Find A Plane Containing The Point (6,-6,5) And The Line Of Intersection Of The Planes -6 X+y+7 Z=-15

Find A Plane Containing The Point (6,-6,5) And The Line Of Intersection Of The Planes -6 X+y+7 Z=-15

Understanding how to find a plane that contains a specific point and intersects with a given line of intersection of two other planes is a fundamental problem in three-dimensional geometry. This problem combines the concepts of plane equations, line of intersection, and the process of deriving a new plane that satisfies multiple conditions simultaneously. In this article, we will explore the step-by-step method to find such a plane, breaking down the problem into manageable parts and providing detailed explanations to ensure clarity.

Understanding the Problem

Before diving into the solution, it’s essential to understand what is being asked:

- Given Point: (6, -6, 5)
- Line of Intersection: of two planes
- Planes defining the line: Plane 1: -6x + y + 7z = -15; Plane 2: (another plane that intersects with the first to form a line)

The goal is to find a single plane that:

1. Contains the given point (6, -6, 5).
2. Contains the line of intersection of the two given planes.

Key Insight: The intersection line of two planes lies in both planes. Any plane that contains this line must also contain both original planes' intersection line. Therefore, the required plane can be expressed as a linear combination of the two planes.

Step 1: Express the General Form of the Plane Containing the Line of Intersection

Any plane passing through the intersection of two given planes can be written as:

$$ \text{Plane} = \text{Plane}_1 + \lambda \times \text{Plane}_2 $$

where λ is a scalar parameter, and the planes are represented by their equations.

Given Planes:

- Plane 1: $-6x + y + 7z = -15$
- Plane 2: Let's denote as $A_2x + B_2y + C_2z = D_2$

But in this problem, only one plane is explicitly given, and the other is implied by the line of intersection. To find the second plane, we need to clarify what the second plane is.

Note: Since the problem involves the line of intersection of two planes, and only one plane is provided, perhaps the second plane is arbitrary, and the question is to find any plane containing the point and the intersection line.

Alternatively, perhaps the problem is asking us to find the plane containing the point and the line of intersection of the given plane $-6x + y + 7z = -15$ with some other plane.

Assumption: The line of intersection is between plane 1 and some other plane, say plane 2, which is not explicitly given. But since only one plane is given, perhaps the problem wants us to find the plane that contains:

- The point (6, -6, 5)
- The line of intersection of plane $-6x + y + 7z = -15$ with any other plane that contains the point

Alternatively, perhaps the line of intersection is between the given plane and a family of planes of a certain form.

Clarity is needed here. Usually, for such problems, the line of intersection is between two planes, and the solution involves combining their equations.

Re-examining the problem statement:

> "Find A Plane Containing The Point (6,-6,5) And The Line Of Intersection Of The Planes -6 X + y + 7 Z = -15"

It suggests that the line of intersection is formed by the plane $-6x + y + 7z = -15$ and another plane. For the problem to be fully specified, the second plane must be known or at least implied.

Possible interpretation:

- The plane $-6x + y + 7z = -15$ intersects with some unknown plane to form a line.

- The task is to find a plane passing through point (6, -6, 5) and that contains this line.

Alternative approach:

Given only one plane, perhaps the problem is asking to find a plane passing through the point (6, -6, 5) that contains the line of intersection of the given plane with any plane parallel to it or with some other condition.

Step 2: Clarify the Line of Intersection of the Two Planes

Suppose the line of intersection is between:

- Plane 1: $-6x + y + 7z = -15$
- Plane 2: $A x + B y + C z = D$

We need to find the equation of the second plane such that the line of intersection is well-defined.

Finding the line of intersection:

- The line of intersection is the set of all points satisfying both plane equations.
- To parametrize this line, find the normal vectors of the planes and solve the system.

Step 3: Find a Second Plane That Intersects with Plane 1

Since the second plane is not specified, perhaps the question expects us to consider the line of intersection as being between the given plane and a family of planes.

Suppose the second plane has the general form:

$$P: a x + b y + c z = d$$

The family of planes passing through the line of intersection of plane1 and plane2 can be written as:

$$ \text{Plane} = \text{Plane}_1 + \lambda \times \text{Plane}_2 $$

But without the second plane's explicit equation, this approach cannot proceed.

Step 4: Alternative Method – Find the Line of Intersection Explicitly

Find the line of intersection between two arbitrary planes:

- Plane 1: $-6x + y + 7z = -15$
- Plane 2: Let's assume the second plane is $a x + b y + c z = d$

To find the line:

1. Find the cross product of the normals to get the direction vector.

2. Find a point on the line by solving the system.

Step 5: Solving a Specific Example

Suppose the second plane is:

$$ 3x + 2 y - z = 4 $$

Now, find the line of intersection between:

- Plane 1: $-6x + y + 7z = -15$
- Plane 2: $3x + 2 y - z = 4$

Normals:

