Determine Whether The Series Is Convergent Or Divergent. If It Is Convergent, Find Its Sum.1/3 + 2/9

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Understanding the behavior of infinite series is a fundamental aspect of calculus and mathematical analysis. Infinite series emerge naturally when summing an infinite sequence of numbers, such as geometric series, harmonic series, or other special series. Determining whether such a series converges (approaches a finite limit) or diverges (grows without bound or fails to settle to a limit) is crucial for many applications in mathematics, physics, engineering, and economics.

This article explores the process of analyzing the convergence or divergence of a specific series: 1/3 + 2/9. While this particular series appears simple, it provides an excellent opportunity to delve into the principles of series convergence, the methods used to analyze them, and how to compute their sums if they are convergent.

Understanding Infinite Series

An infinite series is the sum of infinitely many terms. It is written in the form:

\[
S = \sum_{n=1}^{\infty} a_n
\]

where \(a_n\) represents the nth term of the sequence.

Key concepts include:

- Partial sums: The sum of the first \(n\) terms:

\[
S_n = \sum_{k=1}^{n} a_k
\]

- Convergence: The series converges if the sequence of partial sums \(\{S_n\}\) approaches a finite limit as \(n \to \infty\). Formally,

$$
\lim_{n \to \infty} S_n = S < \infty
$$

- Divergence: The series diverges if the limit does not exist or is infinite.

Analyzing the Series: 1/3 + 2/9

The series in question is:

$$
\frac{1}{3} + \frac{2}{9}
$$

At first glance, this appears to be a finite sum of just two terms, rather than an infinite series. However, to evaluate the convergence or divergence of an entire series, it is essential to understand the structure of the series.

Is this a finite series or an infinite series?
If it's a finite sum, then convergence is trivial, and the sum is simply the addition of these terms. But if the series continues beyond these two terms, we need to analyze its pattern.

Clarifying the Series Structure

Suppose the series is constructed as:

$$
\sum_{n=1}^{\infty} a_n
$$

where the terms are:

$$
a_1 = \frac{1}{3}, \quad a_2 = \frac{2}{9}
$$

and perhaps the subsequent terms follow a pattern. For example, maybe the series continues with:

$$
a_3 = \frac{3}{27} = \frac{1}{9}, \quad a_4 = \frac{4}{81}

\]

and so on.

Alternatively, the series might be a geometric series with initial term a and common ratio r.

Identifying the Series Type

Given the terms:

$$
a_1 = \frac{1}{3}, \quad a_2 = \frac{2}{9}
$$

we observe that:

$$
a_2 = \frac{2}{9} = 2 \times \frac{1}{9}
$$

and $a_1 = \frac{1}{3}$.

If the pattern continues as:

$$
a_n = \frac{n}{3^n}
$$

then the series becomes:

$$
\sum_{n=1}^{\infty} \frac{n}{3^n}
$$

which is a well-known type of power series whose convergence and sum can be computed.

Alternatively, if the series is just the sum of the two terms, then it is finite and trivially convergent with sum $1/3 + 2/9 = 3/9 + 2/9 = 5/9$.

For the purpose of this article, we will assume that the series is:

$$
\sum_{n=1}^{\infty} \frac{n}{3^n}
$$

which generalizes the given initial terms and allows for a comprehensive analysis of convergence and sum calculation.

Convergence of the Series \(\sum_{n=1}^{\infty} \frac{n}{3^n}\)

This series is a power series, specifically a series involving \(n\) multiplied by a geometric factor.

Key points:

- The series is convergent because the terms tend to zero as \(n \to \infty\).
- To confirm convergence, we can apply the Root Test or Ratio Test.

Applying the Ratio Test

The Ratio Test states that for a series \(\sum a_n\):

- If

\[
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right| < 1
\]

then the series converges absolutely.

For \(a_n = \frac{n}{3^n}\):

\[
\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{3^{n+1}}}{\frac{n}{3^n}} = \frac{n+1}{n} \times \frac{1}{3}
\]

Simplify:

\[
\frac{a_{n+1}}{a_n} = \left(1 + \frac{1}{n}\right) \times \frac{1}{3}
\]

Taking the limit as \(n \to \infty\):

\[
L = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right) \times \frac{1}{3} = 1
\times \frac{1}{3} = \frac{1}{3}
\]

Since \(\frac{1}{3} < 1\), the series converges absolutely.

Conclusion: The series \(\sum_{n=1}^{\infty} \frac{n}{3^n}\) converges.

Calculating the Sum of the Series \(\sum_{n=1}^\infty \frac{n}{3^n}\)

Knowing that the series converges, the next step is to determine its sum.

Method 1: Using Known Power Series Formulae

There is a standard power series:

\[
\sum_{n=1}^\infty n x^{n} = \frac{x}{(1 - x)^2}, \quad \text{for} \quad |x| <
1
\]

In our case, \(x = \frac{1}{3}\), which satisfies \(|x| < 1\).

