Evaluate The Work Done Between Point 1 And Point 2 For The Conservative Field F. F= (y+z)i + Xj + Xk

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Understanding how to evaluate work done by a vector field between two points is a fundamental concept in vector calculus, particularly in the study of conservative fields. In this article, we will thoroughly analyze the work done by the vector field F = (y + z)i + xj + xk as it moves from Point 1 to Point 2. We will explore the properties of the field, verify whether it is conservative, and then compute the work done accordingly.

Understanding the Vector Field F

Before diving into the calculations, it is essential to understand the given vector field:

F = (y + z)i + xj + xk

This means that at any point (x, y, z) in space, the vector field assigns a vector:

- In the x-direction: (y + z)
- In the y-direction: x
- In the z-direction: x

This field combines variables in a way that suggests the potential for a scalar potential function, which would classify it as a conservative field.

Verifying Whether F is a Conservative Field

A key step in evaluating the work done by a vector field between two points is to determine if the field is conservative. A conservative field F is one where there exists a scalar potential function ϕ(x, y, z) such that:

$F = \nabla\phi = (\partial\phi/\partial x)i + (\partial\phi/\partial y)j + (\partial\phi/\partial z)k$

If F is conservative, the work done moving from one point to another is path-independent and can be computed as the difference in potential function values at those points.

Step 1: Check Curl of F

For a vector field to be conservative (in simply connected regions), its curl must be zero:

$\nabla \times F = 0$

Calculate the curl:

$$
\nabla \times F = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
y + z & x & x
\end{vmatrix}
$$

Compute each component:

- i-component:

$$
\frac{\partial}{\partial y} (x) - \frac{\partial}{\partial z} (x) = 0 - 0 = 0
$$

- j-component:

$$
\frac{\partial}{\partial z} (y + z) - \frac{\partial}{\partial x} (x) = 1 - 1 = 0
$$

- k-component:

$$
\frac{\partial}{\partial x} (x) - \frac{\partial}{\partial y} (y + z) = 1 - 1 = 0
$$

Since all components are zero, $\nabla \times F = 0$, confirming that F is a conservative field in a simply connected region.

Conclusion: The vector field F is conservative. Therefore, there exists a potential function $\phi(x, y, z)$ such that $F = \nabla\phi$.

Finding the Scalar Potential Function ϕ(x, y,

z)

Given:

$$
\nabla \varphi = \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z} \right) = (y + z, x, x)
$$

We now solve for ϕ:

Step 1: Integrate with respect to x:

$$
\frac{\partial \varphi}{\partial x} = y + z
$$

Integrate:

$$
\varphi(x, y, z) = x(y + z) + C(y, z)
$$

where C(y, z) is an arbitrary function of y and z.

Step 2: Find partial derivatives with respect to y and z to determine C(y, z):

- From the ∂ϕ/∂y component:

$$
\frac{\partial \varphi}{\partial y} = x + \frac{\partial C}{\partial y}
$$

But from the vector field:

$$
\frac{\partial \varphi}{\partial y} = x
$$

Set equal:

$$
x + \frac{\partial C}{\partial y} = x \Rightarrow \frac{\partial C}{\partial y} = 0
$$

Thus, C(y, z) does not depend on y; it is a function of z only:

$$
C(y, z) = C(z)
$$

- From the $\partial\phi/\partial z$ component:

$$
\frac{\partial \varphi}{\partial z} = x + \frac{d C(z)}{d z}
$$

From the vector field:

$$
\frac{\partial \varphi}{\partial z} = x
$$

Set equal:

$$
x + \frac{d C(z)}{d z} = x \Rightarrow \frac{d C(z)}{d z} = 0
$$

Therefore, C(z) is a constant; without loss of generality, we can set it to zero.

Final potential function:

$$
\boxed{
\varphi(x, y, z) = x(y + z)
}
$$

Calculating the Work Done Between Two Points

Since F is conservative, the work done W moving from Point 1 (x_1, y_1, z_1) to Point 2 (x_2, y_2, z_2) is:

$$
W = \varphi(x_2, y_2, z_2) - \varphi(x_1, y_1, z_1)
$$

Step 1: Identify Points

Suppose the points are:

- Point 1: (x_1, y_1, z_1)

- Point 2: (x_2, y_2, z_2)

If specific points are given, substitute their coordinates into the potential function to find the work.

Step 2: Compute the difference

$$
W = x_2(y_2 + z_2) - x_1(y_1 + z_1)
$$

This expression gives the total work done by the field F in moving an object from Point 1 to Point 2.

Practical Example: Calculating Work with Specific Points

Suppose:

- Point 1: $(1, 2, 3)$
- Point 2: $(4, 5, 6)$

Calculate:

$$
W = 4(5 + 6) - 1(2 + 3) = 4 \times 11 - 1 \times 5 = 44 - 5 = 39
$$

Thus, the work done by the field F in moving from $(1, 2, 3)$ to $(4, 5, 6)$ is 39 units.

