Find A Polynomial F(x) Of Degree 4 With Real Coefficients And The Following Zeros.3 (multiplicity 2)

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Creating a polynomial with specified zeros is a fundamental concept in algebra, especially in understanding polynomial functions and their roots. When the problem specifies that a polynomial is of degree 4 with real coefficients and includes a zero at 3 with multiplicity 2, it provides essential information to construct the polynomial explicitly. This guide aims to walk you through the process of deriving such a polynomial step-by-step, clarifying the underlying principles, and offering tips for generalization.

Understanding Polynomial Zeros and Multiplicities

Before diving into the construction of the polynomial, it’s important to clarify some key concepts.

Zeros of a Polynomial

A zero (or root) of a polynomial $F(x)$ is a value c such that $F(c) = 0$. These zeros can be real or complex numbers and are fundamental in defining the polynomial.

Multiplicity of a Zero

The multiplicity of a zero refers to the number of times that zero appears as a root. For example, if $(x - 3)^2$ is a factor of the polynomial, then 3 is a zero with multiplicity 2.

Implications for the Polynomial’s Form

- The degree of the polynomial is the sum of the multiplicities of all zeros.
- Real coefficients imply that complex roots occur in conjugate pairs, but since only a real zero is specified, this consideration is not necessary here.
- The multiplicity affects the shape of the graph near the zero; for even multiplicities, the graph touches the x-axis and turns around, while for odd multiplicities, it crosses the x-axis.

Constructing the Polynomial from the Given Zero and Degree

Given the information:

- The polynomial $F(x)$ has degree 4.
- The polynomial has a zero at $x = 3$ with multiplicity 2.
- The coefficients are real, which aligns with the zeros being real or complex conjugate pairs (not specified here).

To fully determine the polynomial, we need to identify the remaining zeros.

Step 1: Identify Known Factors

Since zero 3 has multiplicity 2, the corresponding factor is:

$$
(x - 3)^2
$$

Because the polynomial is degree 4, and the multiplicity of zero 3 accounts for degree 2, the remaining degree 2 must come from other zeros.

Step 2: Decide on the Remaining Zeros

Options include:
- Another real zero a with multiplicity 2.
- Two distinct real zeros a and b, each with multiplicity 1.
- Complex conjugate zeros (if any), which would be pairs like $p \pm qi$.

Since the problem specifies only the zero at 3, and no other zeros are given, the simplest approach is to assume the remaining zeros are real and distinct or repeated, but for clarity, we will assume the remaining zeros are real and distinct.

Most straightforward assumption:
- The remaining degree 2 zeros are at $x = a$ and $x = b$, each with multiplicity 1.

Thus, the polynomial factors are:

$$
F(x) = k (x - 3)^2 (x - a)(x - b)
$$

where k is a leading coefficient, which we can assume to be 1 for simplicity unless specified otherwise.

Explicit Construction of the Polynomial

Assuming the simplest case with $k = 1$, the polynomial becomes:

$$
F(x) = (x - 3)^2 (x - a)(x - b)
$$

where a and b are real numbers.

Choosing specific zeros for illustrative purposes:

- Let's pick $a = 1$ and $b = 5$, for example.

The polynomial is:
$$
F(x) = (x - 3)^2 (x - 1)(x - 5)
$$

Expanding step-by-step:

1. Expand $(x - 3)^2$:
$$
(x - 3)^2 = x^2 - 6x + 9
$$

2. Expand $(x - 1)(x - 5)$:
$$
(x - 1)(x - 5) = x^2 - 5x - x + 5 = x^2 - 6x + 5
$$

3. Multiply the two quadratic factors:
$$
F(x) = (x^2 - 6x + 9)(x^2 - 6x + 5)
$$

4. Use distributive multiplication:
$$
F(x) = x^2 \cdot x^2 - 6x \cdot x^2 + 9 \cdot x^2 - 6x \cdot (-6x) + 36x^2 - 54x + 5x^2 - 30x + 45
$$