Thus,

\[
\sum_{n=1}^\infty n \left(\frac{1}{3}\right)^n = \frac{\frac{1}{3}}{(1 -
\frac{1}{3})^2}
\]

Calculate denominator:

\[
1 - \frac{1}{3} = \frac{2}{3}
\]

Square it:

\[
\left(\frac{2}{3}\right)^2 = \frac{4}{9}

\]

Calculate numerator:

\[
\frac{1}{3}
\]

Putting it all together:

\[
\text{Sum} = \frac{\frac{1}{3}}{\frac{4}{9}} = \frac{1}{3} \times \frac{9}{4} = \frac{9}{12} = \frac{3}{4}
\]

Result:

\[
\boxed{
\sum_{n=1}^\infty \frac{n}{3^n} = \frac{3}{4}
}
\]

Summary and Final Remarks

- The series \(\sum_{n=1}^\infty \frac{n}{3^n}\) converges because the ratio of successive terms approaches \(\frac{1}{3}\), which is less than 1.
- The convergence can be confirmed by the Ratio Test.
- The sum of the series is \(\frac{3}{4}\), calculated using the power series sum formula.

Additional Context: Why Is This Important?

Understanding the convergence and sum of series like \(\sum_{n=1}^\infty \frac{n}{3^n}\) is fundamental in various scientific fields:

- Mathematics: Series are used to define functions, approximate solutions, and analyze behaviors.
- Physics: Series help in modeling phenomena like oscillations, wave behavior, and quantum mechanics.
- Engineering: Signal processing, control systems, and electrical engineering often involve series sums.

- Economics: Series are used to calculate present and future values in financial models.

Practical Applications of Series Convergence

- Series in Financial Calculations: Present value calculations often involve geometric

Frequently Asked Questions

Is the series 1/3 + 2/9 convergent or divergent?

The series is convergent because it is a geometric series with ratio r = 2/3, which has an absolute value less than 1.

How do we determine if the series 1/3 + 2/9 converges?

By recognizing it as a geometric series and checking if the common ratio's absolute value is less than 1, which indicates convergence.

What is the common ratio of the series 1/3 + 2/9?

The common ratio is (2/9) ÷ (1/3) = (2/9) (3/1) = 2/3.

Can we find the sum of the series 1/3 + 2/9?

Yes, since it's a geometric series with |r| < 1, the sum is given by a / (1 - r), where a = 1/3 and r = 2/3.

What is the sum of the series 1/3 + 2/9?

The sum is (1/3) / (1 - 2/3) = (1/3) / (1/3) = 1.

Are there any other terms in the series beyond 2/9?

Based on the given terms, the series appears to be only two terms, but if extended as an infinite geometric series, the sum converges to 1.

How do you verify if a geometric series converges?

Check if the absolute value of the common ratio |r| < 1; if so, the series converges, and the sum can be calculated using the formula a / (1 - r).

What is the significance of the sum being 1 for this series?

It indicates that the infinite sum of the geometric series approaches 1, meaning the total accumulated value converges to 1.

Is the series 1/3 + 2/9 an example of a finite or infinite series?

If only these two terms are considered, it is a finite series; if extended infinitely with the same ratio, it becomes an infinite convergent series with sum 1.

Additional Resources

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Introduction

Mathematics is often regarded as the language of patterns, structure, and logical reasoning. Among its many fascinating branches, infinite series stand out as a fundamental concept that bridges the finite with the infinite, opening doors to understanding complex phenomena in mathematics, physics, economics, and engineering. When analyzing an infinite series, the crucial question is: Does the series converge or diverge? If it converges, what is its sum? These questions are not only theoretical; they have practical implications in fields such as signal processing, quantum mechanics, and financial modeling.

In this article, we undertake a detailed exploration of the series:

\[
\frac{1}{3} + \frac{2}{9}
\]

and its potential extension into an infinite series. We aim to determine whether this series converges or diverges, and if it converges, to compute its sum. This examination involves a systematic approach, including identifying the series type, applying convergence tests, and performing precise calculations. Through this analysis, we will illustrate key concepts in series convergence and demonstrate practical methods for problem-solving in mathematical analysis.

Understanding the Series: Initial Observations

Before delving into the convergence tests, it is essential to understand the structure of the series in question. The series:

$$
\frac{1}{3} + \frac{2}{9}
$$

appears to be a partial sum of an infinite series if we consider extending it beyond these two terms. To analyze whether the entire series converges, we need to understand the pattern of its terms.

Recognizing the Pattern

Let's examine the given terms:

- First term: $a_1 = \frac{1}{3}$
- Second term: $a_2 = \frac{2}{9}$

Notice that:

$$
a_2 = \frac{2}{9} = \frac{2}{3^2}
$$

Similarly, the first term is:

$$
a_1 = \frac{1}{3} = \frac{1}{3^1}
$$

This suggests that the series might be a geometric series with a certain pattern in the numerator and denominator.

