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Introduction to Work in Vector Fields

Understanding the work done by a vector field is fundamental in fields such as physics, engineering, and mathematics. Specifically, when dealing with conservative vector fields, calculating the work simplifies significantly due to their properties. In this article, we will explore how to evaluate the work done by the vector field F = 6x I + 6y J + 6z between two points, Point 1 and Point 2. This process involves understanding the nature of conservative fields, potential functions, and the path-independent characteristics of such fields.

Understanding the Given Vector Field F = 6x I + 6y J + 6z

Description of the Vector Field

The vector field provided is:

$$
\mathbf{F} = 6x \mathbf{I} + 6y \mathbf{J} + 6z \mathbf{K}
$$

which, in component form, can be written as:

$$
\mathbf{F}(x, y, z) = (6x, 6y, 6z)
$$

This field assigns a vector to every point in 3D space based on the coordinates, and each component is proportional to the coordinate in that direction.

Physical Interpretation

This vector field resembles a radial field pointing outward from the origin, with magnitudes increasing linearly with distance from the origin. Such a field could represent, for example, a symmetric force field where the force increases proportionally with displacement, similar to a spring-like or harmonic force field.

Properties of the Vector Field

Is the Field Conservative?

To evaluate the work done, it is crucial to determine whether the field is conservative. A conservative vector field has two key properties:

- The curl of the field is zero: $\nabla \times \mathbf{F} = \mathbf{0}$
- The field can be expressed as the gradient of a scalar potential function: $\mathbf{F} = \nabla \phi$

Let's verify whether $\mathbf{F}$ is conservative.

Calculating the Curl of $\mathbf{F}$

The curl of $\mathbf{F}$ is:

$$
\nabla \times \mathbf{F} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
6x & 6y & 6z
\end{vmatrix}
$$

Calculating each component:

- $\left(\nabla \times \mathbf{F} \right)_x = \frac{\partial 6z}{\partial y} - \frac{\partial 6y}{\partial z} = 0 -$

$0 = 0$

- $\left(\nabla \times \mathbf{F} \right)_y = \frac{\partial 6x}{\partial z} - \frac{\partial 6z}{\partial x} = 0 - 0 = 0$
- $\left(\nabla \times \mathbf{F} \right)_z = \frac{\partial 6y}{\partial x} - \frac{\partial 6x}{\partial y} = 0 - 0 = 0$

Since all components are zero, $\nabla \times \mathbf{F} = \mathbf{0}$, confirming that $\mathbf{F}$ is a conservative field.

Finding the Potential Function ϕ

Given the conservative nature of $\mathbf{F}$, we can find a scalar potential function $\phi(x, y, z)$ such that:

$$
\nabla \phi = \mathbf{F}
$$

which translates to:

$$
\frac{\partial \phi}{\partial x} = 6x, \quad
\frac{\partial \phi}{\partial y} = 6y, \quad
\frac{\partial \phi}{\partial z} = 6z
$$

Integrating each component:

- For x:

$$
\phi(x, y, z) = \int 6x\, dx = 3x^2 + C_1(y,z)
$$

- For y:

$$
\frac{\partial \phi}{\partial y} = 6y \Rightarrow \phi(x, y, z) = 3y^2 + C_2(x,z)
$$

- For z:

$$
\frac{\partial \phi}{\partial z} = 6z \Rightarrow \phi(x, y, z) = 3z^2 + C_3(x,y)
$$

Combining these, the potential function is:

$$
\phi(x, y, z) = 3x^2 + 3y^2 + 3z^2 + \text{constant}
$$

The constant can be ignored for work calculations. Therefore,

$$
\boxed{
\phi(x, y, z) = 3(x^2 + y^2 + z^2)
}
$$

Calculating Work Done Between Two Points

Work in a Conservative Field

In a conservative vector field, the work done when moving from point A to point B is independent of the path taken and is given by the difference in the potential function:

$$
W = \phi(B) - \phi(A)
$$

where:

- $A = (x_1, y_1, z_1)$ (Point 1)
- $B = (x_2, y_2, z_2)$ (Point 2)

Step-by-Step Calculation

1. Identify the coordinates of Point 1 and Point 2:

For example, suppose:

$$
\textbf{Point 1} = (x_1, y_1, z_1)
$$

$$
\textbf{Point 2} = (x_2, y_2, z_2)
$$

(In real problems, these points are specified explicitly.)

2. Calculate the potential at each point:

$$
\phi_1 = 3(x_1^2 + y_1^2 + z_1^2)
$$

$$
\phi_2 = 3(x_2^2 + y_2^2 + z_2^2)
$$

3. Determine the work done:

$$
W = \phi_2 - \phi_1 = 3[(x_2^2 + y_2^2 + z_2^2) - (x_1^2 + y_1^2 + z_1^2)]
$$

This formula highlights the path independence inherent to conservative fields, simplifying calculations significantly.

