Evaluate The Double Integral To The Given Region R. Sxy E Xy DA; R = {(x,y)| 8 X 15, 0 Y 1} R Ssxy E

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Understanding how to evaluate double integrals over specific regions is a fundamental skill in calculus. This process involves integrating a function of two variables over a defined area in the xy-plane. In this article, we will explore the step-by-step approach to evaluating the double integral of the function $Sxy = xy$ over the region $R = \{(x, y) \mid 8 \leq x \leq 15,\, 0 \leq y \leq 1 \}$. By the end of this guide, you will gain a clear understanding of the methods involved, including setting up the integral, choosing the order of integration, and performing the calculations accurately.

Understanding the Region R and the Function

Before diving into the integration process, it’s essential to understand the region R and the function $Sxy = xy$.

The Region R

- The region R is defined as:

$$
R = \{(x, y) \mid 8 \leq x \leq 15,\, 0 \leq y \leq 1 \}
$$

- This describes a rectangular area in the xy-plane with:
- Horizontal bounds from $x = 8$ to $x = 15$
- Vertical bounds from $y = 0$ to $y = 1$

The Function Sxy = xy

- The integrand xy is a simple polynomial function that multiplies the x and y coordinates.
- This function is continuous over the entire region R, making it suitable for standard double integration techniques.

Setting Up the Double Integral

To evaluate the double integral over the region R, we need to set up the integral in terms of the limits for x and y.

Choosing the Order of Integration

- Since the region R is a rectangle, either order of integration—integrating with respect to y first or x first—is straightforward.
- For clarity, we will perform the integration with respect to y first, then x.

Formulating the Integral

- The double integral over R:

$$
\iint_{R} xy \, dA
$$

- When integrating with respect to y first:

$$
\int_{x=8}^{15} \left(\int_{y=0}^{1} xy \, dy \right) dx
$$

- Alternatively, integrating with respect to x first:

$$
\int_{y=0}^{1} \left(\int_{x=8}^{15} xy \, dx \right) dy
$$

- We will proceed with the first approach: integrating y first.

Calculating the Inner Integral

The inner integral involves integrating xy with respect to y over $0 \leq y \leq 1$.

Inner Integral Calculation

$$
\int_{0}^{1} xy \, dy
$$

- Since x is treated as a constant with respect to y:

$$
x \int_{0}^{1} y \, dy
$$

- Computing the integral:

$$
x \left[\frac{y^2}{2} \right]_0^1 = x \left(\frac{1^2}{2} - 0 \right) = x \times \frac{1}{2} = \frac{x}{2}
$$

Evaluating the Outer Integral

Now, substitute the result of the inner integral into the outer integral:

$$
\int_{8}^{15} \frac{x}{2} \, dx
$$

Outer Integral Calculation

- Factor out $\frac{1}{2}$:
$$
\frac{1}{2} \int_{8}^{15} x \, dx
$$
- Compute the integral:
$$
\frac{1}{2} \left[\frac{x^2}{2} \right]_8^{15} = \frac{1}{2} \left(\frac{15^2}{2} - \frac{8^2}{2} \right)
$$
- Simplify:
$$
\frac{1}{2} \left(\frac{225}{2} - \frac{64}{2} \right) = \frac{1}{2} \times \frac{225 - 64}{2} = \frac{1}{2} \times \frac{161}{2} = \frac{161}{4}
$$

Final Result of the Double Integral

The value of the double integral over the region R is:

$$
\boxed{\frac{161}{4}}
$$

which is equivalent to 40.25 in decimal form.

Summary of Steps for Evaluating Double Integrals Over Rectangular Regions

- Identify the region R: Determine the bounds for x and y.
- Choose the order of integration: Decide whether to integrate with respect to y or x first based on simplicity.
- Set up the integral: Write the iterated integral with proper limits.
- Perform the inner integral: Integrate the function treating the outer variable as a constant.
- Perform the outer integral: Integrate the resulting expression with respect to the outer variable.
- Simplify and interpret the result.

Additional Tips for Evaluating Double Integrals

- Always carefully determine the bounds based on the description of the region.
- When the region is a rectangle, the limits are constants; for more complex regions, consider rewriting the bounds in terms of inequalities.
- Use substitution or recognizing standard integrals to simplify calculations.
- Double-check your limits and integrand before performing calculations to avoid errors.

Applications of Double Integrals in Real-World Problems

Double integrals are vital in various fields such as physics, engineering, and economics. Some common applications include:

- Calculating the area of a region.
- Determining the mass of a lamina with variable density.
- Computing the center of mass or center of gravity.
- Finding the electric charge or heat content over a region.
- Evaluating probability distributions in two dimensions.

