

Find An Equation For The Perpendicular Bisector Of The Line Segment Whose Endpoints Are $(-1,-7)$ and $(1,7)$

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When working with coordinate geometry, understanding how to find the perpendicular bisector of a line segment is a fundamental skill. In this comprehensive guide, we will walk through the step-by-step process of deriving the equation for the perpendicular bisector of the segment with endpoints at $(-1, -7)$ and $(1, 7)$. Whether you're a student preparing for an exam, a teacher creating lesson plans, or a math enthusiast sharpening your skills, this article provides detailed explanations, formulas, and tips to master the concept. Read on to discover clear methods, key points, and practical examples to enhance your understanding of perpendicular bisectors in coordinate geometry.

Understanding the Concept of Perpendicular Bisectors in Coordinate Geometry

What Is a Perpendicular Bisector?

A perpendicular bisector of a line segment is a line that:

- Divides the segment into two equal parts (bisects it).
- Is perpendicular to the segment at the point of intersection (forms a 90-degree angle).

In coordinate geometry, the perpendicular bisector is crucial for locating centers of circles, solving geometric problems, and understanding symmetry.

Why Is the Perpendicular Bisector Important?

Knowing how to find the perpendicular bisector allows you to:

- Find the center of a circle passing through two points.
- Determine the locus of points equidistant from two points.
- Solve problems involving triangle centers such as the circumcenter.
- Understand geometric constructions with straightedge and compass.

Step-by-Step Process to Find the Perpendicular Bisector of a Line Segment

To find the equation of the perpendicular bisector of the segment with endpoints $(-1, -7)$ and $(1, 7)$, follow these key steps:

1. Calculate the Midpoint of the Segment

The midpoint is the point exactly halfway between the endpoints. The formula for the midpoint $M(x_m, y_m)$ is:

$$x_m = \frac{x_1 + x_2}{2}, \quad y_m = \frac{y_1 + y_2}{2}$$

Applying to our points:

$$x_1 = -1, \quad y_1 = -7$$

$$x_2 = 1, \quad y_2 = 7$$

Calculate:

$$x_m = \frac{-1 + 1}{2} = \frac{0}{2} = 0$$

$$y_m = \frac{-7 + 7}{2} = \frac{0}{2} = 0$$

Midpoint: $(0, 0)$

2. Find the Slope of the Original Segment

The slope m_{AB} between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

Plugging in our points:

$$m_{AB} = \frac{7 - (-7)}{1 - (-1)} = \frac{14}{2} = 7$$

$$m_{AB} = \frac{7 - (-7)}{1 - (-1)} = \frac{14}{2} = 7$$

Slope of the segment: 7

3. Determine the Slope of the Perpendicular Bisector

The perpendicular bisector is perpendicular to the original segment, so its slope (m_{PB}) is the negative reciprocal of (m_{AB}) :

$$m_{PB} = -\frac{1}{m_{AB}} = -\frac{1}{7}$$

Perpendicular bisector slope: $(-\frac{1}{7})$

4. Write the Equation of the Perpendicular Bisector

Using the point-slope form of a line:

$$y - y_1 = m(x - x_1)$$

Where:

- (x_1, y_1) is the midpoint $(0, 0)$,
- (m) is the slope $(-\frac{1}{7})$.

Plug in the values:

$$y - 0 = -\frac{1}{7}(x - 0)$$

Simplify:

$$y = -\frac{1}{7}x$$

Equation of the perpendicular bisector: $(y = -\frac{1}{7}x)$

Key Points and Summary

- The midpoint of the segment with endpoints $(-1, -7)$ and $(1, 7)$ is $(0, 0)$.
- The slope of the segment is 7, so the perpendicular bisector's slope is $(-\frac{1}{7})$.
- The perpendicular bisector passes through the midpoint $(0, 0)$.
- The equation of the perpendicular bisector is $(y = -\frac{1}{7}x)$.

Applications of the Perpendicular Bisector in Geometry

1. Finding the Circumcenter of a Triangle

In triangle geometry, the point where the perpendicular bisectors of the sides intersect is the circumcenter — the center of the circumscribed circle around the triangle.

2. Constructing Circles

Knowing the perpendicular bisector helps in constructing circles with specific points, especially when the circle passes through two points.

3. Solving Locus Problems

The perpendicular bisector represents the locus of points equidistant from the segment's endpoints, vital in various geometric proofs.

Additional Tips for Finding Perpendicular Bisectors

1. Always start by calculating the midpoint to identify the point through which the bisector passes.

2. Determine the slope of the original segment first to find the perpendicular slope.
3. Use the point-slope form for the line equation to write the perpendicular bisector.
4. Remember that the perpendicular bisector is unique for every segment, so precise calculations are essential.
5. Visualize the problem with graphs or drawings for better understanding.