But it's easier to expand systematically:

$$
F(x) = (x^2)(x^2 - 6x + 5) - 6x(x^2 - 6x + 5) + 9(x^2 - 6x + 5)
$$

Calculating each:

- $x^2 \cdot x^2 = x^4$
- $x^2 \cdot (-6x) = -6x^3$
- $x^2 \cdot 5 = 5x^2$

- $-6x \cdot x^2 = -6x^3$
- $-6x \cdot (-6x) = 36x^2$
- $-6x \cdot 5 = -30x$

- $9 \cdot x^2 = 9x^2$
- $9 \cdot (-6x) = -54x$
- $9 \cdot 5 = 45$

Now, combine like terms:

$$
F(x) = x^4 + (-6x^3 - 6x^3) + (5x^2 + 36x^2 + 9x^2) + (-30x - 54x) + 45
$$

Simplify:

$$
F(x) = x^4 - 12x^3 + (5 + 36 + 9) x^2 - (30 + 54) x + 45
$$
$$
F(x) = x^4 - 12x^3 + 50x^2 - 84x + 45
$$

This is a polynomial of degree 4 with real coefficients, zeros at $x = 3$ (multiplicity 2), $x = 1$, and $x = 5$.

General Form and Flexibility

The specific zeros a and b can be any real numbers, giving a family of polynomials:

$$
F(x) = (x - 3)^2 (x - a)(x - b)
$$

which expands to:

$$
F(x) = x^4 - (a + b + 6) x^3 + (ab + 6a + 6b + 9) x^2 - (6ab + 9a + 9b) x + 9ab
$$

Key points:
- The coefficients depend on the choice of a and b.
- The roots at 3 have multiplicity 2, as specified.
- The degree is 4, matching the problem statement.
- The coefficients are real if a and b are real.

Alternative Scenarios and Considerations

While the above example assumes two additional real zeros, other configurations are possible, depending on the problem context.

Scenario 1: Zero at 3 with multiplicity 2, and a complex conjugate pair

- Since the coefficients are real, complex roots must come in conjugate pairs, e.g., $p \pm qi$.
- The factors corresponding to these zeros would be quadratic:

$$
x^2 - 2px + (p^2 + q^2)
$$

- The polynomial would then be:

$$
F(x) = (x - 3)^2 \left(x^2 - 2px + p^2 + q^2 \right)
$$

- Expanding this form yields a degree 4 polynomial with real coefficients.

Scenario 2: Zero at 3 with multiplicity 2 and other zeros at zero or negative values

- For example, zeros at 3 (multiplicity 2) and 0 (multiplicity 2):

$$
F(x) = (x)^2 (x - 3)^2
$$

- Which expands to:

$$
x^2 (x^2 - 6x + 9) = x^4 - 6x^3 + 9x^2
$$

Summary and Final Remarks

Constructing

Frequently Asked Questions

How do I find a degree 4 polynomial with real coefficients given a zero at 3 with multiplicity 2?

Since 3 has multiplicity 2, $(x - 3)^2$ is a factor. To find the polynomial, choose other factors as needed, but if only this zero is given, the polynomial can be expressed as $F(x) = a(x - 3)^2$, where a is any real number. For a degree 4 polynomial, you need four roots or factors; if none are specified, you can include factors to reach degree 4, such as $(x - 3)^2$ multiplied by quadratic factors with real coefficients.

What is the general form of the polynomial F(x) with a zero at 3 with multiplicity 2 and degree 4?

A general form would be F(x) = a(x - 3)^2 (x^2 + bx + c), where a, b, and c are real numbers chosen to satisfy additional conditions or constraints. The quadratic factor ensures the total degree is 4, and the multiplicity 2 at zero 3 is reflected in (x - 3)^2.

Can you give an example of a degree 4 polynomial with real coefficients having zero at 3 with multiplicity 2?

Yes. For example, choosing a = 1, b = 0, c = -1, the polynomial is F(x) = (x - 3)^2 (x^2 - 1) = (x - 3)^2 (x - 1)(x + 1).

