Find All Other Zeros Of P(x) = X3 5x + 12x + 18, Given That 3 + 3i Is A Zero. (If There Is More Than

Find All Other Zeros Of P(x) = x^3 - 5x + 12x + 18, Given That 3 + 3i Is A Zero. (If There Is More Than

Understanding polynomial functions and their zeros is a fundamental aspect of algebra and calculus. When working with cubic polynomials, particularly those with complex roots, it becomes crucial to utilize various algebraic techniques, including polynomial division, the conjugate root theorem, and synthetic division, to find all zeros efficiently. In this article, we will explore how to determine all other zeros of the polynomial P(x) = x^3 - 5x + 12x + 18, given that one zero is 3 + 3i. This process involves understanding complex conjugate roots, simplifying polynomial division, and factoring the polynomial completely.

Understanding the Polynomial and Its Zeros

Before delving into the methods for finding the remaining zeros, let's clarify the polynomial's structure and the significance of the given zero.

Analyzing the Polynomial P(x)

Given the polynomial:

$$P(x) = x^3 - 5x + 12x + 18$$

Let's simplify it first:

$$P(x) = x^3 + (-5x + 12x) + 18$$

$$P(x) = x^3 + 7x + 18$$

Thus, the simplified form of the polynomial is:

$$P(x) = x^3 + 7x + 18$$

This cubic polynomial has real coefficients, which means that any complex roots will occur in conjugate pairs.

Given Zero: 3 + 3i

We are told that $3 + 3i$ is a zero of P(x). Since polynomial coefficients are real, the conjugate of this root, $3 - 3i$, must also be a root. Therefore, the two complex roots are:

- $r_1 = 3 + 3i$
- $r_2 = 3 - 3i$

Our goal is to find the remaining zero r_3.

Applying the Conjugate Root Theorem

The conjugate root theorem states that if a polynomial with real coefficients has a complex root $a + bi$, then its conjugate $a - bi$ is also a root. Since we are given one complex zero, the conjugate root must also be a zero.

This knowledge allows us to factor the polynomial as:

$$P(x) = (x - r_1)(x - r_2)(x - r_3)$$

or, more specifically,

$$P(x) = (x - (3 + 3i))(x - (3 - 3i))(x - r_3)$$

Our task now is to find r_3.

Finding the Quadratic Factor from the Complex Roots

Given roots $3 + 3i$ and $3 - 3i$, the quadratic factor formed by these roots is:

$$(x - (3 + 3i))(x - (3 - 3i))$$

Let's expand this quadratic:

1. Write as:

$$[x - 3 - 3i][x - 3 + 3i]$$

2. Recognize this as a difference of squares:

$$[(x - 3) - 3i][(x - 3) + 3i] = (x - 3)^2 - (3i)^2$$

3. Simplify:

$$ (x - 3)^2 - (3i)^2 $$

Note that:

$$ (x - 3)^2 = x^2 - 6x + 9 $$
$$ (3i)^2 = 9i^2 = 9(-1) = -9 $$

So,

$$ (x - 3)^2 - (3i)^2 = x^2 - 6x + 9 - (-9) = x^2 - 6x + 9 + 9 = x^2 - 6x + 18 $$

Therefore, the quadratic factor corresponding to the complex roots is:

$$ x^2 - 6x + 18 $$

Dividing the Polynomial by the Quadratic Factor

Since $P(x)$ is of degree 3, and we've identified a quadratic factor $x^2 - 6x + 18$, dividing $P(x)$ by this quadratic will yield the remaining linear factor corresponding to the third zero.

Recall that:

$$ P(x) = x^3 + 7x + 18 $$

We'll perform polynomial division of $P(x)$ by $x^2 - 6x + 18$.

Polynomial Division Steps

1. Set up the division:

Divide $x^3 + 0x^2 + 7x + 18$ by $x^2 - 6x + 18$.

2. First term:

$$ x^3 \div x^2 = x $$

3. Multiply divisor by x:

$$ x(x^2 - 6x + 18) = x^3 - 6x^2 + 18x $$

4. Subtract:

\[

\begin{aligned}
&(x^3 + 0x^2 + 7x + 18) \\
- &(x^3 - 6x^2 + 18x) \\
= & 0x^3 + 6x^2 - 11x + 18
\end{aligned}
\]

5. Now, divide the new leading term:

\[6x^2 \div x^2 = 6 \]

6. Multiply divisor by 6:

\[6(x^2 - 6x + 18) = 6x^2 - 36x + 108 \]

7. Subtract:

\[
\begin{aligned}
&(6x^2 - 11x + 18) \\
- &(6x^2 - 36x + 108) \\
= & 0x^2 + 25x - 90
\end{aligned}
\]

Since the degree of the remainder \(25x - 90 \) is less than the divisor's degree, the division process stops.

Thus, the quotient is:

\[x + 6 \]

And the remainder is:

\[25x - 90 \]

But for \(P(x) \) to be divisible by \(x^2 - 6x + 18 \), the remainder must be zero. Since it isn't, the division indicates that the quadratic \(x^2 - 6x + 18 \) is not a factor of \(P(x) \) as initially assumed.

