Find An Explicit Description Of Nul A By Listing Vectors That Span The Null Space. [1 0 0 -3 0 0 0 1

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Understanding the null space of a matrix is fundamental in linear algebra, particularly when analyzing solutions to homogeneous systems. In this article, we will explore how to explicitly describe the null space (nullity) of a given matrix by listing vectors that span it. Focusing on the specific matrix represented by the partial row, [1 0 0 -3 0 0 0 1], we will guide you through the process of identifying the null space, computing the basis vectors, and expressing the null space explicitly. Whether you're a student or a professional, this comprehensive guide will clarify the steps involved and deepen your understanding of null spaces and their spanning vectors.

Understanding the Null Space of a Matrix

Definition of Null Space

The null space (or kernel) of a matrix A is the set of all vectors x such that:

$$A x = 0$$

where 0 is the zero vector. The null space is a subspace of $\mathbb{R}^n$ (if A is an $m \times n$ matrix), and understanding its structure is crucial because it describes all solutions to the homogeneous system associated with A.

Significance of Null Space

- The null space indicates the degree of freedom in solving the homogeneous system.
- It helps in determining the rank and nullity of the matrix, key components of the Rank-Nullity Theorem.
- It provides insight into the linear independence of the columns of A.
- Applications include solving linear equations, understanding linear transformations, and more.

Analyzing the Given Matrix

The partial matrix provided is:

$$
A = [1 \quad 0 \quad 0 \quad -3 \quad 0 \quad 0 \quad 0 \quad 1]
$$

which appears to be a 1×8 matrix. However, for a meaningful null space analysis, we generally consider matrices with more than one row. Assuming that this represents a row of a larger matrix or that the matrix is:

$$
A = \begin{bmatrix}
1 & 0 & 0 & -3 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

then the null space consists of all vectors $x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T$ satisfying:

$$
1 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 - 3 x_4 + 0 \cdot x_5 + 0 \cdot x_6 + 0 \cdot x_7 + 1 \cdot x_8 = 0
$$

which simplifies to:

$$
x_1 - 3 x_4 + x_8 = 0
$$

Given this, the null space can be described explicitly in terms of free variables.

Steps to Find Vectors That Span the Null Space

Step 1: Set Up the Homogeneous Equation

Write the equation derived from the matrix:

$$
x_1 - 3 x_4 + x_8 = 0
$$

Express x_1 in terms of free variables:

$$
x_1 = 3 x_4 - x_8
$$

The other variables x_2, x_3, x_5, x_6, x_7 are free, as they do not appear in the equation.

Step 2: Identify Free and Leading Variables

- Leading variable: x_1, as it is expressed explicitly.
- Free variables: $x_2, x_3, x_4, x_5, x_6, x_7, x_8$.

However, since only x_1, x_4, x_8 are involved in the equation, the remaining variables are free.

Step 3: Assign Parameters to Free Variables

Let:

$$
x_2 = s, \quad x_3 = t, \quad x_4 = u, \quad x_5 = v, \quad x_6 = w, \quad x_7 = x, \quad x_8 = y
$$

where s, t, u, v, w, x, y are real parameters.

Express x_1 in terms of u and y:

$$
x_1 = 3 u - y
$$

Note: To be consistent with notation, we can rename parameters to avoid confusion:

- $x_4 = u$
- $x_8 = y$

Remaining variables:

$$
x_2 = s, \quad x_3 = t, \quad x_5 = v, \quad x_6 = w, \quad x_7 = x
$$

Step 4: Write the General Solution Vector

The general vector in the null space is:

$$
x = (x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8)^T
$$

which becomes:

$$
x = (3 u - y, s, t, u, v, w, x, y)^T
$$

\]

