Find An Equation Of The Plane Tangent To The Following Surface At The Given Point. 7xy +yz +4xz - 48

Find An Equation Of The Plane Tangent To The Following Surface At The Given Point. 7xy + yz + 4xz - 48

Understanding how to find the tangent plane to a surface at a specific point is a fundamental concept in multivariable calculus. It has applications across physics, engineering, computer graphics, and more. In this article, we will explore in detail the process of deriving the equation of the tangent plane to the surface defined by the equation $7xy + yz + 4xz - 48 = 0$ at a given point. This comprehensive guide aims to clarify each step, ensuring you understand not only the final answer but also the reasoning behind each process.

Introduction to Tangent Planes and Surfaces

What Is a Tangent Plane?

A tangent plane to a surface at a particular point is a flat, two-dimensional plane that "just touches" the surface at that point. It is the best linear approximation of the surface near that point. If you imagine a curved surface like a sphere, the tangent plane at a point on the sphere is the plane that "kisses" the sphere at that point and is perpendicular to the surface's normal vector at that point.

Why Find Tangent Planes?

Finding the tangent plane is crucial for:
- Approximating functions locally
- Understanding the behavior of surfaces
- Optimization problems involving constraints
- Visualizing surfaces in three dimensions

Understanding the Surface Equation

The surface in question is given by:

$$
7xy + yz + 4xz - 48 = 0

\]

This is a multivariable equation involving three variables x, y, z. It defines a surface in three-dimensional space.

Rewriting the Equation

For clarity:

$$
F(x, y, z) = 7xy + yz + 4xz - 48 = 0
$$

Our goal is to find the tangent plane to this surface at a specific point, say (x_0, y_0, z_0). The problem statement should specify the point; if not provided, we can demonstrate the process using an example point that satisfies the surface equation.

Note: To proceed, let's assume a point (x_0, y_0, z_0) lies on the surface, satisfying:

$$
7x_0 y_0 + y_0 z_0 + 4 x_0 z_0 = 48
$$

If a specific point is given in the original problem, replace it accordingly.

Step-by-Step Procedure to Find the Tangent Plane

The general approach involves the following steps:

1. Identify the function $F(x, y, z)$ defining the surface.
2. Calculate the gradient vector ∇F at the point of interest.
3. Use the gradient vector as the normal vector for the tangent plane.
4. Formulate the equation of the tangent plane using the point-normal form.

Step 1: Define the Function $F(x, y, z)$

As established,

$$
F(x, y, z) = 7xy + yz + 4xz - 48
$$

The surface corresponds to the set of points where $F(x, y, z) = 0$.

Step 2: Compute the Gradient Vector ∇F

The gradient vector is composed of the partial derivatives of F with respect to each variable:

$$
\nabla F(x, y, z) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
$$

Calculating each component:

- $\frac{\partial F}{\partial x} = 7y + 4z$
- $\frac{\partial F}{\partial y} = 7x + z$
- $\frac{\partial F}{\partial z} = y + 4x$

Therefore,

$$
\nabla F(x, y, z) = (7y + 4z,\, 7x + z,\, y + 4x)
$$

Step 3: Evaluate the Gradient at the Point

Suppose we are given a specific point (x_0, y_0, z_0). For illustration, let's choose an example point that satisfies the surface equation:

Example Point:
Let’s select $(x_0, y_0, z_0) = (3, 2, 4)$.

Verify if it lies on the surface:

$$
7 \times 3 \times 2 + 2 \times 4 + 4 \times 3 \times 4 = 7 \times 6 + 8 + 48 = 42 + 8 + 48 = 98
$$

Since $98 \neq 48$, this point does not satisfy the surface equation. Let's find a point that does.

Finding a Point on the Surface:

Suppose $x=2$, $y=3$:

$$
7 \times 2 \times 3 + 3 \times z + 4 \times 2 \times z = 0
$$

$$
42 + 3z + 8z = 0
$$

$$
42 + 11z = 0
$$

$$
11z = -42
$$

$$
z = -\frac{42}{11}
$$

Thus, a point on the surface is $(2, 3, -\frac{42}{11})$.

Now, evaluate the gradient at this point:

$$
\frac{\partial F}{\partial x} = 7 \times 3 + 4 \times \left(-\frac{42}{11}\right) = 21 - \frac{168}{11} = \frac{231 - 168}{11} = \frac{63}{11}
$$

$$
\frac{\partial F}{\partial y} = 7 \times 2 + \left(-\frac{42}{11}\right) = 14 - \frac{42}{11} = \frac{154 - 42}{11} = \frac{112}{11}
$$

$$
\frac{\partial F}{\partial z} = 3 + 4 \times 2 = 3 + 8 = 11
$$

Therefore, the gradient vector at $(2, 3, -\frac{42}{11})$ is:

$$
\nabla F = \left(\frac{63}{11},\, \frac{112}{11},\, 11 \right)
$$

This vector serves as the normal vector to the tangent plane.

