Determine The Wavelength Of The Line In The Hydrogen Atom Spectrum Corresponding To The N = 4 To N =

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Determine The Wavelength Of The Line In The Hydrogen Atom Spectrum Corresponding To The N = 4 To N = transition is a fundamental concept in atomic physics and spectroscopy. Understanding how to calculate the wavelength associated with electronic transitions in the hydrogen atom allows scientists to analyze spectral lines, identify elements in distant stars, and deepen our understanding of atomic structure. This article provides a comprehensive explanation of the process, including the necessary formulas, step-by-step calculations, and contextual insights into the hydrogen atom spectrum.

Understanding the Hydrogen Atom Spectrum

The Bohr Model and Energy Levels

The hydrogen atom spectrum arises from electrons transitioning between discrete energy levels. The Bohr model describes these levels as quantized orbits characterized by an integer quantum number, N. Each transition between energy levels results in the emission or absorption of light with a specific wavelength, creating a spectral line.

The Significance of Spectral Lines

- **Emission Lines:** When an electron drops from a higher energy level (e.g., N=4) to a lower one (e.g., N=3), it emits a photon with a wavelength corresponding to that transition.
- **Absorption Lines:** Conversely, when an electron absorbs a photon and jumps to a higher level, a dark line appears in the spectrum.

The Spectral Series in Hydrogen

Common Spectral Series

The hydrogen spectrum features several series, each corresponding to transitions ending at a specific energy level:

1. **Lyman Series:** N ≥ 2 to N=1 (ultraviolet region)
2. **Balmer Series:** N ≥ 3 to N=2 (visible region)
3. **Paschen Series:** N ≥ 4 to N=3 (infrared region)
4. **Brackett Series:** N ≥ 5 to N=4 (infrared region)
5. **Pfund Series:** N ≥ 6 to N=5 (infrared region)

Since the transition from N=4 to N=? falls into the Paschen series (N ≥ 4 to N=3), but your question specifies N=4 to N=?, it indicates the transition from N=4 to N=3, which is in the Paschen series.

Calculating the Wavelength of the N=4 to N=3 Transition

The Rydberg Formula

The wavelength (\(\lambda\)) of the spectral line corresponding to an electron transition in a hydrogen atom is given by the Rydberg formula:

$$
\frac{1}{\lambda} = R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)
$$

where:
- \(R \) is the Rydberg constant (\(R \approx 1.097 \times 10^7\, \text{m}^{-1} \))
- \(n_1 \) and \(n_2 \) are the principal quantum numbers of the lower and higher energy levels, respectively, with \(n_2 > n_1 \).

For the transition from N=4 to N=3:
- \(n_2 = 4 \)
- \(n_1 = 3 \)

Step-by-Step Calculation

1. Identify the quantum numbers:

    - Initial level: \(n_2 = 4 \)

- Final level: $n_1 = 3$

2. Apply the Rydberg formula:

$$
\frac{1}{\lambda} = R \left(\frac{1}{3^2} - \frac{1}{4^2} \right)
$$

$$
\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{9} - \frac{1}{16} \right)
$$

$$
\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{16 - 9}{144} \right)
$$

$$
\frac{1}{\lambda} = 1.097 \times 10^7 \times \frac{7}{144}
$$

$$
\frac{1}{\lambda} \approx 1.097 \times 10^7 \times 0.048611
$$

$$
\frac{1}{\lambda} \approx 533,236\, \text{m}^{-1}
$$

\item Calculate wavelength:

$$
\lambda = \frac{1}{533,236} \approx 1.876 \times 10^{-6}\, \text{m}
$$

or in nanometers:

$$
\lambda \approx 1876\, \text{nm}
$$

Interpreting the Results and Significance

Wavelength in the Infrared Region

The calculated wavelength (~1876 nm) falls within the infrared region of the electromagnetic spectrum. This aligns with the Paschen series, which involves transitions ending at N=3.

Implications for Spectroscopy

- Such calculations allow astronomers and physicists to identify hydrogen lines in stellar spectra.
- The precise measurement of these lines can reveal physical conditions in astrophysical objects, such as temperature and density.
- Understanding these transitions also supports quantum mechanics and atomic theory development.

Additional Transitions and Their Wavelengths

Other Notable Transitions in Hydrogen

To broaden your understanding, here are some other transitions and their approximate wavelengths:

Transition	Wavelength (nm)	Series	Notes
N=3 to N=2	656.3	Balmer	Visible (H-alpha line)
N=4 to N=2	486.1	Balmer	Blue-green
N=5 to N=2	434.0	Balmer	Violet
N=4 to N=3	1876	Paschen	Infrared (as calculated)
N=6 to N=3	1282	Paschen	Infrared

Conclusion

Determining the wavelength of spectral lines in the hydrogen atom involves understanding the quantized nature of electron energy levels, applying the Rydberg formula, and performing precise calculations. The transition from N=4 to N=3, which falls in the Paschen series, results in an infrared photon with an approximate wavelength of 1876 nm. Mastery of these calculations is essential for spectroscopy, astrophysics, and quantum physics, providing insights into the fundamental properties of matter and the universe.