Example Calculation

Suppose:

- Point 1: $(1, 2, 3)$
- Point 2: $(4, 5, 6)$

Calculating potential at each point:

$$
\phi(1, 2, 3) = 3(1^2 + 2^2 + 3^2) = 3(1 + 4 + 9) = 3 \times 14 = 42
$$

$$
\phi(4, 5, 6) = 3(4^2 + 5^2 + 6^2) = 3(16 + 25 + 36) = 3 \times 77 = 231
$$

Work done:

$$
W = 231 - 42 = 189
$$

Thus, the work done by the field when moving from Point 1 to Point 2 is 189 units.

Significance and Applications

Understanding how to evaluate work in a conservative field has multiple practical implications:

- Physics: Calculating work done by gravitational or electrostatic fields.
- Engineering: Analyzing energy transfer in systems governed by conservative forces.
- Mathematics: Solving potential problems in vector calculus.

This fundamental principle simplifies complex path-dependent calculations to straightforward evaluations of potential functions.

Summary and Key Takeaways

- The given vector field $\mathbf{F} = 6x I + 6y J + 6z K$ is conservative, as confirmed by zero curl.
- The potential function associated with $\mathbf{F}$ is $\phi(x, y, z) = 3(x^2 + y^2 + z^2)$.
- The work done when moving between two points in a conservative field is path-independent and calculated by the difference in potential values at those points.
- The general formula for work is:

\[
W = 3[(x_2^

Frequently Asked Questions

What is the work done by the conservative vector field F = 6x i + 6y j + 6z k between point 1 and point 2?

The work done is equal to the line integral of F between the two points, which can be evaluated using the potential function since F is conservative.

How do you verify that the vector field F = 6x i + 6y j + 6z k is conservative?

A vector field is conservative if its curl is zero. Calculating curl F yields zero, confirming that F is conservative.

What is the potential function associated with the vector field F = 6x i + 6y j + 6z k?

The potential function $\square(x,y,z)$ is given by $\square = 3x^2 + 3y^2 + 3z^2 + C$, where C is an arbitrary constant.

How do you compute the work done by the field F between two points using the potential function?

The work done is the difference in the potential function's values at the two points: $W = \square(\text{point } 2) - \square(\text{point } 1)$.

If point 1 is at $(x\square, y\square, z\square)$ and point 2 is at $(x\square, y\square, z\square)$, how is the work calculated for this field?

Calculate the potential at each point: $\square\square = 3x\square^2 + 3y\square^2 + 3z\square^2$ and $\square\square = 3x\square^2 + 3y\square^2 + 3z\square^2$. The work done is $\square\square - \square\square$.

Does the path between point 1 and point 2 affect the work done in this conservative field?

No, in a conservative field, the work done depends only on the endpoints, not the path taken.

What is the significance of the field being conservative in computing work between two points?

It simplifies calculations because the work can be directly obtained from the potential function values at the endpoints.

Can you provide an example calculation of the work done between points (1, 2, 3) and (4, 5, 6)?

Yes. Calculate □ at each: □□ = 3(1)^2 + 3(2)^2 + 3(3)^2 = 3 + 12 + 27 = 42; □□ = 3(4)^2 + 3(5)^2 + 3(6)^2 = 3(16 + 25 + 36) = 3(77) = 231. Work = 231 - 42 = 189.

What is the general formula for the work done by the field F = 6x i + 6y j + 6z k between any two points?

The work done is $W = 3(x□^2 - x□^2) + 3(y□^2 - y□^2) + 3(z□^2 - z□^2)$, based on the potential function derived.

Additional Resources

Evaluate The Work Done Between Point 1 And Point 2 For The Conservative Field F.F = 6x i + 6y j + 6z

When analyzing vector fields in physics and engineering, one fundamental concept is understanding the work done by a field when moving an object along a specific path. In this context, evaluate the work done between point 1 and point 2 for the conservative field F.F = 6x i + 6y j + 6z is a crucial problem that demonstrates the principles of vector calculus and the properties of conservative fields. This guide aims to walk you through the process step-by-step, ensuring a clear understanding of the techniques involved, from identifying the nature of the vector field to computing the line integral and interpreting the result.

Understanding the Vector Field F.F = 6x i + 6y j + 6z

Before delving into the evaluation of work, it’s essential to understand the characteristics of the vector field in question:

- Vector Field Definition: $F(x, y, z) = 6x\, i + 6y\, j + 6z\, k$
- Component-wise Explanation:
- The i-component (x-direction) is 6x.
- The j-component (y-direction) is 6y.
- The k-component (z-direction) is 6z.

- Type of Field:
- The given field is a linear vector field.
- It is conservative, which means it can be expressed as the gradient of a scalar potential function.