How do the multiplicity of zero at 3 and the degree of the polynomial relate?

The multiplicity of zero at 3 indicates how many times (x - 3) appears as a factor. For degree 4 with zero at 3 of multiplicity 2, the remaining degree 2 must be accounted for by other factors (quadratic with real coefficients). So, the sum of multiplicities of zeros equals the degree of the polynomial.

Why must the polynomial have real coefficients if the zero at 3 has multiplicity 2?

Because zeros with multiplicity 2 at 3 are real zeros, and the polynomial has real coefficients, the conjugate roots theorem does not require additional complex roots. The factor (x - 3)^2 ensures the zero at 3 is real and of multiplicity 2, consistent with the polynomial having real coefficients.

What additional zeros or factors are needed to complete a degree 4 polynomial with the given zero at 3?

You need two more roots or factors. These can be real zeros, such as (x - r1) and (x - r2), or complex conjugate zeros if they exist in pairs. If no other zeros are specified, you can choose quadratic factors with real coefficients, such as (x^2 + px + q), to reach degree 4.

Additional Resources

Polynomial Construction

Creating a polynomial function with specific properties, such as a certain degree and prescribed zeros, is a fundamental task in algebra that combines theoretical insight with practical application. When tasked with finding a degree-4 polynomial, \(F(x) \), with real coefficients and a known zero at \(x = 3 \) with multiplicity 2, the process involves understanding key concepts like roots, multiplicities, complex conjugate pairs, and polynomial expansion. This article delves deeply into the step-by-step methodology, underlying principles, and nuances involved in constructing such a polynomial, providing clarity and thoroughness for students, educators, and enthusiasts alike.

Understanding the Basic Concepts: Zeros, Multiplicity, and Polynomial Structure

Before embarking on the construction of the polynomial, it is crucial to grasp the fundamental ideas that govern polynomial roots and their multiplicities.

Zeros of a Polynomial

A zero (or root) of a polynomial $F(x)$ is a value c such that $F(c) = 0$. Zeros are intimately linked to the factors of the polynomial via the Factor Theorem, which states that if c is a zero, then $(x - c)$ is a factor of $F(x)$.

Multiplicity of a Zero

The multiplicity of a zero indicates how many times that zero appears as a root. For example, if $(x - 3)^2$ divides $F(x)$, then $x=3$ is a zero with multiplicity 2. The multiplicity affects the behavior of the polynomial at that root, influencing the graph's shape — such as whether it touches or crosses the x-axis at that zero.

Degree and Its Implications

The degree of the polynomial reflects the highest power of x. For our task, the degree is 4, which constrains the total multiplicities of all zeros (considering multiplicities). The sum of the multiplicities of all zeros must equal 4.

Given Conditions and Their Consequences

Our goal is to find a polynomial $F(x)$:

- Of degree 4
- With real coefficients
- Having a zero at $x=3$ with multiplicity 2

The key implications are:

- Since $x=3$ has multiplicity 2, $(x - 3)^2$ is a factor.
- The remaining factors must multiply to a quadratic polynomial with real coefficients, since the total degree is 4.
- The other zeros could be real or complex conjugate pairs, but the polynomial must have real coefficients, so complex roots come in conjugate pairs.

Step-by-Step Construction of $F(x)$

Let's break down the process into manageable steps, considering options for the remaining roots and ensuring the polynomial is well-formed.

Step 1: Incorporate the Known Zero with Its Multiplicity

Given the zero at $x=3$ with multiplicity 2, the initial factor is:

$$
(x - 3)^2
$$

This factor guarantees the zero at 3 with the specified multiplicity.

Step 2: Determine the Remaining Roots

Since the polynomial is degree 4, and we've accounted for two roots (via $(x - 3)^2$), the remaining quadratic factor must encompass the other two roots.

Possible scenarios:

- Both roots are real and distinct, leading to two linear factors.
- Roots are complex conjugates, leading to an irreducible quadratic factor over the reals.