Example Applications

- Physics: Calculating the total mass of a rectangular plate with density xy.
- Economics: Determining the total profit over a specific region where profit density varies with x and y.
- Engineering: Analyzing heat distribution over a surface.

Conclusion

Evaluating double integrals over a region like $R = \{(x, y) \mid 8 \leq x \leq 15,\, 0 \leq y \leq 1 \}$ involves understanding the geometric bounds, setting up the integral properly, and performing the integration step-by-step. In this case, integrating the function xy over the specified rectangle yields a result of $\frac{161}{4}$, or 40.25. Mastery of these techniques enables solving complex problems involving areas, masses, and other quantities dependent on two variables, making double integrals a powerful tool in calculus and applied mathematics.

Frequently Asked Questions

How do I set up the double integral for the region R where $8 \leq x \leq 15$ and $0 \leq y \leq 1$ when evaluating $\int\int_R xy\, dA$?

Since the region R is described by $8 \leq x \leq 15$ and $0 \leq y \leq 1$, the double integral can be set up as $\int$

from x=8 to 15 and y=0 to 1 of xy dy dx, that is, $\int_8^{15} \int_0^1 xy \, dy \, dx$.

What is the step-by-step process to evaluate the double integral $\int_8^{15} \int_0^1 xy \, dy \, dx$?

First, integrate with respect to y: $\int_0^1 xy \, dy = x \int_0^1 y \, dy = x \, (1/2) \, y^2 |_0^1 = x \, (1/2)$. Then, integrate with respect to x: $\int_8^{15} (x \, 1/2) \, dx = (1/2) \int_8^{15} x \, dx = (1/2)(1/2) \, x^2 |_8^{15} = (1/4)(225 - 64) = (1/4) \, 161 = 40.25$.

What is the value of the double integral ∫∫_R xy dA over the region R = { (x,y) | 8 ≤ x ≤ 15, 0 ≤ y ≤ 1 }?

The value of the double integral is 40.25.

Why is it valid to integrate xy with respect to y first and then x over the given rectangular region?

Because the region R is a rectangle with constant bounds, the order of integration can be interchanged without affecting the result, making it valid to integrate with respect to y first, then x.

Can the same result be obtained by changing the order of integration in this problem?

Yes, since the region is rectangular, changing the order of integration to integrate with respect to x first and then y will give the same result, though the setup will differ accordingly.

Additional Resources

Comprehensive Evaluation of the Double Integral over Region R: An In-Depth Analysis

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Introduction

The evaluation of double integrals over specific regions is a fundamental concept in multivariable calculus, providing insights into areas, volumes, and more complex quantities such as moments and centroids. In this detailed review, we focus on the integral:

$$
\iint_R xy \, dA
$$

where the region R is defined as:

$$
R = \{ (x, y) \mid 8 \leq x \leq 15, \quad 0 \leq y \leq 1 \}
$$

This region is a rectangle in the xy-plane, stretching from $x=8$ to $x=15$, and from $y=0$ to $y=1$. The integrand xy is a simple polynomial function, but evaluating its integral over R offers valuable insights into the process of setting up, simplifying, and calculating double integrals over rectangular regions.

Understanding the Region R

Geometric Description

- Shape: The region R is a rectangle, an uncomplicated shape in the xy-plane.
- Bounds:
- Horizontal bounds: from $x=8$ to $x=15$
- Vertical bounds: from $y=0$ to $y=1$

Significance of the Region

- Since the region is rectangular, the limits are constants, which simplifies integration.
- The region covers a finite, bounded area, making the integral well-defined and finite.
- Geometrically, this corresponds to a rectangle with length $15 - 8 = 7$ along the x-axis and height $1 - 0 = 1$ along the y-axis.

Setting up the Double Integral

General Form

The double integral over the region R can be expressed as:

$$
\iint_R xy \, dA = \int_{x=8}^{15} \int_{y=0}^{1} xy \, dy \, dx
$$

or equivalently,

$$
\iint_R xy \, dA = \int_{y=0}^{1} \int_{x=8}^{15} xy \, dx \, dy
$$

Given the rectangular bounds, the order of integration can be chosen based on convenience.

Choosing the Order of Integration

- Integrating with respect to y first:

$$
\int_{x=8}^{15} \left(\int_{y=0}^{1} xy \, dy \right) dx
$$

- Integrating with respect to x first:

$$
\int_{y=0}^{1} \left(\int_{x=8}^{15} xy \, dx \right) dy
$$

Both approaches are valid; the choice depends on which simplifies calculations more efficiently.