Step 1: Confirming that the Field is Conservative

Since the problem asks us to evaluate work done in a conservative field, verifying this property is crucial:

- Definition of Conservative Field: A vector field F is conservative if there exists a scalar potential function □(x, y, z) such that F = □□.
- Check Curl: For a field to be conservative in simply connected regions, its curl must be zero:

\[
\nabla \times \mathbf{F} = \mathbf{0}
\]

- Compute curl of F:

\[
\nabla \times \mathbf{F} = \left|

\begin{matrix}

\mathbf{i} & \mathbf{j} & \mathbf{k} \\

\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\

6x & 6y & 6z

\end{matrix}

\right|

\]

Calculating component-wise:

- i-component:

\[

\frac{\partial}{\partial y} (6z) - \frac{\partial}{\partial z} (6y) = 0 - 0 = 0

\]

- j-component:

\[

\frac{\partial}{\partial z} (6x) - \frac{\partial}{\partial x} (6z) = 0 - 0 = 0

\]

- k-component:

\[

\frac{\partial}{\partial x} (6y) - \frac{\partial}{\partial y} (6x) = 0 - 0 = 0

\]

Since all components are zero, $\nabla \times F = 0$, confirming that the field is conservative.

Conclusion: Because the curl of F is zero, F is a conservative vector field, and the work done depends only on the endpoints, not on the path taken.

Step 2: Finding the Scalar Potential Function $\phi(x, y, z)$

Given that $F = \nabla\phi$, find ϕ such that:

$$
\nabla \phi = \left(\frac{\partial \phi}{\partial x}, \frac{\partial \phi}{\partial y}, \frac{\partial \phi}{\partial z} \right) = (6x, 6y, 6z)
$$

- Integrate each component to find ϕ:

- From $\partial\phi/\partial x = 6x$:

$$
\phi(x, y, z) = 3x^2 + C(y, z)
$$

- From $\partial\phi/\partial y = 6y$:

$$
\frac{\partial \phi}{\partial y} = 6y \Rightarrow \phi = 3y^2 + D(x, z)
$$

Since the previous expression for ϕ includes $C(y, z)$, and now a separate expression involves $D(x, z)$, consistency suggests that:

$$

$$
\phi(x, y, z) = 3x^2 + 3y^2 + \text{(terms involving z)}
$$

- From $\partial\phi/\partial z = 6z$:

$$
\phi = 3z^2 + E(x, y)
$$

- Combining all:

$$
\boxed{
\phi(x, y, z) = 3x^2 + 3y^2 + 3z^2 + \text{constant}
}
$$

Constants can be ignored for differences in potential, so $\phi(x, y, z) = 3x^2 + 3y^2 + 3z^2$.

Step 3: Evaluating the Work Done Between Two Points

The work done by a conservative vector field in moving an object from point A to point B is given by the difference in the potential function evaluated at these points:

$$
W = \phi(\mathbf{r}_2) - \phi(\mathbf{r}_1)
$$

where:

- $\mathbf{r}_1 = (x_1, y_1, z_1)$ is the initial point (Point 1),
- $\mathbf{r}_2 = (x_2, y_2, z_2)$ is the terminal point (Point 2).

Note: The actual coordinates for Point 1 and Point 2 are not provided in the problem statement. For demonstration purposes, assume:

- Point 1: $\mathbf{r}_1 = (x_1, y_1, z_1)$
- Point 2: $\mathbf{r}_2 = (x_2, y_2, z_2)$

The work calculation is thus:

$$
W = \left[3x_2^2 + 3y_2^2 + 3z_2^2 \right] - \left[3x_1^2 + 3y_1^2 + 3z_1^2 \right]
$$

Step-by-Step Example Calculation

Suppose:

- Point 1: (1, 2, 3)
- Point 2: (4, 5, 6)

Then:

$$
\phi(4, 5, 6) = 3(4)^2 + 3(5)^2 + 3(6)^2 = 3(16) + 3(25) + 3(36) = 48 + 75 + 108 = 231
$$

$$

$$\phi(1, 2, 3) = 3(1)^2 + 3(2)^2 + 3(3)^2 = 3(1) + 3(4) + 3(9) = 3 + 12 + 27 = 42$$

Work done:

$$W = 231 - 42 = 189$$

This value represents the work done by the field in moving an object from Point 1 to Point 2.

Key Takeaways and Summary

Why is this calculation important?

- It illustrates how the properties of a vector field (being conservative) simplify work calculations.
- The potential function provides a straightforward way to evaluate line integrals without explicitly performing the integral over a path.
- Understanding the scalar potential function associated with a vector field is fundamental in physics, especially in fields like electromagnetism, gravitation, and fluid dynamics.

Summary of the process:

1. Verify if the field is conservative by calculating curl.
2. Find the scalar potential function such that $F = \nabla\phi$.
3. Evaluate the potential function at the endpoints.
4. Calculate the work as the difference in potential at the endpoint points.

Additional Tips and Considerations

- If the points are given explicitly, plug their coordinates directly into the potential function.
- Remember that for conservative fields, the work is path-independent.
- Be cautious that the potential function is determined up to an additive constant; differences cancel out the constant.

Final Remarks

Evaluating the work done in a conservative field is a fundamental skill in vector calculus, with practical implications in physics and engineering. By systematically confirming the field’s properties and leveraging the potential function, complex line integrals are reduced to simple algebraic calculations. This approach not only simplifies the process but also deepens the understanding of the underlying physical principles, such as energy

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