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In algebra and polynomial theory, one common problem is to construct a polynomial function based on given roots or zeros. The zeros of a polynomial are the values of x for which the polynomial equals zero. When a polynomial has specific zeros, these zeros directly influence its factors. This article explores the process of finding a degree 4 polynomial, F(x), that has the zeros 0, 7, -4, and 5. We will systematically derive the polynomial, ensuring it aligns with the given roots and understanding the principles behind the construction.

Understanding Zeros and Polynomial Factors

Zeros of a Polynomial

The zeros of a polynomial are the solutions to the equation $F(x) = 0$. For example, if $x = a$ is a zero of the polynomial, then substituting $x = a$ into the polynomial yields zero: $F(a) = 0$. The zeros define the roots of the polynomial and are fundamental in constructing the polynomial function.

From Zeros to Factors

Each zero corresponds to a factor of the polynomial. Specifically, if a is a zero of multiplicity 1, then $(x - a)$ is a factor. For multiple zeros, the corresponding factors are raised to the appropriate power, reflecting the multiplicity.

For the zeros 0, 7, -4, and 5, the factors are:

- $x - 0$ (which simplifies to x)
- $x - 7$

- $x + 4$ (since zero is at -4, the factor is $x + 4$)
- $x - 5$

Constructing the Polynomial

Step 1: Write the Basic Factors

Given the zeros, the basic polynomial factors are:

- x
- $x - 7$
- $x + 4$
- $x - 5$

Step 2: Form the Polynomial as a Product of Factors

The polynomial $F(x)$ can be expressed as the product of these factors, possibly multiplied by a leading coefficient, c:

$$F(x) = c \quad x \quad (x - 7) \quad (x + 4) \quad (x - 5)$$

The leading coefficient c can be any non-zero constant. Usually, for simplicity and standard form, we take $c = 1$.

Step 3: Expand the Polynomial

To write the polynomial explicitly, we need to expand the product of factors:

$$F(x) = x \quad (x - 7) \quad (x + 4) \quad (x - 5)$$

We proceed step-by-step, expanding the factors systematically.

Expanding the Polynomial

Step 4: Expand $(x - 7)(x + 4)$

$(x - 7)(x + 4) = x^2 + 4x - 7x - 28 = x^2 - 3x - 28$

Step 5: Expand the result with $(x - 5)$

$(x^2 - 3x - 28)(x - 5) = x^2(x - 5) - 3x(x - 5) - 28(x - 5)$

Calculating each term:

$x^2 \;\; x = x^3$
$x^2 \;\; (-5) = -5x^2$
$-3x \;\; x = -3x^2$
$-3x \;\; (-5) = 15x$
$-28 \;\; x = -28x$
$-28 \;\; (-5) = 140$

Now, sum these:

$x^3 - 5x^2 - 3x^2 + 15x - 28x + 140 = x^3 - 8x^2 - 13x + 140$

Step 6: Multiply by x (the initial factor)

$F(x) = x \;\; (x^3 - 8x^2 - 13x + 140) = x^4 - 8x^3 - 13x^2 + 140x$

Final Polynomial Expression

Putting it all together, the polynomial with the specified zeros is:

$F(x) = x^4 - 8x^3 - 13x^2 + 140x$

Additional Considerations

Leading Coefficient

In the derivation above, we assumed the leading coefficient $c = 1$. If desired, you can multiply the entire polynomial by any non-zero constant to scale the polynomial. For example, if you want the polynomial to have a leading coefficient of 3, multiply all terms by 3:

$F(x) = 3x^4 - 24x^3 - 39x^2 + 420x$

Multiplicity of Zeros

In this problem, each zero is assumed to have multiplicity 1. If zeros have higher multiplicity, the corresponding factors are raised to higher powers. For example, if 7 had multiplicity 2, the factor $(x - 7)^2$ would be used, and the degree of the polynomial would increase accordingly.

Conclusion

In summary, to find a degree 4 polynomial with given zeros 0, 7, -4, and 5, you start by translating each zero into a factor, multiply these factors, and expand the product to get the polynomial in standard form. The resulting polynomial is:

$F(x) = x^4 - 8x^3 - 13x^2 + 140x$

This polynomial has the specified zeros, and its degree is 4, matching the number of zeros provided. Adjustments to the leading coefficient can be made depending on specific requirements or normalization preferences. This process exemplifies the fundamental link between zeros and polynomial factors, illustrating how algebraic roots directly inform the structure of polynomial functions.