Conclusion

Finding the equation of the perpendicular bisector of a line segment with specific endpoints — in this case, $(-1, -7)$ and $(1, 7)$ — involves straightforward steps: calculating the midpoint, determining the slope of the segment, finding the negative reciprocal for the perpendicular slope, and then using the point-slope form to derive the line's equation. The resulting perpendicular bisector, $(y = -\frac{1}{7}x)$, is a critical concept in coordinate geometry, with applications ranging from circle construction to solving geometric problems involving symmetry and distances. Mastering these techniques enhances your problem-solving skills and deepens your understanding of geometric relationships on the coordinate plane.

Keywords for SEO optimization:

perpendicular bisector, coordinate geometry, line segment endpoints, midpoint calculation, slope of line, negative reciprocal, equation of line, geometric constructions, circle center, locus of points, triangle circumcenter, coordinate plane, math tutorials, geometry problems

Frequently Asked Questions

How do you find the midpoint of the segment with endpoints $(-1, -7)$ and $(1, 7)$?

To find the midpoint, average the x-coordinates and the y-coordinates: midpoint $M = ((-1 + 1)/2, (-7 + 7)/2) = (0, 0)$.

What is the slope of the line segment connecting $(-1, -7)$ and $(1, 7)$?

The slope $m = (7 - (-7)) / (1 - (-1)) = (14) / (2) = 7$.

How do you find the slope of the perpendicular bisector of a line segment?

The slope of the perpendicular bisector is the negative reciprocal of the original slope. Since the original slope is 7, the perpendicular slope is $-1/7$.

What is the equation of the perpendicular bisector passing through the midpoint (0, 0) with slope $-1/7$?

Using point-slope form: $y - 0 = (-1/7)(x - 0)$, so the equation is $y = (-1/7)x$.

Why does the perpendicular bisector pass through the midpoint of the segment?

Because the perpendicular bisector is the line that divides the segment into two equal parts at a right angle, it always passes through the segment's midpoint.

Can the equation $y = (-1/7)x$ be rewritten in standard form?

Yes, multiplying both sides by 7 gives: $7y = -x$, or $x + 7y = 0$.

What is the overall process to find the perpendicular bisector of a line segment given endpoints?

First, find the midpoint of the segment. Then, determine the slope of the segment and find its negative reciprocal. Lastly, use the midpoint and this slope in point-slope form to write the equation of the perpendicular bisector.

Additional Resources

Find An Equation For The Perpendicular Bisector Of The Line Segment Whose Endpoints Are $(-1,-7)$ and $(1,7)$

Understanding how to find the perpendicular bisector of a line segment is an essential skill in coordinate geometry. It not only helps in solving geometric problems but also forms the foundation for more complex topics such as constructing geometric figures, analyzing symmetry, and solving for unknown points in geometric configurations. In this article, we will thoroughly explore the process of finding the perpendicular bisector of the line segment with endpoints at $(-1, -7)$ and $(1, 7)$. We will dissect each step, discuss relevant concepts, and highlight the features and potential pitfalls involved.

Introduction to the Perpendicular Bisector

What is the Perpendicular Bisector?

The perpendicular bisector of a line segment is a line that divides the segment into two equal parts at a right angle (90 degrees). In other words, it is a line that is perpendicular to the segment and passes through its midpoint. This concept is fundamental in geometry because it helps determine equidistant points from the segment's endpoints, which is useful in triangle constructions, circle definitions, and coordinate problems.

Features of the Perpendicular Bisector:

- Passes through the midpoint of the segment.
- Is perpendicular to the segment.
- Contains all points equidistant from the segment's endpoints.
- Is a fundamental component in constructing circumcircles of triangles.

Applications:

- Finding the center of a circle passing through two points.
- Constructing the circumcenter of a triangle.
- Solving geometric problems involving symmetry and congruence.

Step 1: Find the Midpoint of the Segment

The first step in constructing the perpendicular bisector is to determine the midpoint of the segment connecting the given endpoints, (-1, -7) and (1, 7). The midpoint formula is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Applying this formula:

$$x_m = \frac{-1 + 1}{2} = \frac{0}{2} = 0$$

$$y_m = \frac{-7 + 7}{2} = \frac{0}{2} = 0$$

Thus, the midpoint is at (0, 0).

Features:

- The midpoint provides the point through which the perpendicular bisector passes.
- It guarantees the bisecting property, dividing the segment into two equal lengths.