Summary of Key Points

- The hydrogen spectrum is characterized by discrete spectral lines corresponding to electron transitions between energy levels.

- The Rydberg formula is used to calculate the wavelength of these lines.
- Transitions from N=4 to N=3 produce an infrared spectral line at approximately 1876 nm.
- Understanding these spectral lines aids in astrophysical research and validates quantum mechanical models.

By mastering the process of calculating spectral line wavelengths, scientists can decode the information carried by light from distant celestial objects, unlocking the secrets of the universe.

Frequently Asked Questions

What is the formula to determine the wavelength of a spectral line in the hydrogen atom for a transition from N=4 to a higher level?

The wavelength λ can be determined using the Rydberg formula:

$1/\lambda = R\ (1/n_1^2 - 1/n_2^2)$,

where R is the Rydberg constant (~$1.097 \times 10^7\ m^{-1}$), n_1 is the lower energy level (here 4), and n_2 is the higher energy level.

How do you calculate the wavelength for the transition from N=4 to N=5 in the hydrogen atom?

Using the Rydberg formula:

$1/\lambda = R\ (1/4^2 - 1/5^2) = R\ (1/16 - 1/25)$.

Plug in $R = 1.097 \times 10^7$ m^{-1} and solve for λ.

What is the wavelength of the spectral line resulting from the N=4 to N=3 transition in hydrogen?

Calculate using:

$1/\lambda = R (1/3^2 - 1/4^2) = R (1/9 - 1/16)$.

Substitute R and compute λ accordingly.

Why is the transition from N=4 significant in hydrogen's emission spectrum?

Transitions from N=4 produce visible spectral lines, notably the Balmer series, which are important for spectral analysis and understanding hydrogen's energy levels.

How does the wavelength change when hydrogen transitions from N=4 to higher levels like N=5 or N=6?

As the upper level increases, the energy difference decreases, leading to longer wavelengths (redshift). Therefore, N=4 to N=5 or N=6 transitions produce lines at longer wavelengths than N=4 to N=3.

What is the significance of the N=4 to N=2 transition in hydrogen's spectrum?

The N=4 to N=2 transition corresponds to the Balmer-alpha line (H-alpha), a prominent red line at approximately 656.3 nm, crucial in astronomy and spectroscopy.

Can the wavelength of the line from N=4 to N=1 be calculated using the Rydberg formula?

Yes, but since N=1 is the lowest energy level, the transition from N=4 to N=1 results in a very short wavelength in the ultraviolet or X-ray region, calculated similarly with N□=1 and N□=4.

What is the approximate wavelength of the spectral line for the N=4 to N=3 transition in the hydrogen atom?

Using the Rydberg formula:

$1/\square = R (1/3^2 - 1/4^2) = R (1/9 - 1/16)$.

Calculating gives □ approximately 434 nm, which is in the blue-violet region of the spectrum.

How does knowing the wavelength of the N=4 transition help in scientific research?

Determining this wavelength helps identify spectral lines in astronomical observations, analyze atomic structure, and test quantum theories related to atomic energy levels.

Additional Resources

Determine The Wavelength Of The Line In The Hydrogen Atom Spectrum Corresponding To The N = 4 To N = 3 Transition

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Introduction

The hydrogen atom, the simplest atom in the universe, has served as a cornerstone in the development of atomic physics and quantum mechanics. Its spectral lines—distinct wavelengths of light emitted or absorbed during electron transitions—provide critical insights into atomic structure and quantum behavior. Among these, the spectral line corresponding to an electron transitioning from the N = 4 energy level to the N = 3 level is of particular interest, as it lies within the visible spectrum and is part of the Balmer series.

This article aims to thoroughly analyze how to determine the wavelength associated with this specific transition. It explores the fundamental physics principles involved, the mathematical derivation using the Bohr model, and the implications of this calculation. By understanding this process, one gains deeper insight into atomic spectra and the quantum mechanics that govern microscopic phenomena.

Theoretical Background

Atomic Spectra and Electron Transitions

Atoms emit or absorb photons when electrons transition between discrete energy levels. These energy levels are quantized, meaning electrons can only occupy specific energies. When an electron drops from a higher energy level (say, N = 4) to a lower one (N = 3), a photon with energy equal to the difference between these levels is emitted. Conversely, absorption involves the electron moving from a lower to a higher energy state.

The spectral lines are characterized by their wavelengths or frequencies, which are directly related to the energy of the emitted or absorbed photon. Understanding these wavelengths not only helps identify elements but also tests the validity of quantum theories.

The Bohr Model of the Hydrogen Atom

Niels Bohr proposed a model where electrons orbit the nucleus in quantized orbits with specific energies. The energies of these orbits are given by:

$$
E_n = - \frac{13.6\, \text{eV}}{n^2}
$$

where:

- E_n is the energy of the level with principal quantum number n,
- 13.6 eV is the ionization energy of hydrogen.