Frequently Asked Questions

How do I find a degree 4 polynomial with zeros at 0, 7, -4, and 5?

Since the zeros are given, the polynomial can be written as $F(x) = a \cdot x \cdot (x - 7) \cdot (x + 4) \cdot (x - 5)$, where a is any non-zero constant. To find a specific polynomial, choose a value for a, typically 1, and expand the factors.

What is the general form of a degree 4 polynomial with the zeros 0, 7, -4, and 5?

The general form is $F(x) = a \cdot x \cdot (x - 7) \cdot (x + 4) \cdot (x - 5)$, where $a \neq 0$. If you set $a=1$, then $F(x) = x \cdot (x - 7) \cdot (x + 4) \cdot (x - 5)$.

How do I expand the polynomial F(x) = x·(x - 7)·(x + 4)·(x - 5) to standard form?

First, multiply pairs of factors: (x - 7)(x + 4) = x^2 - 3x - 28, then multiply (x^2 - 3x - 28) by (x - 5), and finally multiply the result by x. Carefully expand step-by-step to get the polynomial in standard form.

Can I choose a different value for 'a' when finding the polynomial?

Yes, 'a' can be any non-zero constant. Setting a=1 gives a monic polynomial, but choosing other values will scale the polynomial accordingly.

What is the expanded form of the polynomial with zeros 0, 7, -4, and 5, assuming a=1?

The expanded polynomial is F(x) = x^4 - 3x^3 - 84x^2 + 420x.

Why is the zero at x=0 important when constructing the polynomial?

The zero at x=0 indicates that the polynomial has a factor of x, ensuring the polynomial equals zero when x=0. This simplifies the construction process.

How do I verify that the polynomial I found has the given zeros?

Substitute each zero into the polynomial. If the result is zero for each, then the polynomial has those zeros. For example, check that F(0)=0, F(7)=0, F(-4)=0, and F(5)=0.

What does the phrase 'Leave Your Answer In The' mean in this context?

It appears to be an incomplete instruction, likely meaning 'Leave your answer in the factored form' or 'standard form.' Usually, the problem asks you to express the polynomial fully expanded or factored, depending on the context.

Additional Resources

Polynomial F(x) of Degree 4 with Zeros 0, 7, -4, 5: An In-Depth Exploration

When delving into the fascinating world of polynomials, one of the fundamental tasks is constructing a polynomial function that aligns with a given set of zeros (roots). This process, while seemingly straightforward, involves a nuanced understanding of how zeros translate into factors, the importance of multiplicities, and how to express the final polynomial in a simplified, canonical form. In this article, we'll explore in detail how to find a polynomial $F(x)$ of degree 4 that has the zeros 0, 7, -4, and 5, leaving the answer in the factored form.

Understanding Zeros and Their Relationship to Polynomials

Before jumping into the construction process, it's essential to clarify what zeros are and how they connect to polynomial functions.

What Are Zeros of a Polynomial?

Zeros, also known as roots, are the values of x for which the polynomial evaluates to zero:

$$
F(x) = 0
$$

For example, if $x = 7$ is a zero of $F(x)$, then substituting $x = 7$ into $F(x)$ yields zero:

$$
F(7) = 0
$$

Zeros can be real or complex, and they may have multiplicities greater than one. In our case, the zeros are 0, 7, -4, and 5, each with implied multiplicity of one unless specified otherwise.

From Zeros to Factors: Building Blocks of the Polynomial

The fundamental theorem of algebra states that every polynomial of degree n has exactly n roots in the complex plane (counting multiplicities). Conversely, each zero corresponds to a factor of the polynomial.

Constructing Factors from Zeros

Given a zero a, the corresponding factor is $(x - a)$. Therefore:

- Zero at 0 corresponds to $x - 0 = x$
- Zero at 7 corresponds to $x - 7$
- Zero at -4 corresponds to $x - (-4) = x + 4$
- Zero at 5 corresponds to $x - 5$

Since each zero is of multiplicity one, the polynomial's factored form is the product of these linear factors:

$$
F(x) = k \times x \times (x - 7) \times (x + 4) \times (x - 5)
$$

where k is a non-zero constant coefficient. The choice of k affects the scaling but does not influence the zeros.

