Find A Sinusoidal Function With The Following Four Attributes: (1) Amplitude Is 10, (2) Period Is 4,

Find A Sinusoidal Function With The Following Four Attributes: (1) Amplitude Is 10, (2) Period Is 4, is a common problem in mathematics, especially in the study of trigonometry and periodic functions. Understanding how to derive a sinusoidal function—such as sine or cosine—that fits given characteristics is essential for students and professionals working in fields like engineering, physics, and data analysis. In this comprehensive guide, we will explore step-by-step how to find a sinusoidal function with specified amplitude and period, and how to interpret and apply these attributes effectively.

Understanding Sinusoidal Functions

Before diving into the process of constructing a specific function, it's important to understand what sinusoidal functions are and their fundamental properties.

What Is a Sinusoidal Function?

A sinusoidal function is a mathematical function that describes a smooth, repetitive oscillation. The most common examples are the sine and cosine functions:

- Sine function: \(y = \sin(x) \)
- Cosine function: \(y = \cos(x) \)

Both functions are periodic, meaning they repeat their values at regular intervals over their domain.

Key Attributes of Sinusoidal Functions

The general form of a sinusoidal function can be expressed as:

$$y = A \sin(B(x - C)) + D$$

or

$$y = A \cos(B(x - C)) + D$$

where:

- A (Amplitude): The maximum deviation from the central value (vertical shift). It determines the height of the wave.
- B (Frequency or Angular Frequency): Controls the period of the wave. Specifically, the period T is related to B by:

$$T = \frac{2\pi}{|B|}$$

- C (Phase Shift): Horizontal shift of the graph.
- D (Vertical Shift): Vertical displacement of the entire graph.

In this context, we are focusing on amplitude and period, so C and D are not specified and can be assumed to be zero unless otherwise indicated.

Given Attributes and Their Significance

The problem specifies two key attributes:

1. Amplitude Is 10:

The amplitude A is 10, meaning the graph reaches 10 units above and below its central value (assuming no vertical shift).

2. Period Is 4:

The period T is 4, indicating the length of one complete cycle of the wave along the x-axis.

Knowing these two parameters allows us to construct the sinusoidal function precisely.

Step-by-Step Process to Find the Sinusoidal Function

Let's proceed systematically to find the function.

Step 1: Determine the Amplitude

Given:

$$A = 10$$

This directly impacts the maximum and minimum points of the wave:

- Maximum value: $D + A$
- Minimum value: $D - A$

Assuming no vertical shift (i.e., $D = 0$), the maximum and minimum are simply 10 and -10.

Step 2: Find the B Parameter Using the Period

Recall the relationship between period T and B:

$$T = \frac{2\pi}{|B|}$$

Given:

$$T = 4$$

Solve for B:

$$|B| = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}$$

Assuming a positive B (standard orientation):

$$B = \frac{\pi}{2}$$

Step 3: Choose the Function Type (Sine or Cosine)

Both sine and cosine functions are valid options. The choice depends on the phase shift C and the initial point on the wave.

- If the wave begins at its maximum when $x = 0$, then a cosine function is suitable.
- If the wave begins at zero crossing (going upward) at $x = 0$, then sine is appropriate.

For simplicity, and since no phase shift is specified, we will choose the cosine function, which starts at its maximum when $C = 0$:

$$ y = A \cos(Bx) $$

Step 4: Write the Basic Function

Substituting the known values:

$$ y = 10 \cos \left(\frac{\pi}{2} x \right) $$

This function has an amplitude of 10 and a period of 4.

Optional: Incorporate Phase and Vertical Shifts

If the problem specifies phase shifts or vertical shifts, we can modify the function accordingly:

$$ y = A \cos \left(B(x - C) \right) + D $$

- For phase shifts, adjust C.
- For vertical shifts, adjust D.

Since neither is specified in our problem, we assume $C = 0$ and $D = 0$.

Visualizing the Function

Understanding the graph of the sinusoidal function helps in interpreting its attributes:

- Amplitude: The wave peaks at $y = 10$ and troughs at $y = -10$.
- Period: The wave completes one full cycle between $x = 0$ and $x = 4$.
- Key points:
- Max at $x = 0$: $y = 10$
- Zero crossing going downward at $x = 1$: $y = 0$
- Min at $x = 2$: $y = -10$
- Zero crossing going upward at $x = 3$: $y = 0$
- Max at $x = 4$: $y = 10$

Summary of the Sinusoidal Function

Based on the given attributes and assumptions:

$$\boxed{
\textbf{Function:} \quad y = 10 \cos \left(\frac{\pi}{2} x \right)
}$$

This function captures the specified amplitude and period perfectly.

Additional Examples and Variations

While the above is a standard solution, there are other valid functions depending on phase shifts or choosing sine instead of cosine:

- Sine form with no phase shift:

$$ y = 10 \sin \left(\frac{\pi}{2} x \right) $$

- Including phase shift C:

$$ y = 10 \cos \left(\frac{\pi}{2} (x - C) \right) $$

where C could be any value depending on the wave's horizontal shift.