To keep the polynomial general and flexible, we'll consider both options.

Option A: Both remaining roots are real and distinct

Suppose the other roots are r and s, both real numbers. Then, the quadratic factor is:

$$
(x - r)(x - s) = x^2 - (r + s)x + rs
$$

Thus, the full polynomial:

$$
F(x) = (x - 3)^2 [x^2 - (r + s)x + rs]
$$

where r and s are real numbers to be chosen.

Option B: Remaining roots are complex conjugates

Suppose the other roots are $a + bi$ and $a - bi$, with $b \neq 0$. Their quadratic factor is:

$$
(x - (a + bi))(x - (a - bi)) = x^2 - 2ax + (a^2 + b^2)
$$

which is a quadratic with real coefficients.

Therefore, the general form in this case:

$$
F(x) = (x - 3)^2 [x^2 - 2ax + (a^2 + b^2)]
$$

where $a, b \in \mathbb{R}$, $b \neq 0$.

Constructing a Specific Polynomial: An Example Approach

To illustrate the process concretely, let's choose specific values for the remaining roots in each scenario. This approach helps clarify how different choices influence the polynomial's form.

Example 1: Real Roots $r=0$ and $s=2$

- The quadratic factor:

$$
(x - 0)(x - 2) = x(x - 2) = x^2 - 2x
$$

- The polynomial:

$$
F(x) = (x - 3)^2 \cdot (x^2 - 2x)
$$

- Expanding:

$$
F(x) = (x^2 - 6x + 9)(x^2 - 2x)
$$

- Multiplying out:

$$
F(x) = x^2 \cdot x^2 - 2x \cdot x^2 - 6x \cdot x^2 + 12x^2 + 9 \cdot x^2 - 18x
$$

Calculations:

$$
F(x) = x^4 - 2x^3 - 6x^3 + 12x^2 + 9x^2 - 18x
$$

Combine like terms:

$$
F(x) = x^4 - 8x^3 + 21x^2 - 18x
$$

This polynomial has degree 4, real coefficients, and zeros at $x=3$ (multiplicity 2), $x=0$, and $x=2$.

Example 2: Complex Conjugate Roots $a=1, b=2$

- The quadratic factor:

$$
(x - (1 + 2i))(x - (1 - 2i)) = x^2 - 2 \cdot 1 \cdot x + (1^2 + 2^2) = x^2 - 2x + 5
$$

- The polynomial:

$$
F(x) = (x - 3)^2 (x^2 - 2x + 5)
$$

- Expanding:

$$
F(x) = (x^2 - 6x + 9)(x^2 - 2x + 5)

\]

- Multiply term-by-term:

\[
x^2 \cdot x^2 = x^4
\]
\[
x^2 \cdot (-2x) = -2x^3
\]
\[
x^2 \cdot 5 = 5x^2
\]
\[
-6x \cdot x^2 = -6x^3
\]
\[
-6x \cdot (-2x) = 12x^2
\]
\[
-6x \cdot 5 = -30x
\]
\[
9 \cdot x^2 = 9x^2
\]
\[
9 \cdot (-2x) = -18x
\]
\[
9 \cdot 5 = 45
\]

Adding all these:

\[
F(x) = x^4 - 2x^3 + 5x^2 - 6x^3 + 12x^2 - 30x + 9x^2 - 18x + 45
\]

Combine like terms:

\[
F(x) = x^4 - (2x^3 + 6x^3) + (5x^2 + 12x^2 + 9x^2) - (30x + 18x) + 45
\]
\[
F(x) = x^4 - 8x^3 + 26x^2 - 48x + 45
\]

This polynomial features the zero at \(x=3 \) with multiplicity 2, and complex conjugate roots \(1 \pm 2i \).

General Form and Flexibility in Construction

The above examples demonstrate the flexibility in choosing the remaining roots:

- You can select any real numbers for r and s in the case of real roots.
- You can choose any a and b (with $b \neq 0$) to represent complex conjugate roots.

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