Formulating the Polynomial: The Factored Form

Given the above, the general form of the polynomial is:

$$
F(x) = k \times x \times (x - 7) \times (x + 4) \times (x - 5)
$$

This expression guarantees the zeros at 0, 7, -4, and 5, each with multiplicity one. Since the problem specifies a degree 4 polynomial, and the product involves four factors, this aligns perfectly.

Choosing the Leading Coefficient k

The constant k scales the polynomial vertically. In most standard problems, unless specified, the leading coefficient is taken to be 1 for simplicity, resulting in a monic polynomial.

Why choose $k = 1$?

- Simplifies calculations
- Provides a standard form
- Makes the polynomial easier to analyze

If no additional conditions are given (such as the polynomial passing through a particular point), then choosing $k = 1$ is appropriate.

Thus, our polynomial simplifies to:

$$
F(x) = x \times (x - 7) \times (x + 4) \times (x - 5)
$$

Expanding the Polynomial: From Factored to Standard Form

While the factored form is often the most informative—especially for identifying zeros—it's also helpful to expand the polynomial into standard form for various applications, such as graphing or analyzing coefficients.

Step-by-Step Expansion

Let's expand:

$$
F(x) = x \times (x - 7) \times (x + 4) \times (x - 5)
$$

Step 1: Expand the quadratic factors first

- Multiply $(x - 7)$ and $(x + 4)$:

$$
(x - 7)(x + 4) = x \times x + x \times 4 - 7 \times x - 7 \times 4 = x^2 + 4x - 7x - 28 = x^2 - 3x - 28
$$

- Multiply $(x^2 - 3x - 28)$ by $(x - 5)$:

$$
(x^2 - 3x - 28)(x - 5)
$$

Applying distributive property:

$$
x^2 \times x = x^3
$$

$$
x^2 \times (-5) = -5x^2
$$
$$
-3x \times x = -3x^2
$$
$$
-3x \times (-5) = +15x
$$
$$
-28 \times x = -28x
$$
$$
-28 \times (-5) = +140
$$

Summing all terms:

$$
x^3 - 5x^2 - 3x^2 + 15x - 28x + 140 = x^3 - 8x^2 - 13x + 140
$$

Step 2: Multiply the result by x:

$$
F(x) = x \times (x^3 - 8x^2 - 13x + 140)
$$

Distribute:

$$
x \times x^3 = x^4
$$
$$
x \times (-8x^2) = -8x^3
$$
$$
x \times (-13x) = -13x^2
$$
$$
x \times 140 = 140x
$$

Final expanded form:

$$
F(x) = x^4 - 8x^3 - 13x^2 + 140x
$$

Summary of the Polynomial

Factored form:

$$
\boxed{
F(x) = x \times (x - 7) \times (x + 4) \times (x - 5)
}
$$

Expanded form:

$$
\boxed{
F(x) = x^4 - 8x^3 - 13x^2 + 140x
}
$$

Additional Considerations and Variations

While the primary goal is to find a polynomial of degree 4 with the given zeros, here are some additional points to consider:

1. Multiplicities of Zeros

- If any zero had multiplicity greater than one, the corresponding factor would be raised to that power.
- For example, if zero at 7 has multiplicity 2, the factor $(x - 7)^2$ would be included.

2. Leading Coefficient Variations

- If you want the polynomial to have a different leading coefficient, multiply the entire polynomial by that constant.
- For example, choosing $k = 3$:

$$
F(x) = 3 \times x \times (x - 7) \times (x + 4) \times (x - 5)
$$

and expanding accordingly.

3. Graphical Implications

- The zeros determine the x-intercepts of the graph.
- The multiplicities influence the behavior of the graph at each zero (e.g.,

whether it touches or crosses the x-axis).

Conclusion: The Art of Polynomial Construction

Constructing a polynomial function based on specified zeros is a foundational skill in algebra, with applications spanning from solving equations to modeling real-world phenomena. By understanding the relationship between zeros and factors, selecting an appropriate leading coefficient, and expanding or maintaining the factored form, you can craft precise polynomial functions tailored to your needs.

In this specific case, the polynomial of degree 4 with zeros at 0, 7, -4, and 5 is most straightforwardly expressed as:

$$
F(x) = x \times (x - 7) \times (x + 4) \times (x - 5)
$$

which can be left in factored form for clarity and ease of interpretation, or expanded into standard polynomial form for detailed analysis.

Whether for academic purposes or real-world modeling, mastering this process enhances your mathematical toolkit and deepens your understanding of polynomial behavior.

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