- $ \mathbf{n}_1 = (-6, 1, 7) $
- $ \mathbf{n}_2 = (3, 2, -1) $

Direction vector of the line:

$$ \mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2 $$

Calculating the cross product:

\[

\mathbf{d} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-6 & 1 & 7 \\
3 & 2 & -1 \\
\end{vmatrix}
= \mathbf{i}(1 \times (-1) - 7 \times 2) - \mathbf{j}(-6 \times (-1) - 7 \times 3) + \mathbf{k}(-6 \times 2 - 1 \times 3)
\]

Compute each component:

- \(\mathbf{i}: (1 \times -1) - (7 \times 2) = -1 - 14 = -15 \)
- \(\mathbf{j}: - [(-6 \times -1) - (7 \times 3)] = - [6 - 21] = - (-15) = 15 \)
- \(\mathbf{k}: (-6 \times 2) - (1 \times 3) = -12 - 3 = -15 \)

Thus,

\[
\mathbf{d} = (-15, 15, -15)
\]

which can be simplified to:

\[
\mathbf{d} = (-1, 1, -1)
\]

Step 6: Find a Point on the Line

To find a point \((x_0, y_0, z_0)\) on the line, solve the system:

\[
\begin{cases}
-6x + y + 7z = -15 \\
3x + 2 y - z = 4
\end{cases}
\]

Let's assign a parameter \(t\) to \(z\):

Set \(z = t\).

From the second equation:

\[

3x + 2 y - t = 4 \Rightarrow 3x + 2 y = 4 + t
\]

From the first equation:

\[
-6x + y + 7 t = -15
\]

Express \(y\) from the first:

\[
y = -15 + 6x - 7t
\]

Substitute into the second:

\[
3x + 2(-15 + 6x - 7t) = 4 + t
\]

Simplify

Frequently Asked Questions

How do I find a plane that contains the point (6, -6, 5) and intersects the given planes?

To find such a plane, you can express it as a linear combination of the given planes and ensure it passes through the point (6, -6, 5). This involves setting up equations based on the plane's normal vector and the point's coordinates.

What is the significance of the line of intersection between the planes -6x + y + 7z = -15 in determining the required plane?

The line of intersection represents all points common to both planes. Any plane that contains this line also contains all points on it, so our desired plane must include this intersection line along with the point (6, -6, 5).

How can I verify if a given plane contains both the point (6, -6, 5) and the intersection line of the two planes?

You verify by substituting the point's coordinates into the plane equation to see if it satisfies it, and check if the line's parametric equations satisfy the plane equation as well. If both do, the plane contains the point and the line.

Is there a systematic method to find all planes passing through a point and a line of intersection?

Yes. You can express the general plane as a linear combination of the two given planes and impose the condition that the plane passes through the given point. Solving the resulting equations yields the family of planes satisfying the conditions.

Can I find a specific equation for the plane containing (6, -6, 5) and the intersection line of the given planes?

Yes. By representing the plane as a combination of the two given planes and applying the point condition, you can derive the explicit equation for that plane.

What is the role of the parameter in the general equation of the plane containing the intersection line?

The parameter reflects the family of planes that contain the intersection line. Adjusting this parameter yields different planes passing through the line, and selecting the one passing through the specific point completes the solution.

Are there any special cases or considerations I should be aware of when finding this plane?

Yes. Ensure that the point (6, -6, 5) does not lie on the intersection line unless the plane is the same as one of the original planes. Also, verify that the derived plane equations satisfy all conditions to avoid errors.

Additional Resources

Find A Plane Containing The Point (6, -6, 5) And The Line Of Intersection Of The Planes $-6x + y + 7z = -15$

When approaching the problem of finding a plane that contains a specific point and the line of intersection of two given planes, it’s essential to understand the fundamental concepts of planes, lines, and their intersections in three-dimensional space. This problem combines several core ideas in analytic geometry, including the equations of planes, the line of intersection of two planes, and the conditions for a point to lie on a plane. The detailed process involves algebraic manipulation, geometric intuition, and sometimes parametrization, making it an interesting and instructive exercise for students and practitioners alike.

Understanding the Fundamentals: Planes, Points, and Lines

Planes in Three-Dimensional Space

A plane in three-dimensional space can be represented by a linear equation of the form:

$$ Ax + By + Cz + D = 0 $$

where (A, B, C) is the normal vector to the plane, and D is a scalar constant. The key characteristics include:

- The normal vector indicates the orientation of the plane.
- The plane extends infinitely in all directions within its two-dimensional surface.
- Any point (x, y, z) satisfying the plane's equation lies on the plane.

Line of Intersection of Two Planes

Given two planes:

$$
\begin{cases}
A_1x + B_1y + C_1z + D_1 = 0 \\
A_2x + B_2y + C_2z + D_2 = 0
\end{cases}
$$

Their intersection, if not empty or a single point, is a line. This line can be described parametrically or via vector equations. The line's direction vector $\mathbf{d}$ is the cross product of the normals of the two planes:

$$
\mathbf{d} = \mathbf{n}_1 \times \mathbf{n}_2
$$

where:

\[
\mathbf{n}_1 = (A_1, B_1, C_1), \quad \mathbf{n}_2 = (A_2, B_2, C_2)
\]

The line of intersection is then characterized by a point lying on both planes and the direction vector.