The energy difference between two levels n_i and n_f (initial and final states) determines the photon emitted or absorbed:

$$
\Delta E = E_{n_f} - E_{n_i}
$$

This energy difference relates to the photon wavelength via Planck's equation.

Mathematical Derivation

Step 1: Calculating the Energy Difference

The energies for levels n are:

$$
E_n = - \frac{13.6\, \text{eV}}{n^2}
$$

For the transition from $N = 4$ to $N = 3$:

$$
\Delta E = E_3 - E_4 = - \frac{13.6}{3^2} - \left(- \frac{13.6}{4^2} \right)
$$

Calculating each term:

$$
E_3 = - \frac{13.6}{9} \approx -1.511\, \text{eV}
$$

$$
E_4 = - \frac{13.6}{16} = -0.85\, \text{eV}
$$

Thus,

$$
\Delta E = -1.511\, \text{eV} - (-0.85\, \text{eV}) = -1.511 + 0.85 = -0.661\, \text{eV}
$$

The negative sign indicates energy release; the photon carries away this energy magnitude:

$$
|\Delta E| = 0.661\, \text{eV}
$$

Step 2: Converting Energy to Wavelength

The relation between photon energy and wavelength is:

$$
E = h \nu = \frac{hc}{\lambda}
$$

where:

- h is Planck's constant $(6.626 \times 10^{-34}\, \text{Js})$,
- c is the speed of light $(3.00 \times 10^8\, \text{m/s})$,
- λ is the wavelength.

Rearranged:

$$
\lambda = \frac{hc}{E}
$$

Since E is in electronvolts, convert to joules:

$$
1\, \text{eV} = 1.602 \times 10^{-19}\, \text{J}
$$

Therefore,

$$
E = 0.661\, \text{eV} \times 1.602 \times 10^{-19} \approx 1.059 \times 10^{-19}\, \text{J}
$$

Now, compute λ:

\[

$$
\lambda = \frac{6.626 \times 10^{-34} \times 3.00 \times 10^{8}}{1.059 \times 10^{-19}} \approx \frac{1.9878 \times 10^{-25}}{1.059 \times 10^{-19}} \approx 1.877 \times 10^{-6}\, \text{m}
$$

Expressed in nanometers:

$$
\lambda \approx 1877\, \text{nm}
$$

Analysis of Results

Significance of the Wavelength

The calculated wavelength of approximately 1877 nm situates this spectral line in the infrared region of the electromagnetic spectrum, outside the visible range. This insight aligns with the known spectral series of hydrogen.

The Balmer Series Context

The Balmer series involves transitions where the final level is $n=2$, producing visible light. Transitions ending at $n=3$ (like from $n=4$ to $n=3$) produce spectral lines in the infrared region, often designated as part of the Paschen series. Therefore, the transition from $N=4$ to $N=3$ is characteristic of the Paschen series.

Validation Against Empirical Data

The theoretical calculation aligns with empirical spectral data. In practice, the spectral line corresponding to the $4 \to 3$ transition in hydrogen is observed around 1875 nm, matching the derived value closely. Such consistency underscores the Bohr model's effectiveness in explaining hydrogen's spectral lines, despite its limitations for more complex atoms.

Broader Implications and Advanced Considerations

Limitations of the Bohr Model

While the Bohr model accurately predicts hydrogen spectral lines, it neglects finer effects such as:

- Fine structure caused by relativistic effects,
- Hyperfine splitting due to nuclear spin,
- Quantum electrodynamics (QED) corrections.

Modern quantum mechanics and QED provide more accurate calculations, especially for more complex atoms.

Applications in Astrophysics and Spectroscopy

Knowledge of hydrogen spectral lines, including those from the $N=4 \to 3$ transition, is vital in astrophysics. For example:

- Stellar atmospheres analysis involves identifying spectral lines to determine composition and temperature.
- Interstellar medium studies utilize infrared lines for regions obscured by dust.
- Spectroscopic techniques rely on precise wavelength data for material analysis.

Technological Relevance

Understanding these spectral lines assists in developing:

- Laser technologies based on hydrogen transitions,
- Spectroscopic sensors for environmental monitoring,
- Calibration standards in optical instrumentation.

Conclusion

Determining the wavelength of the spectral line corresponding to the transition from $N=4$ to $N=3$ in the hydrogen atom exemplifies the intersection of quantum physics, mathematics, and spectroscopy. Through the application of the Bohr model, the calculation reveals a wavelength of approximately 1877 nanometers, placing this transition within the infrared spectrum and illustrating the diversity of hydrogen's spectral features.

This analysis underscores the importance of fundamental physics in understanding the universe's building blocks and demonstrates how classical models, despite their limitations, continue to provide valuable insights into atomic behavior. As spectroscopy techniques advance, precise measurements of such lines continue to refine our knowledge of atomic and molecular structures, enabling groundbreaking discoveries across scientific disciplines.

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