremely low solubility, as expected for such a sparingly soluble salt.

Note on Approximation

Given the initial assumption that $s \ll 0.0500$, the approximation holds true. For more precise calculations, the quadratic or iterative methods can be employed, considering the actual addition of s to 0.0500 M.

Influence of the Common Ion Effect on Solubility

Quantitative Analysis

The presence of 0.0500 M Al^{3+} ions from $AlCl_3$ significantly suppresses the solubility of $Al(OH)_3$

compared to pure water. Without the common ion, the molar solubility would be higher, calculated solely based on Ksp:

$$ s_{pure} = \sqrt[3]{\frac{K_{sp}}{3^3}} $$

which would be:

$$ s_{pure} \approx \sqrt[3]{\frac{1.9 \times 10^{-33}}{27}} \approx 1.21 \times 10^{-11} \, \text{M} $$

This is very similar to the value with the common ion, demonstrating that the initial Al^{3+} concentration from $AlCl_3$ dominates the system.

Effect Summary

- The common ion (Al^{3+}) from $AlCl_3$ reduces the solubility of $Al(OH)_3$.
- The higher the initial Al^{3+} concentration, the lower the molar solubility.
- This effect is crucial in industrial processes where controlling solubility by adjusting ion concentrations can optimize product recovery or prevent unwanted precipitation.

Broader Implications and Applications

Environmental Chemistry

In natural waters, aluminum hydroxides often precipitate or dissolve depending on pH and ionic composition. Understanding the molar solubility helps predict aluminum mobility, potential toxicity, and the formation of mineral phases.

Industrial Processes

In water treatment, the removal of aluminum contaminants relies on controlling pH and ionic strength to precipitate aluminum hydroxides efficiently. Accurate solubility data ensures optimal dosing and minimal environmental impact.

Materials Science

Aluminum hydroxides are used as flame retardants and in the production of alumina. Knowledge of their solubility behavior influences manufacturing processes and material stability.

Conclusion

Determining the molar solubility of $Al(OH)_3$ in a solution containing 0.0500 M $AlCl_3$ involves a nuanced understanding

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