Find All Values Of H Such That Y Will Be In The Subspace Of ?3 Spanned By V1, V2, V3 Ifv1 = [1 2 -4]

Find All Values Of H Such That Y Will Be In The Subspace Of ℝ³ Spanned By V_1, V_2, V_3 If V_1 = [1 2 -4]

Introduction

In the realm of linear algebra, understanding the relationship between vectors and subspaces is fundamental. One common problem involves determining the conditions under which a given vector lies within a specific subspace of $\mathbb{R}^3$, especially when that subspace is spanned by other vectors. This task often requires solving systems of linear equations, analyzing linear combinations, and utilizing concepts such as span, basis, and linear independence.

In this article, we focus on a particular problem: Find all values of H such that a vector Y will be in the subspace of $\mathbb{R}^3$ spanned by vectors V_1, V_2, and V_3, given that V_1 = [1, 2, -4]. This problem involves understanding how vectors combine to form subspaces and the conditions necessary for a vector to be expressed as a linear combination of basis vectors.

We will explore this topic in detail, covering the definitions of subspaces, span, and linear combinations, and then proceed step-by-step to solve the problem. By the end, you will have a comprehensive understanding of how to determine the values of H that satisfy the condition, along with practical methods for solving similar problems.

Background Concepts and Definitions

Subspace of $\mathbb{R}^3$

A subspace of $\mathbb{R}^3$ is a subset of $\mathbb{R}^3$ that itself forms a vector space under vector addition and scalar multiplication. Examples include:

- The zero vector {[0, 0, 0]}

- Lines through the origin
- Planes through the origin
- The entire space $\mathbb{R}^3$ itself

To verify that a set of vectors forms a subspace, it must satisfy three conditions:

1. The zero vector is in the set.
2. The set is closed under vector addition.
3. The set is closed under scalar multiplication.

Span of Vectors

The span of a set of vectors $\{V_1, V_2, ..., V_n\}$ in $\mathbb{R}^3$ is the set of all linear combinations of these vectors:

$$
\text{Span}\{V_1, V_2, ..., V_n\} = \left\{ a_1V_1 + a_2V_2 + ... + a_nV_n \mid a_i \in \mathbb{R} \right\}
$$

The span forms a subspace of $\mathbb{R}^3$. If the vectors are linearly independent, they form a basis for that subspace.

Linear Combination and Dependence

A vector Y can be expressed as a linear combination of vectors V_1, V_2, V_3 if there exist scalars (such as H) satisfying:

$$
Y = c_1V_1 + c_2V_2 + c_3V_3
$$

Determining whether Y is in the span of V_1, V_2, V_3 involves solving for these scalars. If solutions exist for the scalars, then Y belongs to that subspace.

Problem Setup and Approach

Given:

- $V_1 = [1, 2, -4]$
- V_2 and V_3 (unknown vectors)
- Y (unknown vector, possibly depending on H)

The goal:

- Find all values of H such that $Y \in \text{span}\{V_1, V_2, V_3\}$.

Note: Since V_2 and V_3 are not specified, the problem likely involves a general form for Y or some relation involving H.

Suppose we are given a specific vector Y that depends on H, such as:

$$
Y = [a(H), b(H), c(H)]
$$

or perhaps a general vector with H as a parameter, e.g.:

$$
Y = [H, 2H, -4H]
$$

To proceed, we need to specify Y and V_2, V_3 to understand the problem fully.

For the purpose of this explanation, let's assume:

- The vector Y is given as $Y = [H, 2H, -4H]$.

- V_2 and V_3 are arbitrary vectors that form a basis along with V_1.

Step-by-Step Solution

Step 1: Express the problem algebraically

We want to find all H such that:

$$
Y = [H, 2H, -4H] \in \text{span}\{V_1, V_2, V_3\}
$$

This means:

$$
[H, 2H, -4H] = c_1 V_1 + c_2 V_2 + c_3 V_3
$$

for some scalars c_1, c_2, c_3.

Step 2: Understand the subspace structure

If V_1, V_2, and V_3 are linearly independent, they form a basis for a 3D subspace, which is the entire $\mathbb{R}^3$, meaning any vector Y in $\mathbb{R}^3$ can be expressed as a linear combination of them.

If V_1, V_2, and V_3 are linearly dependent, then their span is a subspace of dimension less than 3, and only vectors within that subspace can be represented as their linear combinations.

In many problems, V_2 and V_3 are given explicitly or constructed to complement V_1.

Step 3: Constructing an example with specific V_2 and V_3

Let's assume:

- V_2 = [0, 1, 0]
- V_3 = [0, 0, 1]

This is a common choice to simplify the problem, creating a standard basis alongside V_1.

Now, the span of V_1, V_2, and V_3 is $\mathbb{R}^3$, because these vectors are linearly independent.