However, this suggests a misstep: earlier, we directly used the roots to form the quadratic. Let's verify the roots again.

Verifying the Roots and Correcting the Approach

Given the original polynomial:

\[P(x) = x^3 + 7x + 18 \]

and the known complex zero $3 + 3i$, its conjugate $3 - 3i$, the quadratic factor is:

$$ (x - (3 + 3i))(x - (3 - 3i)) = x^2 - 6x + 18 $$

Now, to check whether $P(x)$ is divisible by $x^2 - 6x + 18$, perform polynomial division again or evaluate $P(x)$ at the roots.

Let's verify if $3 + 3i$ is a zero:

$$ P(3 + 3i) = (3 + 3i)^3 + 7(3 + 3i) + 18 $$

Calculate $(3 + 3i)^3$:

- First, $(3 + 3i)^2$:

$$
(3 + 3i)^2 = 3^2 + 2 \times 3 \times 3i + (3i)^2 = 9 + 18i + 9i^2
$$

Since $i^2 = -1$:

$$
= 9 + 18i - 9 = 0 + 18i
$$

- Next, $(3 + 3i)^3 = (3 + 3i) \times (3 + 3i)^2 = (3 + 3i) \times 18i$

$$
= 3 \times 18i + 3i \times 18i = 54i + 54i^2
$$

Again, $i^2 = -1$:

$$
= 54i - 54 = -54 + 54i
$$

Now, compute $P(3 + 3i)$:

$$
(-54 + 54i) + 7(3 + 3i) + 18
$$

Calculate $7(3 + 3i)$:

$$
21 + 21i
$$

Add all:

\[
-54 + 54i + 21 + 21i + 18 = (-54 + 21 + 18)

Frequently Asked Questions

How do I find all zeros of the polynomial p(x) = x^3 - 5x + 12x + 18, given that 3 + 3i is a zero?

First, recognize that since 3 + 3i is a zero and the polynomial has real coefficients, its conjugate 3 - 3i is also a zero. Use these roots to factor the polynomial and then find the remaining zero by dividing the polynomial by the quadratic factor formed from these roots.

What is the process for finding the other zeros of p(x) after knowing one complex zero?

Once you know a complex zero, you can construct the quadratic factor (x - (3 + 3i))(x - (3 - 3i)), expand it to get a quadratic polynomial, and divide the original polynomial by this quadratic to find the remaining linear factor, which gives the other zero.

Can synthetic division be used to find the remaining zeros after knowing one zero of p(x)?

Synthetic division can be used after selecting a real zero. However, since the known zero is complex, it's easier to factor out the quadratic corresponding to complex conjugate roots directly, rather than using synthetic division, which is more straightforward with real zeros.

How do I verify that 3 + 3i is a zero of p(x)?

Substitute x = 3 + 3i into p(x) and simplify. If the result is zero, then 3 + 3i is indeed a zero. For complex roots, this often involves calculating powers and simplifying complex expressions carefully.

What is the final step to find all zeros of p(x) after factoring out known roots?

After dividing the polynomial by the quadratic factor derived from the complex conjugate roots, you'll get a linear factor. Solve this linear factor to find the remaining zero. All roots are then identified by combining the complex roots and the linear root.

Additional Resources

Find All Other Zeros Of P(x) = x^3 + 5x + 12x + 18, Given That 3 + 3i Is A Zero

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Introduction

When dealing with polynomial functions, one of the fundamental tasks in algebra is finding all the zeros of the polynomial. These zeros, or roots, are the values of x for which $P(x) = 0$. This process often involves leveraging known zeros to factor the polynomial and then solving the resulting lower-degree polynomial equations.

In this detailed exploration, we will analyze the polynomial:

$$
P(x) = x^3 + 5x + 12x + 18
$$

and determine all its zeros, given that $3 + 3i$ is already a zero.

This problem encompasses several important concepts in algebra, including complex roots, conjugate pairs, polynomial division, factorization, and solving cubic equations. Our goal is to systematically find all zeros, explaining each step thoroughly for clarity and depth.

Clarification and Simplification of the Polynomial

Before proceeding, it's essential to clarify the polynomial's form, as the given expression appears inconsistent:

$$
P(x) = x^3 + 5x + 12x + 18
$$

Combining like terms:

$$
P(x) = x^3 + (5x + 12x) + 18 = x^3 + 17x + 18
$$

Therefore, the polynomial simplifies to:

$$
\boxed{P(x) = x^3 + 17x + 18}
$$

Note: The initial expression might have been a typo or misprint; based on typical polynomial forms, this is the most logical interpretation.

Recognizing the Polynomial Type

The polynomial

$$
P(x) = x^3 + 17x + 18
$$

is a cubic polynomial with real coefficients. Our task is to find all roots, knowing that:

$$
3 + 3i \quad \text{is a zero}
$$

Understanding Zeroes and Their Properties

1. Complex Zeros and Conjugate Roots

- Fundamental Theorem of Algebra states that every non-constant polynomial with real coefficients has at least one complex root.
- Conjugate Pair Theorem: If a polynomial has real coefficients, then complex roots come in conjugate pairs.

