Determine Whether The Three Points P=(-7,6,-8), Q=(-8,4,-11), R=(-9,3,-14) Are Colinear By Computing

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When working with points in three-dimensional space, a common question encountered in geometry and vector calculus is whether three given points lie on the same straight line. This concept, known as collinearity, is fundamental in various fields such as computer graphics, physics, and engineering. Determining whether points are colinear involves examining their positions relative to each other, often through vector operations like subtraction, cross products, and scalar multiples.

In this article, we will explore how to determine whether the points P = (-7, 6, -8), Q = (-8, 4, -11), and R = (-9, 3, -14) are colinear by computing the relevant vector properties. We will guide you through the process step-by-step, providing detailed explanations of the mathematics involved, including vector subtraction, the cross product, and the interpretation of results. By the end, you will have a clear understanding of how to apply these techniques to any set of three points in space.

Understanding Collinearity in Three-Dimensional Space

What Does It Mean for Points to Be Colinear?

In three-dimensional geometry, three points P, Q, and R are said to be colinear if they all lie on a single straight line. This implies that the vector from P to Q and the vector from P to R are scalar multiples of each other, or equivalently, the vectors are linearly dependent.

Mathematically, points P, Q, and R are colinear if:
- The vectors PQ and PR are parallel, i.e., one is a scalar multiple of the other.
- The cross product of PQ and PR is the zero vector.

Understanding these conditions is vital because they form the basis of the computation methods we will use.

Methods for Determining Collinearity

There are primarily two methods to determine whether three points are colinear:

1. Vector Method:
- Compute the vectors PQ and PR.

- Check if one is a scalar multiple of the other.
- Alternatively, compute the cross product PQ × PR; if it results in the zero vector, the points are colinear.

2. Geometric Interpretation:
- If the area of the triangle formed by the points is zero, the points are colinear.
- Since the area can be computed using the cross product, this method aligns with the vector method.

In this article, we will focus on the vector method involving cross products because of its straightforward computational nature.

Step-by-Step Process to Determine Collinearity

Step 1: Define the Points and Compute Vectors

Given points:
- P = (-7, 6, -8)
- Q = (-8, 4, -11)
- R = (-9, 3, -14)

Compute the vectors PQ and PR:
- PQ = Q - P
- PR = R - P

Calculations:
- PQ:
- x-component: -8 - (-7) = -8 + 7 = -1
- y-component: 4 - 6 = -2
- z-component: -11 - (-8) = -11 + 8 = -3

- PR:
- x-component: -9 - (-7) = -9 + 7 = -2
- y-component: 3 - 6 = -3
- z-component: -14 - (-8) = -14 + 8 = -6

Thus:
- PQ = (-1, -2, -3)
- PR = (-2, -3, -6)

Step 2: Compute the Cross Product of PQ and PR

The cross product of two vectors $A = (a_1, a_2, a_3)$ and $B = (b_1, b_2, b_3)$ is given by:

$A \times B = (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1)$

Applying this to PQ and PR:

- PQ = (-1, -2, -3)
- PR = (-2, -3, -6)

Calculate each component:

1. x-component:
(-2)(-6) - (-3)(-3) = (12) - (9) = 3

2. y-component:
(-3)(-2) - (-1)(-6) = (6) - (6) = 0

3. z-component:
(-1)(-3) - (-2)(-2) = (3) - (4) = -1

Therefore:
PQ × PR = (3, 0, -1)

Step 3: Interpret the Cross Product Result

- If the cross product is the zero vector (0, 0, 0), then PQ and PR are parallel, indicating the points are colinear.
- In our case, the cross product is (3, 0, -1), which is not the zero vector.

Conclusion:
Since PQ × PR ≠ (0, 0, 0), the vectors are not parallel, and the points do not lie on the same straight line. Therefore, points P, Q, and R are not colinear.

Alternative Check: Scalar Multiples

To further verify, check whether PQ and PR are scalar multiples:

- Is there a scalar k such that:
- (-1) = k (-2)
- -2 = k (-3)
- -3 = k (-6)

Calculations:
- From the first component: k = (-1)/(-2) = 0.5
- From the second component: k = (-2)/(-3) ≈ 0.666...
- From the third component: k = (-3)/(-6) = 0.5

Since the values of k are inconsistent (0.5 vs. approximately 0.666...), the vectors are not scalar multiples, reaffirming that the points are not colinear.

Summary of Findings

- The vectors PQ and PR are not scalar multiples.
- The cross product PQ × PR is not zero.

