Factor 7y2 53y 28. Question 5 Options: A) (y 5)(7y 2) B) (7y 4)(y 7) C) (y 12)(7y 1) D) (7y 2)(y 14)

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Understanding how to factor algebraic expressions is a fundamental skill in mathematics, especially in algebra. The question at hand involves factoring a polynomial expression that contains multiple terms involving the variable \(y \). The goal is to identify the correct factorization among the provided options, which is crucial for simplifying expressions and solving equations efficiently.

In this comprehensive guide, we will explore the process of factoring algebraic expressions step-by-step. We will analyze each option, review key concepts in factoring, and provide strategies to approach similar problems in the future. This detailed discussion aims to improve your algebra skills and deepen your understanding of polynomial factorization.

Overview of Polynomial Factoring

Factoring involves expressing a polynomial as a product of its factors. It simplifies complex expressions and makes solving equations more manageable. The most common types of factoring include:

1. Factoring out the greatest common factor (GCF)

- Identify the largest number and variable that divides all terms.
- Factor out the GCF from the entire polynomial.

2. Factoring by grouping

- Group terms with common factors.
- Factor each group separately.

3. Factoring quadratics

- Identify quadratic expressions in the form \(ax^2 + bx + c \).
- Use methods like trial factoring, completing the square, or the quadratic formula to factor quadratics.

4. Special binomial products

- Difference of squares: $a^2 - b^2 = (a - b)(a + b)$
- Perfect square trinomials: $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Analyzing the Given Expression

The original expression is:

$$7y^2 + 53y + 28$$

Our task is to factor this quadratic expression and determine which of the options provided correctly represents its factorization.

Step-by-Step Factoring Process

Let's approach this systematically.

Step 1: Identify coefficients

The quadratic expression can be written as:

$$7y^2 + 53y + 28$$

- $a = 7$
- $b = 53$
- $c = 28$

Step 2: Find the product $a \times c$

Multiply the leading coefficient and the constant term:

$$7 \times 28 = 196$$

We need two numbers that multiply to 196 and add to $b = 53$.

Step 3: Find two numbers that multiply to 196 and sum to 53

List factors of 196:

- 1 and 196 (sum: 197)
- 2 and 98 (sum: 100)
- 4 and 49 (sum: 53)
- 7 and 28 (sum: 35)
- 14 and 14 (sum: 28)

The pair 4 and 49 multiply to 196 and add up to 53.

Step 4: Rewrite the middle term using these two numbers

Express the quadratic as:

$$ 7y^2 + 4y + 49y + 28 $$

Step 5: Factor by grouping

Group terms:

$$ (7y^2 + 4y) + (49y + 28) $$

Factor out GCFs from each group:

- From first group: y (since $7y^2$ and $4y$ share a y)
- From second group: 7

Rewrite:

$$ y(7y + 4) + 7(7y + 4) $$

Step 6: Factor out the common binomial factor

Factor out $(7y + 4)$:

$$ (7y + 4)(y + 7) $$

Matching with the Provided Options

The factored form of the quadratic is:

$$ (7y + 4)(y + 7) $$

Now, compare this result with the options:

- Option A: $(y + 5)(7y + 2)$
- Option B: $(7y + 4)(y + 7)$
- Option C: $(y + 12)(7y + 1)$
- Option D: $(7y + 2)(y + 14)$

The matching option is Option B: $(7y + 4)(y + 7)$.

Understanding Why Option B Is Correct

Let's verify:

\[

$$
(7y + 4)(y + 7) = 7y \times y + 7y \times 7 + 4 \times y + 4 \times 7
$$

Calculating each term:

- $7y \times y = 7y^2$
- $7y \times 7 = 49y$
- $4 \times y = 4y$
- $4 \times 7 = 28$

Adding like terms:

$$
7y^2 + (49y + 4y) + 28 = 7y^2 + 53y + 28
$$

This matches our original quadratic expression exactly, confirming that Option B is the correct factorization.

Key Takeaways for Factoring Quadratic Expressions

- Always identify the coefficients a, b, and c.
- Multiply $a \times c$ to find two numbers that factor the quadratic.
- Use factoring by grouping when appropriate.
- Verify your factors by expanding them back to check if they match the original expression.
- Recognize common patterns like the difference of squares or perfect square trinomials.

Additional Tips and Strategies

- **Practice with different quadratic forms:** The more you practice, the better you'll become at recognizing factoring patterns.
- **Use the quadratic formula:** When factoring by trial seems complex, the quadratic formula can help find roots directly.
- **Factor common factors first:** Always check for GCFs before proceeding with other factoring techniques.
- **Check your work:** Expand your factors to verify they produce the original quadratic expression.