Since $3 + 3i$ is a zero, its conjugate $3 - 3i$ must also be a zero.

2. Implications for Our Polynomial

- Known zeros: $3 + 3i$ and $3 - 3i$.
- These zeros can be used to factor $P(x)$ into quadratic and linear factors.

Step 1: Use Known Zeros to Find the Quadratic Factor

Given zeros $3 + 3i$ and $3 - 3i$, the corresponding quadratic factor is:

$$
(x - (3 + 3i))(x - (3 - 3i))
$$

Expanding this quadratic:

$$
\begin{aligned}
& (x - 3 - 3i)(x - 3 + 3i) \\
= {} & [(x - 3) - 3i][(x - 3) + 3i] \\
= {} & (x - 3)^2 - (3i)^2 \quad \text{(difference of squares)} \\
= {} & (x - 3)^2 - (-9) \quad \text{since } (3i)^2 = -9 \\
= {} & (x - 3)^2 + 9 \\
= {} & (x^2 - 6x + 9) + 9 \\
= {} & x^2 - 6x + 18
\end{aligned}
$$

Thus, the quadratic factor corresponding to the complex roots is:

\[
Q(x) = x^2 - 6x + 18
\]

Step 2: Polynomial Division to Find the Remaining Root

Since \(P(x) \) is cubic and we have a quadratic factor \(Q(x) \), dividing \(P(x) \) by \(Q(x) \) should yield a linear factor:

\[
P(x) = Q(x) \times R(x)
\]

where \(R(x) \) is a linear polynomial.

Perform polynomial division:

\[
\frac{P(x)}{Q(x)} = \frac{x^3 + 17x + 18}{x^2 - 6x + 18}
\]

Step 3: Polynomial Division Process

Set up division:

- Divide \(x^3 \) by \(x^2 \) to get \(x \).
- Multiply \(Q(x) \) by \(x \):

\[
x \times (x^2 - 6x + 18) = x^3 - 6x^2 + 18x
\]

- Subtract this from \(P(x) \):

\[
\begin{aligned}
& \left[x^3 + 0x^2 + 17x + 18\right] - \left[x^3 - 6x^2 + 18x\right] \\
= {} & (x^3 - x^3) + (0x^2 + 6x^2) + (17x - 18x) + 18 \\
= {} & 0 + 6x^2 - x + 18
\end{aligned}
\]

- Now divide \(6x^2 \) by \(x^2 \) to get \(+6 \).

- Multiply \(Q(x) \) by 6:

$$
6 \times (x^2 - 6x + 18) = 6x^2 - 36x + 108
$$

- Subtract:

$$
\begin{aligned}
& (6x^2 - x + 18) - (6x^2 - 36x + 108) \\
= {} & 6x^2 - 6x^2 + (-x + 36x) + (18 - 108) \\
= {} & 0 + 35x - 90
\end{aligned}
$$

The remainder is $35x - 90$, which is linear, indicating that $P(x)$ factors as:

$$
P(x) = Q(x) \times (x + \text{something}) + \text{remainder}
$$

Alternatively, since the division results in a quadratic remainder, this suggests that the polynomial division isn't exact unless the remainder is zero.

Therefore, perform synthetic division or polynomial long division precisely.

Step 4: Polynomial Long Division

Let's perform the division carefully:

Divide $P(x) = x^3 + 17x + 18$ by $Q(x) = x^2 - 6x + 18$.

Set up:

$$
\begin{array}{r|l}
x^2 - 6x + 18 & x^3 + 0x^2 + 17x + 18
\end{array}
$$

- First step:

Divide x^3 by x^2:

$$
x^3 / x^2 = x
$$

- Multiply $Q(x)$ by x:

$$
x(x^2 - 6x + 18) = x^3 - 6x^2 + 18x
$$

- Subtract:

$$
\begin{aligned}
& (x^3 + 0x^2 + 17x + 18) - (x^3 - 6x^2 + 18x) \\
= {} & 0x^3 + (0x^2 + 6x^2) + (17x - 18x) + 18 \\
= {} & 6x^2 - x + 18
\end{aligned}
$$

- Next step:

Divide $6x^2$ by x^2:

$$
6x^2 / x^2 = 6
$$

- Multiply $Q(x)$ by 6:

$$
6(x^2 - 6x + 18) = 6x^2 - 36x + 108
$$

- Subtract:

$$
\begin{aligned}
& (6x^2 - x + 18) - (6x^2 - 36x + 108) \\
= {} & 0x^2 + (- x + 36x) + (18 - 108) \\
= {} & 35x - 90
\end{aligned}
$$

The remainder is not zero, indicating that $P(x)$ is not exactly divisible by $Q(x)$, which contradicts our earlier assumption.

Step 5: Reassessing the Factorization Approach

Given the residual, it suggests the initial assumption that the roots $3 + 3i$ and $3 - 3i$ correspond to a quadratic factor may need verification.

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