Evaluation of the Double Integral

Step 1: Inner Integral Calculation

Let's proceed with the order $dy \, dx$, which appears straightforward:

$$
\iint_R xy \, dA = \int_{x=8}^{15} \left(\int_{y=0}^{1} xy \, dy \right) dx
$$

- Inner integral:

$$
\int_{0}^{1} xy \, dy
$$

- Since x is treated as a constant during the inner integration:

$$
x \int_{0}^{1} y \, dy
$$

- Calculating $\int_{0}^{1} y \, dy$:

$$
\left[\frac{1}{2} y^2 \right]_0^1 = \frac{1}{2} (1)^2 - 0 = \frac{1}{2}
$$

- Result of the inner integral:

$$
x \times \frac{1}{2} = \frac{x}{2}
$$

Step 2: Outer Integral Calculation

Now, the integral reduces to:

$$
\int_{8}^{15} \frac{x}{2} \, dx
$$

- Simplify the integral:

$$
\frac{1}{2} \int_{8}^{15} x \, dx
$$

- Calculating $\int_{8}^{15} x \, dx$:

$$
\left[\frac{1}{2} x^2 \right]_8^{15} = \frac{1}{2} (15)^2 - \frac{1}{2} (8)^2 = \frac{1}{2} (225) - \frac{1}{2} (64) = \frac{225 - 64}{2} = \frac{161}{2}
$$

- Final result:

$$
\frac{1}{2} \times \frac{161}{2} = \frac{161}{4}
$$

Final Answer:

$$
\boxed{
\iint_R xy \, dA = \frac{161}{4} = 40.25
}
$$

This calculation confirms that the value of the double integral over the specified rectangular region is 40.25.

Deep Dive: Analytical Insights and Alternative Approaches

1. Why Choose a Rectangular Region?

Rectangular regions simplify the process of setting bounds because limits are constant, making the integral straightforward. For more complex regions, the bounds depend on the variables, requiring careful consideration of inequalities and sometimes changing the order of integration.

2. Symmetry Considerations

- The integrand xy is a product of two variables.
- The region R is symmetric in y but not in x. Since y spans from 0 to 1 (positive), and x spans from 8 to 15 (positive), the integral remains positive.
- If the region or the integrand were symmetric about axes or lines, properties such as zero integrals over symmetric regions could be exploited.

3. Alternative Integration Order

Choosing to integrate dx first:

$$
\iint_R xy \, dA = \int_{y=0}^{1} \left(\int_{x=8}^{15} xy \, dx \right) dy
$$

- Inner integral:

$$
y \int_{8}^{15} x \, dx = y \left[\frac{1}{2} x^2 \right]_8^{15} = y \left(\frac{1}{2} (225 - 64) \right) = y \times \frac{161}{2}
$$

- Outer integral:

$$
\int_0^1 y \times \frac{161}{2} \, dy = \frac{161}{2} \int_0^1 y \, dy = \frac{161}{2} \times \frac{1}{2} = \frac{161}{4}
$$

- Same result, confirming the consistency of the calculation.

Applications and Significance of the Double Integral

1. Calculating Areas and Volumes

- In this specific case, since the integrand is xy, the integral can be interpreted as a weighted measure over the region, possibly representing moments, mass, or other physical quantities in applied mathematics or physics.

2. Center of Mass and Centroids

- If xy represented density over the region, the integral could help compute the centroid or moments.
- For example, the first moment about the y-axis:

$$
\int_R x \times \text{density} \, dA
$$

- Similarly, the integral of $y \times \text{density}$ would give moments about the x-axis.

3. Probability and Statistics

- If the region and integrand correspond to probability density functions, the integral could measure probabilities or expectations.
- For example, if xy were normalized appropriately, the integral could represent expected values.

Practical Considerations in Computing Double Integrals

Tools and Techniques

- Manual calculation: As demonstrated, straightforward for rectangular regions with polynomial integrands.
- Graphing and visualization: Visualizing the region helps understand the bounds and integrand behavior.
- Software tools: Mathematica, MATLAB, WolframAlpha, and Python libraries (SymPy, NumPy) facilitate symbolic and numeric evaluation of integrals.

Common Pitfalls

- Incorrect bounds: Carefully determine whether to integrate dy or dx first.
- Sign errors: Ensure the limits are ordered correctly and integrand signs are maintained.
- Ignoring the region's shape: Always visualize to confirm bounds.

Summary and Conclusions

- The double integral of xy over the region $R = \{ (x,y) | 8 \leq x \leq 15, 0 \leq y \leq 1 \}$ evaluates to 40.25.
- The process involves setting up the integral with proper bounds, choosing an order of integration, simplifying the integrand where possible, and performing straightforward polynomial integrations.
- The simplicity of the rectangular region facilitates direct calculation, but the fundamental techniques apply to more complex regions with appropriate adjustments.
- Understanding the geometric interpretation, application contexts, and

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