Hypothesizing the General Term

Suppose the n-th term of the series is:

$$
a_n = \frac{n}{3^n}
$$

Let's verify whether this pattern matches the given terms:

- For $n=1$:

$$
a_1 = \frac{1}{3^1} = \frac{1}{3} \quad \checkmark
$$

\]

- For $n=2$:

$$
a_2 = \frac{2}{3^2} = \frac{2}{9} \quad \checkmark
$$

This confirms that the series:

$$
\sum_{n=1}^\infty \frac{n}{3^n}
$$

is consistent with the given partial sum.

Formal Definition of the Series

Based on the above, the infinite series under consideration is:

$$
\boxed{
S = \sum_{n=1}^\infty \frac{n}{3^n}
}
$$

Our goal now is to determine whether this series converges, and if so, to find its sum.

Convergence Analysis

1. Recognizing the Series Type

The series:

$$
S = \sum_{n=1}^\infty \frac{n}{3^n}
$$

is a power series with terms involving n and a common ratio $r = \frac{1}{3}$. It belongs to the class of series known as arithmetic-geometric series, where each term involves a linear factor multiplied by a geometric component.

2. Applying Convergence Tests

The primary test for convergence of a series with positive terms like this is the Ratio Test.

Ratio Test:

Given a series $\sum a_n$, compute:

$$
L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|
$$

- If $L < 1$, the series converges.
- If $L > 1$, the series diverges.
- If $L = 1$, the test is inconclusive.

Let's compute this for our series:

$$
a_n = \frac{n}{3^n}
$$

then

$$
a_{n+1} = \frac{n+1}{3^{n+1}} = \frac{n+1}{3 \cdot 3^{n}}
$$

Now,

$$
\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{3 \cdot 3^{n}}}{\frac{n}{3^{n}}} = \frac{n+1}{3} \times \frac{3^{n}}{n} \times \frac{1}{3^{n}} = \frac{n+1}{3} \times \frac{1}{n} = \frac{n+1}{3n}
$$

Therefore,

$$
L = \lim_{n \to \infty} \frac{n+1}{3n} = \frac{1}{3} \lim_{n \to \infty} \left(1 + \frac{1}{n} \right) = \frac{1}{3} \times 1 = \frac{1}{3}
$$

Since $L = \frac{1}{3} < 1$, the series converges by the Ratio Test.

3. Conclusion on Convergence

Because the limit is less than 1, the series converges absolutely. This means the sum of the infinite series exists and is finite.

Finding the Sum of the Series

Having established convergence, the next step is to compute the sum:

$$
S = \sum_{n=1}^\infty \frac{n}{3^n}
$$

1. Recognizing the Series Sum Formula

There is a well-known result for the sum of a series involving n and a geometric component:

$$
\sum_{n=1}^\infty n r^n = \frac{r}{(1 - r)^2}, \quad \text{for } |r| < 1
$$

This formula can be derived using differentiation of the geometric series sum, but it is a standard result.

2. Applying the Formula

In our case:

$$
r = \frac{1}{3}
$$

Thus,

$$
S = \sum_{n=1}^\infty n \left(\frac{1}{3} \right)^n = \frac{\frac{1}{3}}{(1 - \frac{1}{3})^2}
$$

Calculate the denominator:

$$
1 - \frac{1}{3} = \frac{2}{3}
$$

Therefore,

$$
S = \frac{\frac{1}{3}}{\left(\frac{2}{3} \right)^2} = \frac{\frac{1}{3}}{\frac{4}{9}} = \frac{1}{3} \times \frac{9}{4} = \frac{9}{12} = \frac{3}{4}
$$

3. Final Result

The sum of the infinite series is:

$$
\boxed{
\boxed{\frac{3}{4}}
}
$$

which is approximately 0.75.

Summary and Implications

Key Takeaways:

- The series in question is:

$$
\sum_{n=1}^\infty \frac{n}{3^n}
$$

- It converges absolutely because the ratio test yields a limit less than 1.
- Its sum can be computed using the standard formula for the sum of the series involving $n r^n$:

$$
\sum_{n=1}^\infty n r^n = \frac{r}{(1 - r)^2}
$$

- Substituting $r = \frac{1}{3}$, we find the sum:

$$
S = \frac{3}{4}
$$

Practical Significance:

Understanding the convergence and sum of such series is crucial in various applications:

- Financial mathematics: calculating the present value of an infinite series of payments.
- Physics: modeling decay processes or infinite sums in quantum mechanics.
- Computer science: analyzing algorithms with geometric or arithmetic-geometric complexity.

Additional Considerations

While the specific series analyzed here involves a linear numerator and a geometric denominator, the same principles extend to more complex series. For example, series with factorial terms, alternating signs, or more intricate patterns require tailored tests such as the Comparison Test, Integral Test, or Alternating Series Test. The core idea remains: determine the nature of the terms, apply an appropriate convergence test, and then compute the sum if the series converges.

Conclusion

Determining whether an infinite series converges or diverges is a fundamental skill in mathematical analysis. Through careful examination of the pattern of terms, application of the Ratio Test, and utilization of known sum formulas, we established that the series:

\[

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