Summary and Final Thoughts

Evaluating the work done by a vector field between two points is straightforward once the field is identified as conservative. In our case, the key steps involved:

- Verifying the conservative nature of F via curl calculation.
- Finding the potential function $\phi(x, y, z)$.
- Applying the potential difference formula to compute work.

Since F is conservative, the work is path-independent and depends solely on the initial and final points. This property simplifies many calculations in physics and engineering, especially in fields like electrostatics, gravity, and fluid flow.

Whether dealing with theoretical problems or practical applications, understanding the process of evaluating work in conservative vector fields enhances problem-solving efficiency and deepens comprehension of the underlying physics.

Additional Tips for Students and Practitioners:

- Always verify if a vector field is conservative before attempting to compute work; use curl as a quick test.
- When finding the potential function, integrate component-wise carefully, and determine any integration functions by matching partial derivatives.
- Remember that in non-conservative fields, work depends on the path taken, requiring line integral calculations.

By mastering these steps and concepts, you can confidently evaluate work in a variety of vector fields, streamlining your analysis of physical systems.

Frequently Asked Questions

What is the general approach to evaluate the work done by a conservative vector field between two points?

Since the field is conservative, the work done is equal to the difference in potential function values at the two points, or equivalently, the line integral of the gradient between those points.

Given the vector field F = (y + z)i + xj + xk, how do you verify if it is conservative?

You verify by checking if the curl of F is zero. If curl F = 0, then F is conservative. Alternatively, since the field is defined in a simply connected domain, a zero curl confirms conservativeness.

How do you find the potential function for the given vector field F?

Integrate the components of F with respect to their variables, ensuring consistency across the partial derivatives, to determine a potential function $\varphi(x, y, z)$ such that $\nabla\varphi = F$.

What are the steps to compute the work done between Point 1 and Point 2 for this field?

First, find the potential function φ. Then, evaluate φ at Point 2 and Point 1, and subtract: Work = φ(Point 2) - φ(Point 1).

Can the work done be directly computed by evaluating the line integral of F between the two points?

Yes, but since F is conservative, it's easier to use the potential function difference. The line integral equals this difference, simplifying the calculation.

What are the potential challenges in evaluating the work done in this problem?

Potential challenges include accurately finding the potential function, ensuring the field is indeed conservative, and correctly applying the limits at the given points.

If Point 1 is (x1, y1, z1) and Point 2 is (x2, y2, z2), how do you express the work done explicitly?

Compute φ(x2, y2, z2) and φ(x1, y1, z1), then subtract: Work = φ(x2, y2, z2) - φ(x1, y1, z1).

Additional Resources

Evaluate The Work Done Between Point 1 And Point 2 For The Conservative Field F: An In-Depth Analysis

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Introduction

In the realm of vector calculus, understanding how a vector field influences movement between two points is fundamental. The specific focus of this article is to thoroughly evaluate the work done by the vector field F = (y + z)i + xj + xk along a specified path from Point 1 to Point 2. This investigation not only elucidates the properties of the field but also reinforces core concepts such as conservative fields, potential functions, and line integrals. Through a comprehensive, step-by-step analysis, we aim to provide clarity and insight into the behavior of this vector field, especially in the context of work calculation.

Understanding the Vector Field F

Before proceeding with the work evaluation, it is essential to analyze the nature of the vector field F.

The field is given as:

$$
\mathbf{F} = (y + z)\mathbf{i} + x\mathbf{j} + x\mathbf{k}
$$

Expressed component-wise:

- $F_x = y + z$
- $F_y = x$
- $F_z = x$

Key observations:

- The x-component depends on y and z.
- The y and z components both depend on x.

This structure hints at potential symmetries and the possibility of the field being conservative, which would significantly simplify work calculations.

Step 1: Determining if F is a Conservative Field

A vector field F is conservative if there exists a scalar potential function ϕ such that:

$$
\nabla \phi = \mathbf{F}
$$

or explicitly:

$$
\frac{\partial \phi}{\partial x} = F_x = y + z
$$
$$
\frac{\partial \phi}{\partial y} = F_y = x
$$
$$
\frac{\partial \phi}{\partial z} = F_z = x
$$

Checking for consistency:

- Cross partial derivatives should be equal for a conservative field:

$$
\frac{\partial F_x}{\partial y} = \frac{\partial F_y}{\partial x}
$$
$$
\frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x}
$$
$$
\frac{\partial F_y}{\partial z} = \frac{\partial F_z}{\partial y}
$$

Calculations:

- $\frac{\partial F_x}{\partial y} = 1$
- $\frac{\partial F_y}{\partial x} = 1$

(First condition satisfied)

- $\frac{\partial F_x}{\partial z} = 1$
- $\frac{\partial F_z}{\partial x} = 1$

(Second condition satisfied)

- $\frac{\partial F_y}{\partial z} = 0$
- $\frac{\partial F_z}{\partial y} = 0$

(Third condition satisfied)

Since all cross derivatives match, F is a conservative vector field.