Step 4: Write the Equation of the Tangent Plane

Using the point-normal form:

$$
\text{Equation of tangent plane: } \nabla F \cdot (\mathbf{r} - \mathbf{r}_0) = 0
$$

where $\mathbf{r}_0 = (x_0, y_0, z_0)$ and $\mathbf{r} = (x, y, z)$.

Plugging in the values:

$$
\left(\frac{63}{11},\, \frac{112}{11},\, 11 \right) \cdot \left(x - 2,\, y - 3,\, z + \frac{42}{11} \right) = 0
$$

Expanding:

$$
\frac{63}{11}(x - 2) + \frac{112}{11}(y - 3) + 11 \left(z + \frac{42}{11} \right) = 0
$$

Simplify each term:

$$
\frac{63}{11}x - \frac{126}{11} + \frac{112}{11}y - \frac{336}{11} + 11z + 42 = 0
$$

Combine constants:

$$
-\frac{126}{11} - \frac{336}{11} + 42 = -\frac{462}{11} + 42
$$
$$
= -\frac{462}{11} + \frac{462}{11} = 0
$$

Thus, the equation simplifies to:

$$
\frac{63}{11}x + \frac{112}{11}y + 11z = 0
$$

Multiply through by 11 to clear denominators:

$$
63x + 112y + 121z = 0
$$

This is the equation of the tangent plane at the chosen point.

Summary of the Process

- Step 1: Identify the surface equation $F(x, y, z)$.

- Step 2: Calculate the gradient vector \(\nabla F\).
- Step 3: Determine a point \((x_0, y_0, z_0)\) on the surface.
- Step 4: Evaluate \(\nabla F\) at that point

Frequently Asked Questions

How do you find the equation of the tangent plane to the surface 7xy + yz + 4xz - 48 at a given point?

To find the tangent plane, first compute the gradient vector of the surface function at the point, then use the point-normal form of the plane equation: $(r - r_0) \cdot \nabla F = 0$, where r_0 is the position vector of the point.

What is the role of the gradient vector in determining the tangent plane to a surface?

The gradient vector of the surface function at a point is perpendicular to the tangent plane at that point, serving as the normal vector for the tangent plane equation.

Given the surface 7xy + yz + 4xz - 48, how do you find the partial derivatives needed for the gradient vector?

Calculate the partial derivatives with respect to x, y, and z: $\partial F/\partial x = 7y + 4z$, $\partial F/\partial y = 7x + z$, and $\partial F/\partial z = y + 4x$. Evaluate these at the given point to find the gradient vector.

If the surface is defined by F(x,y,z) = 7xy + yz + 4xz - 48, how do you write the equation of the tangent plane at point (x0, y0, z0)?

The tangent plane equation is: $\nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0$, where ∇F is the gradient vector evaluated at (x_0, y_0, z_0).

What steps are involved in applying the gradient method to find the tangent plane to a surface at a specific point?

First, find the gradient vector of the surface function, evaluate it at the given point, then use the point-normal form of the plane equation with this gradient as the normal vector to determine the tangent plane.

Additional Resources

Find An Equation Of The Plane Tangent To The Following Surface At The Given Point is a fundamental concept in multivariable calculus that beautifully combines geometric intuition with algebraic techniques. Understanding how to determine the tangent plane to a surface at a specific point is crucial for analyzing the local behavior of multivariable functions, optimizing surfaces, and solving real-world problems in engineering, physics, and computer graphics. This article provides a comprehensive exploration of the process, focusing on the surface given by the equation 7xy + yz + 4xz - 48, and demonstrates how to find the tangent plane at a particular point.

Introduction to Tangent Planes and Surfaces

Before diving into the specific problem, it’s essential to understand the fundamental concepts of surfaces and their tangent planes in three-dimensional space.

Surfaces in 3D Space

A surface in three-dimensional space can often be represented implicitly by an equation of the form F(x, y, z) = 0. For example, the surface in question is given by:

$$F(x, y, z) = 7xy + yz + 4xz - 48 = 0$$

This defines a set of points (x, y, z) that satisfy this equation, forming a surface.

Tangent Planes

A tangent plane at a point on a surface is the best linear approximation of the surface near that point. Geometrically, it is the plane that "touches" the surface at exactly one point and has the same orientation as the surface at that point.

Mathematically, if a surface is defined implicitly as F(x, y, z) = 0, then the tangent plane at a point (x_0, y_0, z_0) (where the surface passes through) can be expressed as:

$$
\nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0
$$

where ∇F is the gradient vector of F, evaluated at the point.

Step-by-Step Approach to Find the Tangent Plane

The process involves several key steps:

1. Identify the point of tangency.
2. Compute the gradient vector ∇F at that point.
3. Write the equation of the tangent plane using the gradient.

Let's delve into each step with specifics.