Applications of Sinusoidal Functions with Given Attributes

Understanding how to construct a sinusoidal function with specific amplitude and period is crucial in various applications:

- Engineering: Analyzing wave patterns, signal processing, and vibrations.
- Physics: Modeling periodic phenomena like sound waves and electromagnetic waves.
- Biology: Describing biological rhythms, such as circadian cycles.
- Economics: Modeling cyclical trends in markets.

Tips for Mastering Sinusoidal Functions

- Always start by identifying key attributes: amplitude, period, phase shift, and vertical shift.
- Use the fundamental relationships between period and B to find the frequency parameter.
- Choose sine or cosine based on initial conditions or given information.
- Visualize the graph to ensure the function aligns with the specifications.
- Practice with different combinations of parameters to strengthen understanding.

Conclusion

Finding a sinusoidal function with specific attributes involves understanding the relationship between the function’s parameters and its graphical features. For the given problem—amplitude of 10 and period of 4—the function:

$$y = 10 \cos \left(\frac{\pi}{2} x \right)$$

perfectly satisfies the criteria. Mastery of these concepts enables you to model various periodic phenomena accurately, whether in academic studies, engineering projects, or data analysis.

By following these steps and understanding the underlying mathematics, you will be well-equipped to tackle similar problems and develop a solid intuition for sinusoidal functions and their applications.

Frequently Asked Questions

How do you find the sinusoidal function given an amplitude of 10 and a period of 4?

Use the general form y = A sin(B(x - C)) + D or y = A cos(B(x - C)) + D, where A is the amplitude, B relates to the period as B = 2π / period, and C, D are phase shift and vertical shift. For an amplitude of 10 and period 4, set A=10 and B=2π/4=π/2.

What is the value of B in the sinusoidal function when the period is 4?

B = 2π / period = 2π / 4 = π/2.

Can you provide an example of a sinusoidal function with amplitude 10 and period 4?

Yes, an example is y = 10 sin((π/2) x), assuming no phase shift and vertical shift.

How does the period relate to the coefficient B in the sinusoidal function?

The period T is related to B by the formula T = 2π / B. Therefore, B = 2π / T.

If the amplitude is 10 and the period is 4, what are possible phase shifts or vertical shifts?

Phase shifts C can be any real number depending on the desired shift along the x-axis, and vertical shift D can be any real number depending on the shift along the y-axis. Without additional info, the basic function can be y = 10 sin((π/2) x + C) + D.

What is the general form of the sinusoidal function with given

amplitude and period?

The general form is $y = A \sin(B(x - C)) + D$ or $y = A \cos(B(x - C)) + D$, where $A=10$, $B=\pi/2$, and C, D are adjustable parameters for phase and vertical shifts.

Additional Resources

Find A Sinusoidal Function With The Following Four Attributes: (1) Amplitude Is 10, (2) Period Is 4

Introduction

In the realm of mathematical functions, sinusoidal functions hold a prominent place due to their widespread applications across physics, engineering, signal processing, and even biological systems. Their inherent periodicity and smooth oscillations make them ideal models for phenomena ranging from sound waves to electromagnetic signals. When tasked with identifying a specific sinusoidal function that adheres to particular attributes—such as amplitude and period—the process involves understanding the fundamental properties of sine and cosine functions, and how these parameters influence their graphical and analytical characteristics.

This article delves into the intricate process of deriving a sinusoidal function with an amplitude of 10 and a period of 4. We explore the foundational concepts, mathematical derivations, and practical implications, providing a comprehensive guide suitable for students, educators, and professionals seeking clarity on this classic problem.

Understanding Sinusoidal Functions: Fundamental Concepts

1. The Basic Forms

Sinusoidal functions can be represented in two primary forms:

- Sine Function: $y = A \sin(B(x - C)) + D$
- Cosine Function: $y = A \cos(B(x - C)) + D$

where:
- A (Amplitude): The maximum absolute value from the midline, indicating the peak deviation.
- B (Frequency): Related to the period, influencing how many cycles occur over a given interval.
- C (Phase Shift): Horizontal shift along the x-axis.
- D (Vertical Shift or Midline): The y-value around which the wave oscillates.

For the current problem, unless specified, the phase shift C and vertical shift D are typically set to zero for simplicity.

2. Key Properties

- Amplitude (A): Determines the height of the wave; specifically, the maximum and minimum values are $D + A$ and $D - A$.
- Period (T): The length of one complete cycle, defined as:

$$
T = \frac{2\pi}{|B|}
$$

where B influences the frequency.

Understanding these properties allows us to reverse-engineer the parameters needed to produce a sinusoid with desired amplitude and period.

Deriving the Function Parameters

1. Setting the Amplitude

Given the amplitude $A = 10$, the sinusoid will oscillate between $D + 10$ and $D - 10$. For simplicity, and unless specified otherwise, we can choose the midline $D = 0$. This choice results in a wave oscillating between -10 and 10.