Step 4: Set up the linear combination

Express Y as:

$$
Y = c_1 V_1 + c_2 V_2 + c_3 V_3
$$

which translates to:

$$
[H, 2H, -4H] = c_1 [1, 2, -4] + c_2 [0, 1, 0] + c_3 [0, 0, 1]

or component-wise:

$$
\begin{cases}
H = c_1 \times 1 + c_2 \times 0 + c_3 \times 0 = c_1 \\
2H = c_1 \times 2 + c_2 \times 1 + c_3 \times 0 = 2 c_1 + c_2 \\
-4H = c_1 \times (-4) + c_2 \times 0 + c_3 \times 1 = -4 c_1 + c_3
\end{cases}
$$

Step 5: Solve for the scalars c_1, c_2, c_3

From the first equation:

$$
c_1 = H
$$

Substitute into the second:

$$
2H = 2 c_1 + c_2 \Rightarrow 2H = 2H + c_2 \Rightarrow c_2 = 0
$$

Now, substitute $c_1 = H$ into the third:

$$
-4H = -4 c_1 + c_3 \Rightarrow -4H = -4H + c_3 \Rightarrow c_3 = 0
$$

Thus, the scalars:

$$
c_1 = H, \quad c_2= 0, \quad c_3= 0
$$

are consistent for all real H.

Conclusion from the Example

Since the scalars c_1, c_2, c_3 exist for all H, it follows that:

$$
Y = [H, 2H, -4H]
$$

lies in the span of V_1, V_2, V_3 for every real H.

Therefore, in this specific case, all values of H satisfy the condition.

Generalization and Additional Considerations

In real problems, V_2 and V_3 could be different, and the question might involve more complex relationships. Here's how to approach more general cases:

Case 1: Vectors are linearly independent

If V_1, V_2, V_3 are linearly independent, then they span $\mathbb{R}^3$, and any vector Y in $\mathbb{R}^3$ can be expressed as their linear combination.

In this case, the answer is that all H satisfy the condition.

Case 2: Vectors are linearly dependent

If V_1, V_2, V_3 are linearly dependent, their span is a proper subspace. To determine whether Y is in this subspace:

- Form the matrix

Frequently Asked Questions

What is the main goal when finding all values of H in the given subspace problem?

The main goal is to determine the set of all values of H for which the vector Y can be expressed as a linear combination of the vectors V1, V2, and V3, thereby lying within their span.

Given V1 = [1, 2, -4], what additional information

is needed to find all H such that Y is in the span of V1, V2, and V3?

You need the definitions or components of V2, V3, and the vector Y, as well as any relationships or equations involving H that connect these vectors.

How do you determine whether a vector Y is in the subspace spanned by V1, V2, and V3?

You check if Y can be written as a linear combination of V1, V2, and V3 by solving the system of equations derived from Y = aV1 + bV2 + cV3 for scalars a, b, c, and H.

What role does the parameter H play in establishing whether Y is in the subspace?

H likely appears in the components of Y or V2 and V3, influencing the system of equations; solving these equations with respect to H determines the specific values of H for which Y belongs to the subspace.

Can the problem be approached using matrix methods, and if so, how?

Yes, by forming a matrix with V1, V2, V3 as columns and augmenting it with Y, then using row reduction or rank analysis to find the values of H that satisfy the conditions for Y to be in the span.

What is the significance of the span of V1, V2, and V3 in understanding the subspace?

The span represents all possible linear combinations of V1, V2, and V3, defining the subspace within which we are testing whether Y (dependent on H) resides, which is crucial for understanding the subspace's structure.

If Y is not in the span for certain values of H, what does that imply about the relationship between Y and the vectors V1, V2, V3?

It implies that for those values of H, Y cannot be expressed as a linear combination of V1, V2, and V3, meaning Y lies outside the subspace spanned by these vectors for those H values.

Additional Resources

Find All Values Of H Such That Y Will Be In The Subspace Of $\mathbb{R}^3$ Spanned By V_1, V_2, V_3 If V_1 = [1, 2, -4]

Introduction

In the realm of linear algebra, understanding the relationships between vectors and subspaces is fundamental. One of the essential problems involves determining whether a given vector lies within a specified subspace, and if so, under what conditions. This article delves into a specific scenario: identifying all values of a parameter H such that a vector Y belongs to the subspace of $\mathbb{R}^3$ spanned by three vectors V_1, V_2, V_3, with a given initial vector $V_1 = [1, 2, -4]$. This problem encapsulates core concepts such as vector span, linear dependence, parametric solutions, and the conditions for vector inclusion in subspaces.

Understanding the Core Concepts

Before venturing into the specific problem, it is essential to revisit some foundational ideas in linear algebra.

1. Vector Spaces and Subspaces

A vector space over a field $\mathbb{F}$ (commonly $\mathbb{R}$ or $\mathbb{C}$) is a set of vectors closed under vector addition and scalar multiplication. A subspace is a subset of a vector space that itself forms a vector space under the same operations.

In $\mathbb{R}^3$, typical subspaces include planes, lines, and the zero vector space. The span of a set of vectors $\{V_1, V_2, V_3\}$ is the smallest subspace containing all three vectors. Any vector in this span can be expressed as a linear combination of V_1, V_2, V_3.

2. Linear Dependence and Independence

The vectors V_1, V_2, V_3 may or may not be linearly independent:

- Linearly independent vectors have no non-trivial linear combination equaling the zero vector.
- Linearly dependent vectors have at least one non-trivial linear combination equaling zero, implying redundancy among the vectors.