1. Choosing the Point of Tangency

The problem statement should specify a point (x_0, y_0, z_0) on the surface. If not provided, it must be determined or chosen based on context or additional information. For example, suppose the point is $(x_0, y_0, z_0) = (3, 2, 4)$.

To verify that this point lies on the surface:

$$
F(3, 2, 4) = 7 \times 3 \times 2 + 2 \times 4 + 4 \times 3 \times 4 - 48
$$
$$
= 7 \times 6 + 8 + 4 \times 12 - 48
$$
$$
= 42 + 8 + 48 - 48 = 50
$$

Since $F(3, 2, 4) = 50 \neq 0$, this point does not lie on the surface. Therefore, either choose another point or find a point that satisfies the surface equation.

Suppose instead we look for points satisfying the surface equation. For simplicity, pick $x=3, y=2$, then solve for z:

$$
7 \times 3 \times 2 + 2z + 4 \times 3 \times z - 48 = 0
$$
$$
42 + 2z + 12z - 48 = 0
$$
$$
(14z) - 6 = 0 \Rightarrow 14z=6 \Rightarrow z=\frac{6}{14}=\frac{3}{7}
$$

Thus, the point \((3, 2, 3/7)\) lies on the surface.

---

2. Computing the Gradient Vector \(\nabla F\)

The gradient vector of a scalar function \(F(x, y, z)\) is:

\[
\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
\]

Calculate each partial derivative:

- \(\frac{\partial F}{\partial x}\):

\[
\frac{\partial}{\partial x} (7xy + yz + 4xz - 48) = 7y + 4z
\]

- \(\frac{\partial F}{\partial y}\):

\[
\frac{\partial}{\partial y} (7xy + yz + 4xz - 48) = 7x + z
\]

- \(\frac{\partial F}{\partial z}\):

\[
\frac{\partial}{\partial z} (7xy + yz + 4xz - 48) = y + 4x
\]

Now evaluate at the point \((x, y, z) = (3, 2, 3/7)\):

\[
\nabla F(3, 2, 3/7) = \left(7 \times 2 + 4 \times \frac{3}{7}, \quad 7 \times 3 + \frac{3}{7}, \quad 2 + 4 \times 3 \right)
\]

Compute each component:

- \(x\)-component:

\[
14 + \frac{12}{7} = 14 + \frac{12}{7} = 14 + 1.7142857 \approx 15.7143
\]

- \(y\)-component:

\[
21 + \frac{3}{7} = 21 + 0.4286 \approx 21.4286
\]

- \(z\)-component:

\[
2 + 12 = 14
\]

Thus, the gradient vector at the point is approximately:

\[
\nabla F(3, 2, 3/7) \approx (15.7143, 21.4286, 14)
\]

---

3. Formulating the Equation of the Tangent Plane

The general formula for the tangent plane at \((x_0, y_0, z_0)\) is:

\[
\nabla F(x_0, y_0, z_0) \cdot (x - x_0, y - y_0, z - z_0) = 0
\]

Substituting the values:

\[
15.7143 (x - 3) + 21.4286 (y - 2) + 14 (z - 3/7) = 0
\]

This equation can be expanded and simplified:

\[
15.7143x - 47.1429 + 21.4286y - 42.8572 + 14z - 6 = 0
\]

Combine constants:

\[
15.7143x + 21.4286y + 14z - (47.1429 + 42.8572 + 6) = 0
\]
\[
15.7143x + 21.4286y + 14z - 96.0001 \approx 0
\]

Expressed more cleanly:

\[

15.7143x + 21.4286y + 14z = 96.0001
\]

This is the equation of the tangent plane at the chosen point.

Features, Pros, and Cons of the Method

Features:

- Uses the gradient vector as a normal vector to the tangent plane.
- Applicable to any surface defined implicitly by a scalar function.
- Provides a straightforward formula that generalizes well.

Pros:

- Conceptually intuitive: the gradient points in the direction of maximum increase and is perpendicular to the surface.
- Precise: yields an exact linear approximation at the point.
- Versatile: applicable to complex surfaces with minimal modifications.

Cons:

- Requires the point to be on the surface; otherwise, the method is invalid.
- Computing derivatives can be cumbersome for more complicated functions.
- The approximation is only valid close to the point; inaccuracies grow farther away.

Additional Considerations and Tips

- Verification: Always verify that the point lies on the surface before proceeding.
- Precision: Use exact fractions where possible to avoid rounding errors.
- Visualization: Visualizing the surface and tangent plane can aid understanding, especially for complex surfaces.
- Extensions: The same method applies to find tangent lines to curves or tangent hyperplanes in higher dimensions.

Conclusion

Finding the equation of the tangent plane to a surface at a given point is a vital skill in multivariable calculus, providing insights into the local geometry of surfaces. By understanding the role of the gradient vector as the normal to the tangent plane, and carefully computing derivatives, students and

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