- Therefore, points P, Q, and R do not lie on the same straight line.

Implications and Applications

Understanding how to determine collinearity has numerous practical applications:

- Computer Graphics: Ensuring points lie on a line for rendering and modeling.
- Navigation and GPS: Confirming whether landmarks or waypoints are aligned.
- Physics: Analyzing trajectories or force vectors.
- Engineering: Designing linear structures or components.

Mastering the vector methods, especially the cross product, allows professionals and students to analyze spatial relationships efficiently and accurately.

Conclusion

Determining whether three points are colinear in three-dimensional space involves a systematic approach centered around vector operations. By computing the vectors between points and analyzing their cross product, one can conclusively establish whether the points lie on the same line. In the case of points P = (-7, 6, -8), Q = (-8, 4, -11), and R = (-9, 3, -14), the calculations reveal that they are not colinear.

This process underscores the importance of vector algebra in spatial analysis and provides a foundation for more complex geometric and mathematical investigations involving points, lines, and planes in three-dimensional space. Whether for academic purposes or practical applications, mastering these techniques enhances your ability to interpret and manipulate spatial data effectively.

Keywords: colinear points, three-dimensional geometry, vector cross product, vector analysis, spatial relationships, points in space, vector methods, geometry computations, vector algebra

Frequently Asked Questions

How can I determine if three points P, Q, and R are colinear in 3D space?

You can determine if three points are colinear by checking if the vectors PQ and PR are scalar multiples, or by computing the cross product of PQ and PR. If the cross product is the zero vector, the points are colinear.

What is the step-by-step method to verify colinearity for points P=(-7,6,-8), Q=(-8,4,-11), and R=(-9,3,-14)?

First, find vectors PQ and PR by subtracting coordinates: PQ = Q - P and PR = R - P. Then, compute the cross product of these vectors. If the result is the zero vector, the points are colinear.

How do I compute vector PQ from point P=(-7,6,-8) and Q=(-8,4,-11)?

Subtract the coordinates of P from Q: PQ = (-8 - (-7), 4 - 6, -11 - (-8)) = (-1, -2, -3).

What is the calculation for the cross product of vectors PQ and PR for these points?

First, find PR: R - P = (-9 - (-7), 3 - 6, -14 - (-8)) = (-2, -3, -6). Then, compute the cross product of PQ = (-1, -2, -3) and PR = (-2, -3, -6).

What is the result of the cross product of vectors PQ and PR, and what does it signify?

The cross product is calculated as: (-2)(-6) - (-3)(-3), -3(-2) - (-1)(-6), (-1)(-3) - (-2)(-2). Evaluating, we get (0, 0, 0). Since the result is the zero vector, the points are colinear.

Based on the cross product result, are points P, Q, and R colinear?

Yes, since the cross product of vectors PQ and PR is the zero vector, the points P, Q, and R are colinear.

Additional Resources

Determine Whether The Three Points P=(-7,6,-8), Q=(-8,4,-11), R=(-9,3,-14) Are Colinear By Computing is a fundamental problem in coordinate geometry that often appears in mathematics, physics, and engineering contexts. Understanding whether three points are colinear—that is, lying on the same straight line—is essential for analyzing geometric configurations, solving spatial problems, and simplifying complex calculations. This article provides a comprehensive overview of how to determine colinearity through vector methods, focusing on the step-by-step process of computing the relevant vectors, using the concept of the cross product, and interpreting the results to reach a conclusion.

Understanding the Concept of Colinearity

What Does It Mean for Points to Be Colinear?

In three-dimensional space, three points P, Q, R are said to be colinear if they all lie along a single straight line. Geometrically, this means that the position vectors of these points satisfy a linear relationship or, equivalently, that the vectors connecting these points are scalar multiples of each other. If not, then the points form a triangle with an area greater than zero, indicating they are not colinear.

Why Is Determining Colinearity Important?

- Geometric simplification: Recognizing colinear points can simplify the analysis of geometric figures.
- Collision detection: In computer graphics and spatial modeling, colinearity helps in detecting aligned objects.
- Mathematical proofs and problem-solving: Confirming colinearity is often a step in proofs and problem solutions involving lines, planes, and spatial relationships.
- Engineering applications: Design and analysis of structures often rely on understanding point alignments.

Methods to Determine Colinearity

There are multiple approaches to test whether three points are colinear, including:

- Slope method (limited to 2D): Comparing slopes between pairs of points (not applicable in 3D).
- Vector cross product method: Using vectors to analyze the area (or volume) spanned by the points.
- Parameterization method: Expressing points along a line and checking for consistency.