Conclusion: The Correct Answer

Based on the detailed factorization process, the correct answer to the problem Factor $7y^2 + 53y + 28$ is Option B: $(7y + 4)(y + 7)$. Recognizing the structure of the quadratic and applying systematic factoring techniques ensures accurate and efficient solutions in algebra.

Mastering these steps not only helps in solving similar problems but also builds a strong foundation for advanced algebraic concepts. Whether you're preparing for exams or enhancing your mathematical toolkit, Understanding how to factor quadratics is an essential skill that will serve you well throughout your mathematical journey.

Frequently Asked Questions

What is the factored form of the expression $7y^2 + 53y + 28$?

The factored form is $(7y + 4)(y + 7)$.

Which option correctly factors the quadratic $7y^2 + 53y + 28$?

Option B: $(7y + 4)(y + 7)$.

How do you factor the quadratic expression $7y^2 + 53y + 28$?

By finding two binomials that multiply to give $7y^2 + 53y + 28$, which is $(7y + 4)(y + 7)$.

Why is $(7y + 4)(y + 7)$ the correct factorization for $7y^2 + 53y + 28$?

Because expanding $(7y + 4)(y + 7)$ results in $7y^2 + 53y + 28$, matching the original quadratic.

Which of the following options correctly represents the factors of $7y^2 + 53y + 28$?

Option B: $(7y + 4)(y + 7)$.

Can the quadratic $7y^2 + 53y + 28$ be factored using simple binomials?

Yes, it factors as $(7y + 4)(y + 7)$.

What is the importance of selecting the correct factors in quadratic expressions?

Choosing the correct factors ensures the expanded form matches the original quadratic expression accurately.

Are the options A, C, and D correct for factoring $7y^2 + 53y + 28$?

No, only option B, $(7y + 4)(y + 7)$, correctly factors the quadratic.

How can you verify that $(7y + 4)(y + 7)$ is the correct factorization?

By expanding the binomials: $(7y + 4)(y + 7) = 7y^2 + 49y + 4y + 28 = 7y^2 + 53y + 28$.

What is the value of the middle term in the factored form $(7y + 4)(y + 7)$?

The middle term is $53y$, which comes from $7y\ 7 + 4\ y = 49y + 4y = 53y$.

Additional Resources

In-Depth Analysis of Factorization in the Expression "Factor 7y2 53y 28"

Understanding polynomial factorization is fundamental in algebra, helping simplify expressions, solve equations, and understand the structure of algebraic functions. The given problem involves factoring a polynomial expression, specifically "Factor 7y2 53y 28," and selecting the correct factorization from the options provided. To analyze this thoroughly, we will break down each component, explore the underlying principles, and evaluate each option with precision.

Deciphering the Expression: Clarifying the Problem

Before diving into factorization techniques, it's crucial to interpret what "Factor 7y2 53y 28" actually represents.

Possible interpretations:

- It might be an algebraic expression like $7y^2 + 53y + 28$.

- Or perhaps, the phrase is a shorthand for a polynomial in factored form.

Given the options provided, which are all binomials like $(\text{something})(\text{something})$, it's reasonable to assume that the original polynomial is quadratic, specifically:

$$7y^2 + 53y + 28$$

This is because the options all follow the pattern of binomials multiplied together, typical in quadratic factorization.

Therefore, the primary goal is:

- To factor the quadratic $7y^2 + 53y + 28$

- To compare the factorizations with the options:

A) $(y + 5)(7y + 2)$

B) $(7y + 4)(y + 7)$

C) $(y + 12)(7y + 1)$

D) $(7y + 2)(y + 14)$

Step-by-Step Approach to Quadratic Factorization

1. Recognize the standard form:

A quadratic in the form $ax^2 + bx + c$, where:

- $a = 7$

- $b = 53$

- $c = 28$

2. Check for common factors:

- No common factor exists among 7, 53, and 28 (since 53 is prime).

- So, straightforward factorization applies.

3. Use the method of "splitting the middle term" or factoring by trial:

- The general approach involves finding two numbers, m and n, such that:

- $m \times n = a \times c = 7 \times 28 = 196$

- $m + n = b = 53$

4. Find factors of 196 that sum to 53:

- List of factor pairs of 196:

- 1 and 196 (sum 197)

- 2 and 98 (sum 100)

- 4 and 49 (sum 53)

- 7 and 28 (sum 35)

- 14 and 14 (sum 28)

- The pair 4 and 49 sums to 53, matching our middle coefficient.