Step 2: Finding the Potential Function ϕ

Given:

$$
\frac{\partial \phi}{\partial x} = y + z
$$

Integrate w.r.t. x:

$$
\phi(x, y, z) = xy + xz + C(y, z)
$$

where $C(y, z)$ is an arbitrary function of y and z.

Next, differentiate ϕ w.r.t. y:

$$
\frac{\partial \phi}{\partial y} = x + \frac{\partial C}{\partial y}
$$

$$
$$

But from the field:

$$
\frac{\partial \phi}{\partial y} = F_y = x
$$

Thus,

$$
x + \frac{\partial C}{\partial y} = x \Rightarrow \frac{\partial C}{\partial y} = 0
$$

which implies C does not depend on y.

Similarly, differentiate ϕ w.r.t. z:

$$
\frac{\partial \phi}{\partial z} = x + \frac{\partial C}{\partial z}
$$

Given:

$$
\frac{\partial \phi}{\partial z} = F_z = x
$$

We find:

$$
x + \frac{\partial C}{\partial z} = x \Rightarrow \frac{\partial C}{\partial z} = 0
$$

Therefore, C is a constant function; we can set $C = 0$ for simplicity.

The potential function is then:

$$
\boxed{
\phi(x, y, z) = xy + xz
}
$$

Step 3: Computing the Work Done

Since F is conservative, the work done W by the field along a path from Point

1 $\mathbf{r}_1$ to Point 2 $\mathbf{r}_2$ is:

$$
W = \phi(\mathbf{r}_2) - \phi(\mathbf{r}_1)
$$

Step 4: Identifying Points 1 and 2

To evaluate the work, we need the coordinates of the initial and final points. Suppose:

- Point 1: $\mathbf{r}_1 = (x_1, y_1, z_1)$
- Point 2: $\mathbf{r}_2 = (x_2, y_2, z_2)$

Note: The specific points are not provided in the prompt. For an illustrative purpose, let us assume:

- Point 1: $(x_1, y_1, z_1) = (x_0, y_0, z_0)$
- Point 2: $(x_2, y_2, z_2) = (x_1, y_1, z_1)$

(Replace with actual points if provided)

Step 5: Final Calculation

The work is:

$$
W = \phi(x_2, y_2, z_2) - \phi(x_1, y_1, z_1)
$$

Substituting the potential function:

$$
W = (x_2 y_2 + x_2 z_2) - (x_1 y_1 + x_1 z_1)
$$

This expression provides a straightforward computation of the work done, relying solely on the initial and final coordinates.

Deep Dive: Practical Implications and Theoretical Insights

Why Is Recognizing a Field as Conservative Important?

Identifying that F is conservative drastically simplifies work calculations. Instead of evaluating the line integral directly along potentially complex paths, one can leverage the potential function to compute the difference at endpoints. This property is invaluable in physics and engineering, where energy conservation principles are foundational.

Key points:

- Line integrals in conservative fields are path-independent.
- Potential functions serve as energy-like quantities.
- Simplifies complex path calculations to endpoint evaluations.

Potential Function Derivation and Its Significance

The derivation of $\phi = xy + xz$ underscores the importance of cross-derivative consistency. It demonstrates an elegant method of constructing potential functions by integration, highlighting the interconnectedness of field components.

Implications:

- Validates the conservative nature of F.
- Provides a direct method to compute work.
- Offers insight into the physical interpretation, e.g., potential energy in physics.

Limitations and Assumptions

While the analysis is thorough, it hinges on the assumption that points and the path are known. The actual points of start and end influence the numerical value of work. If specific points are provided, the calculation becomes precise.

Furthermore, the potential function approach assumes the field is defined over a simply connected domain. If the field were defined over a domain with holes or discontinuities, the conservative property might not hold, complicating the analysis.

Extensions and Further Investigations

- Path-dependent fields: For non-conservative fields, the line integral depends on the path taken, leading to more complex evaluations.
- Numerical methods: When potential functions are not readily obtainable, numerical integration along specified paths becomes necessary.
- Physical applications: Fields like gravitational, electrostatic, or magnetic fields often exhibit similar properties, making this analysis applicable in various scientific contexts.

Conclusion

The comprehensive evaluation of the work done by the vector field F = (y + z)i + xj + xk between two points reveals a fundamental characteristic: F is a conservative vector field. The derivation of the potential function \(\phi = xy + xz\) simplifies the work calculation, transforming a potentially complex line integral into a straightforward difference of potential values.

This analysis not only underscores the elegance of vector calculus principles but also emphasizes the importance of identifying conservative fields in practical applications. Whether in physics, engineering, or mathematical modeling, recognizing and exploiting such properties can lead to significant simplifications, deeper understanding, and more efficient problem-solving strategies.

In essence, evaluating work in conservative vector fields is a testament to the power of

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