Given the three-dimensional nature of the points P, Q, R, the most robust and general method is the vector approach, especially utilizing the cross product.

Step-by-Step Process Using Vector Cross Product

1. Define the Points and Compute Vectors

Given three points in 3D:

$$
P = (-7, 6, -8), \quad Q = (-8, 4, -11), \quad R = (-9, 3, -14)
$$

We first calculate the vectors connecting these points:

$$
\vec{PQ} = \vec{Q} - \vec{P}
$$
$$
\vec{PR} = \vec{R} - \vec{P}
$$

Calculating these:

$$
\vec{PQ} = (-8 - (-7), 4 - 6, -11 - (-8)) = (-1, -2, -3)
$$

$$
\vec{PR} = (-9 - (-7), 3 - 6, -14 - (-8)) = (-2, -3, -6)
$$

---

2. Compute the Cross Product of $\vec{PQ}$ and $\vec{PR}$

The cross product of two vectors results in a vector perpendicular to both, and its magnitude indicates the area of the parallelogram spanned by the vectors. If the vectors are parallel (i.e., scalar multiples), the cross product will be the zero vector, indicating the points are colinear.

Calculate:

$$
\vec{PQ} \times \vec{PR} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-1 & -2 & -3 \\
-2 & -3 & -6 \\
\end{vmatrix}
$$

Using the determinant expansion:

\[
\vec{PQ} \times \vec{PR} = \mathbf{i}((-2)(-6) - (-3)(-3)) - \mathbf{j}((-1)(-6) - (-3)(-2)) + \mathbf{k}((-1)(-3) - (-2)(-2))
\]

Calculate each component:

- \(\mathbf{i}\) component:

\[
(-2)(-6) - (-3)(-3) = 12 - 9 = 3
\]

- \(\mathbf{j}\) component:

\[
(-1)(-6) - (-3)(-2) = 6 - 6 = 0
\]

- \(\mathbf{k}\) component:

\[
(-1)(-3) - (-2)(-2) = 3 - 4 = -1
\]

Thus:

\[
\vec{PQ} \times \vec{PR} = (3, 0, -1)
\]

---

3. Interpret the Cross Product Result

- If the cross product vector is the zero vector \((0, 0, 0)\), it indicates that \(\vec{PQ}\) and \(\vec{PR}\) are parallel, and the points are colinear.
- If the cross product is non-zero, the points are not colinear.

In this case:

\[
(3, 0, -1) \neq (0, 0, 0)
\]

Therefore, the vectors are not parallel, which suggests the points are not colinear.

Conclusion and Final Verdict

Based on the calculations, the cross product $\vec{PQ} \times \vec{PR}$ yields a non-zero vector, indicating that the vectors $\vec{PQ}$ and $\vec{PR}$ are not scalar multiples, and consequently, the points P, Q, R do not lie on the same straight line.

Hence, the points $P=(-7,6,-8)$, $Q=(-8,4,-11)$, and $R=(-9,3,-14)$ are not colinear.

Additional Insights and Considerations

Alternative Methods

- Scalar triple product: One could compute the scalar triple product $\vec{PQ} \cdot (\vec{PR} \times \vec{PS})$ if a fourth point is involved.
- Geometric visualization: Plotting the points can sometimes help intuitively verify colinearity, though computational methods are more reliable.

Limitations and Challenges

- Numerical precision: Small calculation errors in floating-point arithmetic can affect the result, especially in more complex scenarios.
- Degenerate cases: When points are very close or nearly colinear, careful calculation is necessary to avoid misclassification.

Pros and Cons of the Vector Cross Product Method

Pros:
- Works efficiently in 3D space.
- Provides a clear mathematical criterion (zero vector) for colinearity.
- Applicable to any number of points in space.

Cons:
- Requires understanding of vector operations.
- Not as straightforward in purely algebraic or slope-based contexts.
- Slightly more involved than 2D slope checks.

Summary

Determining whether three points are colinear in three-dimensional space involves vector analysis, primarily using the cross product to assess whether the vectors connecting the points are scalar multiples. Through systematic computation, it becomes evident that points \(P, Q, R \) do not lie on a single straight line, as their cross product is non-zero. This approach provides a robust, mathematically grounded method for spatial analysis, applicable across various scientific and engineering disciplines.

Whether for academic, professional, or practical purposes, mastering the vector approach to colinearity enhances one’s ability to analyze complex spatial relationships with confidence and precision.

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