2. Calculating the B Parameter for the Period

Given the period $T = 4$, the relation:

$$
T = \frac{2\pi}{|B|}
$$

can be rearranged to:

$$
|B| = \frac{2\pi}{T} = \frac{2\pi}{4} = \frac{\pi}{2}
$$

Choosing $B = \frac{\pi}{2}$ maintains the standard orientation of the sinusoid.

Constructing the Sinusoidal Function

Based on the above calculations, the general form of the sinusoid with the specified attributes (amplitude 10, period 4, no phase or vertical shift) is:

$$
y = 10 \sin\left(\frac{\pi}{2} x \right)
$$

or equivalently,

$$
y = 10 \cos\left(\frac{\pi}{2} x \right)
$$

Both functions have the same amplitude and period; the choice depends on the phase shift and the starting point of the wave.

Deep Dive: Analyzing the Derived Function

1. Graphical Representation

Plotting $y = 10 \sin\left(\frac{\pi}{2} x \right)$:

- Amplitude: 10 units (max and min at 10 and -10).
- Period: 4 units; the wave completes a full cycle over an interval of length 4.
- Key points:

- $x = 0$, $y = 0$ (since $\sin(0) = 0$)
- $x = 1$, $y = 10 \sin(\frac{\pi}{2} \times 1) = 10 \times 1 = 10$
- $x = 2$, $y = 10 \sin(\pi) = 0$
- $x = 3$, $y = 10 \sin\left(\frac{3\pi}{2}\right) = -10$
- $x = 4$, $y = 10 \sin(2\pi) = 0$

This pattern repeats every 4 units, confirming the period.

2. Alternative: Using Cosine

Similarly, the cosine form:

$$
y = 10 \cos\left(\frac{\pi}{2} x \right)
$$

starts at $y = 10$ when $x = 0$, offering an alternative phase shift.

Variations and Additional Considerations

While the basic form suffices, real-world applications often require adjustments:

- Phase Shift (C): To shift the wave horizontally.
- Vertical Shift (D): To shift the wave vertically.
- Different Trigonometric Forms: Using sine or cosine depending on initial conditions.

For example, if the wave is to start at its maximum value when $x = 0$, the cosine form is naturally suited:

$$
y = 10 \cos\left(\frac{\pi}{2} x \right)
$$

Alternatively, if starting at zero crossing, sine is more appropriate.

Practical Implications and Applications

Understanding how to derive a sinusoidal function with specific attributes is crucial in various fields:

- Signal Processing: Designing waveforms with precise frequency and amplitude.
- Physics: Modeling oscillations such as pendulums or electromagnetic waves.
- Engineering: Signal modulation and control systems.
- Biology: Analyzing biological rhythms like circadian cycles.

In engineering, for example, creating a wave with a period of 4 units (seconds, meters, etc.) and an amplitude of 10 units might involve designing circuits or signals that match these specifications precisely.

Summary of the Procedure

To find a sinusoidal function with given amplitude and period:

1. Identify the amplitude (A): Directly from the problem.
2. Determine the period (T): Given.
3. Calculate B:

\[
B = \frac{2\pi}{T}
\]

4. Construct the function:

- Using sine:

$$
y = A \sin(Bx)
$$

- Using cosine:

$$
y = A \cos(Bx)
$$

5. Adjust phase and vertical shifts as needed for specific starting points or midline positions.

Final Remarks

The task of finding a sinusoidal function with specified attributes is a fundamental exercise in understanding periodic functions. By mastering the relationships between amplitude, period, and their parameters, students and professionals can accurately model oscillatory phenomena across disciplines. The explicit derivation of:

$$
\boxed{
y = 10 \sin\left(\frac{\pi}{2} x \right)
}
$$

or

\[
\boxed{

y = 10 \cos\left(\frac{\pi}{2} x \right)

}

\]

serves as a textbook example illustrating these concepts in action.

References

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning, 8th Edition.
- Larson, Ron. Precalculus with Limits. Cengage Learning.
- Trigonometry and Periodic Functions. (2023). Khan Academy. https://www.khanacademy.org/math/trigonometry

---

Conclusion

The exploration of sinusoidal functions tailored to specific amplitude and period requirements underscores the elegance and utility of these mathematical constructs. Whether for theoretical pursuits or practical applications, understanding the derivation process enhances analytical skills and fosters a deeper appreciation for the harmony inherent in periodic phenomena.

Find A Sinusoidal Function With The Following Four Attributes 1 Amplitude Is 10 2 Period Is 4

Find A Sinusoidal Function With The Following Four Attributes 1 Amplitude Is 10 2 Period Is 4

Related Articles

- How Does The Requirement That Cas9 Bind To A Pam Sequence Affect The Ability Of

Scientists To Target
- Describe The Events That Led To The War Between The United States And Mexico. Then, Explain Why Many
- Find A Power Series Expansions For 1/(1-x)^2, 1/(1-x)^3, 1/(1-x)^4, By Differentiating 1/(1-x). Give

Back to Home