Determining the linear dependence of the set is crucial because it affects the dimension of the span and the representation of other vectors within that span.

3. Expressing a Vector as a Linear Combination

Given vectors V_1, V_2, V_3, a vector Y belongs to their span if and only if there exist scalars a, b, c such that:

$$
Y = aV_1 + bV_2 + cV_3
$$

If one of the vectors or the parameters involved (like H in our case) influence Y, the problem becomes one of solving a system of linear equations to find the permissible values of those parameters.

The Specific Problem Setup

In the problem under discussion, the initial vector V_1 is given as:

$$
V_1 = [1, 2, -4]
$$

The main question is: "Find all values of H such that Y will be in the subspace of $\mathbb{R}^3$ spanned by V_1, V_2, V_3."

While the vectors V_2 and V_3 are not explicitly provided in the initial statement, the typical context suggests considering a generic Y that depends on H, or perhaps specific expressions for V_2, V_3 involving H.

Assuming the problem involves a vector Y expressed in terms of H, or perhaps the vectors V_2 and V_3 are functions of H, the goal is to derive the conditions on H for which Y can be written as a linear combination of V_1, V_2, V_3.

Approach to the Solution

To analyze this problem systematically, the following steps are necessary:

1. Clarify the Vectors V_2, V_3, and Y

- If vectors V_2 and V_3 are functions involving H, their explicit forms are crucial.
- The vector Y should also be explicitly defined, potentially as a function of H.

For example, suppose:

$$
V_2 = [a_1(H), a_2(H), a_3(H)]
$$

$$
V_3 = [b_1(H), b_2(H), b_3(H)]
$$

$$
Y = [y_1(H), y_2(H), y_3(H)]
$$

The problem reduces to solving:

$$
Y = \alpha V_1 + \beta V_2 + \gamma V_3
$$

for scalars α, β, γ, and identifying the values of H for which this is feasible.

2. Set Up the System of Equations

Express the above as a matrix equation:

$$
\begin{bmatrix}
1 & a_1(H) & b_1(H) \\
2 & a_2(H) & b_2(H) \\
-4 & a_3(H) & b_3(H)
\end{bmatrix}
\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix}
=
\begin{bmatrix}
y_1(H) \\
y_2(H) \\
y_3(H)
\end{bmatrix}
$$

The system's solvability depends on the rank of the coefficient matrix and the augmented matrix. The key is to analyze when the system is consistent, which involves calculating the determinant or applying row reduction.

3. Determine Conditions on H

- The system has solutions if and only if the rank of the coefficient matrix equals the rank of the augmented matrix.
- The critical step is to derive the values of H that satisfy this

condition—often involving solving equations where determinants are zero or non-zero.

Detailed Analytical Steps

Given the general structure, suppose the vectors are explicitly:

$$
V_2 = [p(H), q(H), r(H)]
$$
$$
V_3 = [s(H), t(H), u(H)]
$$
$$
Y = [Y_1(H), Y_2(H), Y_3(H)]
$$

Then, establishing the conditions involves:

- Calculating the determinant of the 3x3 matrix:

$$
\det
\begin{bmatrix}
1 & p(H) & s(H) \\
2 & q(H) & t(H) \\
-4 & r(H) & u(H)
\end{bmatrix}
$$

- Setting this determinant to zero to find critical H values where the vectors are linearly dependent, or where the subspace's dimension drops.

- Alternatively, solving the linear system directly for α, β, γ in terms of H and verifying the existence of solutions.

Special Cases and Insights

1. When V_2 and V_3 Are Linearly Independent of V_1:

- The subspace spanned by V_1, V_2, V_3 is three-dimensional, i.e., the entire $\mathbb{R}^3$.
- In this case, any Y in $\mathbb{R}^3$ will be in this subspace, and the condition on H becomes irrelevant.

2. When V_2 and V_3 are linearly dependent on V_1:

- The span reduces to a line or plane, and the set of \(Y \) vectors that lie within depends on the parameters.
- The focus shifts to solving for \(H \) such that the vectors are linearly dependent, which restricts the subspace.

Practical Applications and Significance

Understanding the values of \(H \) that determine whether \(Y \) is in the subspace has multiple applications:

- Control Systems: Parameters \(H \) might represent system gains or feedback coefficients, impacting controllability.
- Signal Processing: Adjusting \(H \) could correspond to filter coefficients, affecting signal subspace inclusion.
- Data Science: Parameters influencing the span of feature vectors or data points for dimensionality reduction.

This type of analysis allows engineers and scientists to identify critical thresholds where system properties change, such as transitioning from controllable to uncontrollable states or from full-rank to rank-deficient matrices.

Conclusion

The problem of determining all values of \(H \) such that a vector \(Y \) resides within a subspace spanned by vectors \(V_1, V_2, V_3 \) encapsulates core concepts of linear algebra, including span, linear dependence, and parametric systems. While the initial statement provides only partial information—namely \(V_1 = [1, 2, -4] \

Find All Values Of H Such That Y Will Be In The Subspace Of 3 Spanned By V1 V2 V3 Ifv1 1 2 4

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