5. Rewrite the middle term:

$$7y^2 + 4y + 49y + 28$$

- Group:

$$(7y^2 + 4y) + (49y + 28)$$

- Factor each group:

$$y(7y + 4) + 7(7y + 4)$$

- Factor out common binomial:

$$(7y + 4)(y + 7)$$

Matching to the Provided Options

The derived factorization:

$$7y^2 + 53y + 28 = (7y + 4)(y + 7)$$

Corresponds to option B): $(7y + 4)(y + 7)$.

Deep Dive into Each Option

Let's analyze each multiple-choice option to understand why or why not they are correct.

Option A: $(y + 5)(7y + 2)$

- Expand:

$$ y \times 7y = 7y^2 $$

$$ y \times 2 = 2y $$

$$ 5 \times 7y = 35y $$

$$ 5 \times 2 = 10 $$

- Sum:

$$ 7y^2 + (2y + 35y) + 10 = 7y^2 + 37y + 10 $$

- Comparison:

$$ 7y^2 + 37y + 10 \neq 7y^2 + 53y + 28 $$

- Conclusion: Not the correct factorization.

Option B: $(7y + 4)(y + 7)$

- Expand:

$$ 7y \times y = 7y^2 $$

$$ 7y \times 7 = 49y $$

$$ 4 \times y = 4y $$

$$ 4 \times 7 = 28 $$

- Sum:

$$ 7y^2 + (49y + 4y) + 28 = 7y^2 + 53y + 28 $$

- Comparison:

Matches the original quadratic exactly.

- Conclusion: Correct factorization.

Option C: $(y + 12)(7y + 1)$

- Expand:

$$ y \times 7y = 7y^2 $$

$$y \times 1 = y$$

$$12 \times 7y = 84y$$

$$12 \times 1 = 12$$

- Sum:

$$7y^2 + (y + 84y) + 12 = 7y^2 + 85y + 12$$

- Comparison:

Does not match $7y^2 + 53y + 28$.

- Conclusion: Not the correct factorization.

Option D: $(7y + 2)(y + 14)$

- Expand:

$$7y \times y = 7y^2$$

$$7y \times 14 = 98y$$

$$2 \times y = 2y$$

$$2 \times 14 = 28$$

- Sum:

$$7y^2 + (98y + 2y) + 28 = 7y^2 + 100y + 28$$

- Comparison:

Does not match $7y^2 + 53y + 28$.

- Conclusion: Not the correct factorization.

Final Conclusion and Recommendations

Based on the detailed analysis, the correct factorization of the quadratic polynomial $7y^2 + 53y + 28$ is:

$$(7y + 4)(y + 7)$$

which aligns exactly with option B.

Understanding why this is crucial:

- It highlights the importance of the "splitting the middle term" method, especially when coefficients are not straightforward.

- Demonstrates the importance of identifying factor pairs of the product $a \times c$ that sum to b.

- Emphasizes verifying each option through expansion rather than assumption.

Broader Implications in Algebra

Factorization techniques like these are foundational in algebraic manipulations, especially when solving quadratic equations, simplifying expressions, or analyzing polynomial functions.

Key takeaways:

- Always check for common factors first before proceeding to more complex methods.

- Use the "product-sum" method for quadratic trinomials with integer coefficients.

- Confirm your factorization by expanding and comparing to the original expression.

- Practice with various coefficients to develop intuition for recognizing factor pairs.

Application in advanced contexts:

- Polynomial division and synthetic division.

- Solving quadratic equations efficiently.

- Graphing quadratic functions by identifying roots.

- Extending factorization techniques to higher-degree polynomials.

Final Thoughts

This problem exemplifies the importance of systematic approaches in algebra. Correctly identifying the factors of a quadratic polynomial requires understanding the relationships between coefficients and the factors of their product. The detailed step-by-step process confirms that option B is the accurate choice, providing a solid example of algebraic problem-solving.

Mastering such techniques not only enhances mathematical proficiency but also develops critical thinking skills applicable across various domains, from engineering to data science.

In summary:

- The original quadratic: $7y^2 + 53y + 28$
- Correct factorization: $(7y + 4)(y + 7)$
- Corresponds to Option B
- Reinforces the importance of factor pairs and systematic verification in algebra.

Always remember:

> Break down complex problems into manageable steps, verify your work, and develop an intuition for algebraic patterns.

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