Implication: Since the bisector passes through (0, 0), any line we derive must include this

point.

Step 2: Find the Slope of the Original Segment

Next, determine the slope of the segment connecting the two endpoints. The slope formula is:

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$$

Calculating:

$$m_{AB} = \frac{7 - (-7)}{1 - (-1)} = \frac{14}{2} = 7$$

So, the slope of the segment is 7.

Features:

- The slope indicates the steepness of the line.
- The direction vector from (-1, -7) to (1, 7) is consistent with this slope.

Implication: The perpendicular bisector must be perpendicular to this segment, so its slope will be the negative reciprocal of 7, which is $(-\frac{1}{7})$.

Step 3: Determine the Slope of the Perpendicular Bisector

Since the perpendicular bisector is perpendicular to the original segment, its slope is:

$$m_{\text{perp}} = -\frac{1}{m_{AB}} = -\frac{1}{7}$$

This is a key step because it provides the slope of the line we're trying to find.

Features:

- The negative reciprocal ensures the lines are perpendicular.
- This property is crucial in constructing the perpendicular bisector.

Step 4: Write the Equation of the Perpendicular

Bisector

Having the slope $(m_{\text{perp}} = -\frac{1}{7})$ and passing through the midpoint $(0, 0)$, we use the point-slope form:

$$y - y_1 = m (x - x_1)$$

Plugging in:

$$y - 0 = -\frac{1}{7} (x - 0)$$

Simplifies to:

$$y = -\frac{1}{7} x$$

Final Equation:

$$\boxed{y = -\frac{1}{7} x }$$

This line is the perpendicular bisector of the segment connecting $(-1, -7)$ and $(1, 7)$.

Features and Significance of the Resulting Equation

Features:

- It passes through the origin, consistent with the midpoint.
- Has a slope of $(-\frac{1}{7})$, indicating a gentle downward tilt as x increases.
- Is in slope-intercept form, making it straightforward to interpret and graph.

Significance:

- The line divides the segment into two equal parts and meets it at a right angle.
- Any point on this line is equidistant from the endpoints of the segment, making it critical in circle and triangle constructions.

Additional Considerations and Variations

While the direct algebraic approach yields the equation efficiently, understanding

alternative methods enhances comprehension and problem-solving flexibility.

Using Vector Methods

Instead of relying solely on slopes, vector methods consider the perpendicular direction directly. For example:

- Find the vector from one endpoint to the other.
- Determine a perpendicular vector.
- Use the midpoint and this perpendicular vector to write the line equation.

Pros:

- Useful for more complex 3D problems.
- Provides geometric insight into directions and orientations.

Cons:

- Slightly more involved algebraically.
- May require familiarity with vector operations.

Graphical Approach

Plotting the points and visualizing the midpoint and perpendicular line can be helpful for intuitive understanding.

Pros:

- Good for visual learners.
- Useful in verifying algebraic results.

Cons:

- Less precise for exact solutions.
- Difficult to visualize slope without graphing tools.

Applications of the Perpendicular Bisector Equation

Understanding the equation $y = -\frac{1}{7}x$ extends beyond the immediate problem:

- Finding the Circumcenter: For triangles, the perpendicular bisectors of sides intersect at the circumcenter. Knowing this line helps locate the circle passing through the vertices.
- Constructing Circles: The perpendicular bisector of a segment contains the center of a circle passing through the segment's endpoints.
- Coordinate Geometry Problems: Used in problem-solving involving distances, midpoints, and symmetry.

Summary and Conclusion

Finding the perpendicular bisector of a line segment with given endpoints is a fundamental process in coordinate geometry. Starting with calculating the midpoint and the slope of the segment, then determining the negative reciprocal for the perpendicular slope, leads to the straightforward equation of the bisector. For the segment with endpoints at $(-1, -7)$ and $(1, 7)$, the perpendicular bisector is $(y = -\frac{1}{7}x)$. This line passes through the origin, bisects the segment at right angles, and contains all points equidistant from the endpoints. Mastery of this process not only aids in solving geometric problems but also deepens understanding of the relationship between lines, points, and shapes in the coordinate plane.

Pros of this method:

- Clear, step-by-step approach.
- Utilizes fundamental formulas (midpoint, slope).
- Easily adaptable to other segments and conditions.

Cons:

- Assumes familiarity with algebraic manipulation and slope concepts.
- Less intuitive without visual aid, requiring graphing for full comprehension.

By comprehensively understanding and applying these principles, students and enthusiasts alike can confidently find and utilize perpendicular bisectors in various geometric contexts, enhancing their problem-solving toolkit in coordinate geometry.

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