Formulating the Problem

The problem asks: Find a plane that contains a specific point \((6, -6, 5)\) and the line of intersection of the planes \(-6x + y + 7z = -15\).

This involves several steps:

1. Identify the line of intersection of the given planes.
2. Determine the general form of a plane that contains this line.
3. Ensure the plane passes through the given point \((6, -6, 5)\).

The key is that the sought plane must contain the entire line of intersection of the two given planes, which means it must be a linear combination of these planes.

Step-by-Step Solution

1. Find the line of intersection of the given planes

Given planes:

\[
P_1: -6x + y + 7z = -15
\]
\[
P_2: \text{(second plane)} \quad \text{(Assuming the second plane is given, but in the problem statement, only one plane is explicitly provided. Likely, the problem intended for the line of intersection of the first plane with another, or perhaps the second plane is implied.}

Note: The user’s question mentions "the line of intersection of the planes -6x + y + 7z = -15" but only provides one explicit plane. Usually, to find a line of intersection, two planes are needed. If the problem is to find a plane containing a point and the line of intersection of that plane with another plane, then the second plane must be specified.

Assumption: The problem likely involves the line of intersection of a given plane with a family of planes passing through that line. Alternatively, perhaps the problem intends for us to find a plane passing through the point and the line formed by the intersection of the plane $-6x + y + 7z = -15$ with some other plane, or the line itself is given implicitly.

Clarification needed: Since only one plane is explicitly provided, perhaps the task is to find all planes containing the point and the line of intersection of the given plane with a family of planes.

Alternative interpretation: The line of intersection is defined as the intersection of the given plane with an unspecified second plane; the goal is to find a plane containing the point and that line.

In such problems, a common approach involves:

- Recognizing that the line of intersection of two planes can be represented parametrically.
- The general form of a plane containing that line can be expressed as a linear combination of the two planes.

Let me proceed assuming the second plane is also $-6x + y + 7z = -15$. If this is the case, then the intersection is the entire plane itself, which is trivial.

Alternatively, perhaps the second plane is missing, and the problem is to find a plane containing the point $(6, -6, 5)$ and the line of intersection of the single plane with some other plane, which might be the family of planes.

Given the uncertainty, I will proceed with the common approach:

Suppose the line of intersection is of two planes:

$$
\begin{cases}
-6x + y + 7z = -15 \quad \text{(Plane 1)} \\
A x + B y + C z + D = 0 \quad \text{(Plane 2)}
\end{cases}
$$

Our goal is to find the general plane passing through the line of intersection of these two planes and contains the point $(6, -6, 5)$.

---

2. Express the family of planes containing the line of intersection

Any plane passing through the line of intersection of two planes can be written as:

$$
\text{Plane} = P_1 + \lambda P_2
$$

or, explicitly:

$$
(-6x + y + 7z + 15) + \lambda (A x + B y + C z + D) = 0
$$

where λ is a scalar parameter.

The general equation of the plane is then:

$$
(-6 + \lambda A) x + (1 + \lambda B) y + (7 + \lambda C) z + (15 + \lambda D) = 0
$$

---

3. Find λ such that the plane contains the point (6, -6, 5)

Substitute the point $(6, -6, 5)$ into the above general plane:

$$
(-6 + \lambda A) \times 6 + (1 + \lambda B) \times (-6) + (7 + \lambda C) \times 5 + (15 + \lambda D) = 0
$$

This simplifies to:

$$
6(-6 + \lambda A) -6(1 + \lambda B) + 5(7 + \lambda C) + 15 + \lambda D = 0
$$

Expanding:

$$
-36 + 6 \lambda A - 6 - 6 \lambda B + 35 + 5 \lambda C + 15 + \lambda D = 0

\]

Combine constant terms:

$$
(-36 - 6 + 35 + 15) + (6 \lambda A - 6 \lambda B + 5 \lambda C + \lambda D) = 0
$$

Calculating the constants:

$$
(-36 - 6 + 35 + 15) = 8
$$

So, the condition becomes:

$$
8 + \lambda (6A - 6B + 5C + D) = 0
$$

To satisfy this for some λ, the term in parentheses must be non-zero. Therefore,

$$
\lambda = - \frac{8}{6A - 6B + 5C + D}
$$

Provided the denominator is not zero, this determines λ, and thus the plane.

Special Cases and Simplifications

In many problems, the second plane is either explicitly given or can be chosen conveniently. For example:

- If the second plane is parallel to the first, then the intersection line is undefined or the planes are coincident.
- If the second plane is orthogonal or at a specific angle, the intersection line can be explicitly parametrized.

In the absence of explicit information about the second plane, an alternative approach is to assume the second plane is a general plane passing through the point $(6, -6, 5)$, or to consider the intersection line as given.

Alternative Approach: Using the General Form of the Plane

Suppose we are to find a plane that contains the point \((6, -6, 5)\

Find A Plane Containing The Point 6 6 5 And The Line Of Intersection Of The Planes 6 X Y 7 Z 15

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