Expressed as a linear combination of free variables:

\[
x = u \cdot (3, 0, 0, 1, 0, 0, 0, 0) + y \cdot (-1, 0, 0, 0, 0, 0, 0, 1) + s \cdot (0, 1, 0, 0, 0, 0, 0, 0) + t \cdot (0, 0, 1, 0, 0, 0, 0, 0) + v \cdot (0, 0, 0, 0, 1, 0, 0, 0) + w \cdot (0, 0, 0, 0, 0, 1, 0, 0) + x \cdot (0, 0, 0, 0, 0, 0, 1, 0)
\]

Explicit Spanning Vectors of the Null Space

Based on the previous step, the basis vectors that span the null space are:

1. **Vector associated with \(u\):**

   \[
   \mathbf{v}_1 = (3, 0, 0, 1, 0, 0, 0, 0)
   \]

2. **Vector associated with \(y\):**

   \[
   \mathbf{v}_2 = (-1, 0, 0, 0, 0, 0, 0, 1)
   \]

3. **Vector associated with \(s\):**

   \[
   \mathbf{v}_3 = (0, 1, 0, 0, 0, 0, 0, 0)
   \]

4. **Vector associated with \(t\):**

   \[
   \mathbf{v}_4 = (0, 0, 1, 0, 0, 0, 0, 0)
   \]

5. **Vector associated with \(v\):**

   \[
   \mathbf{v}_5 = (0, 0, 0, 0, 1, 0, 0, 0)
   \]

6. **Vector associated with \(w\):**

\[
\mathbf{v}_6 = (0, 0, 0, 0, 0, 1, 0, 0)
\]

7. **Vector associated with \(x\):**

   \[
   \mathbf{v}_7 = (0, 0, 0, 0, 0, 0, 1, 0)
   \]

The null space is therefore spanned by the set:

\[
\mathcal{N}(A) = \text{span} \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4, \mathbf{v}_5, \mathbf{v}_6, \mathbf{v}_7\}
\]

which forms a basis for the null space.

Summary of the Explicit Null Space Description

- The null space consists of all vectors of the form:

\[
x = u \cdot (3, 0, 0, 1, 0, 0, 0, 0) + y \cdot (-1, 0, 0, 0,

Frequently Asked Questions

How do I find the null space of a matrix given its row or column vectors?

To find the null space of a matrix, set up the homogeneous equation Ax = 0 and solve for x. This involves row reducing the matrix to its reduced row echelon form and expressing free variables in terms of leading variables. The null space consists of all vectors x satisfying these solutions.

What does it mean to list vectors that span the null space of a matrix?

Listing vectors that span the null space means providing a set of vectors such that any vector in the null space can be written as a linear combination of these vectors. These vectors form a basis for the null space.

Given the matrix [1 0 0 -3 0 0 0 1], how can I find its null space?

You set up the equation [1 0 0 -3 0 0 0 1] x = 0 and solve for x. This involves expressing the leading variables in terms of free variables. The resulting vectors form the basis for the null space.

What steps are involved in explicitly describing the null space of a matrix?

The steps include: writing the matrix equation, row reducing to echelon form, identifying free variables, expressing leading variables in terms of free variables, and then writing the null space vectors as linear combinations of the free variables.

How do I interpret the null space vectors once I have them explicitly listed?

Once listed, the null space vectors form a basis. Any vector in the null space can be expressed as a linear combination of these basis vectors, representing all solutions to Ax=0.

Can you provide an explicit basis for the null space of the matrix [1 0 0 -3 0 0 0 1]?

Yes. First, solve the homogeneous system: the null space vectors can be expressed as combinations of free variables. The basis vectors are derived from setting free variables to 1 or 0 in turn. For this matrix, the null space basis vectors are: [3, 0, 0, 1, 0, 0, 0, 0] and [0, 0, 0, 0, 0, 0, 0, 1].

What is the significance of listing vectors that span the null space in linear algebra?

Listing vectors that span the null space helps understand the solution space of homogeneous systems, facilitates solving linear equations, and aids in analyzing the matrix's properties such as rank and nullity.

How does understanding the null space help in solving linear systems or analyzing matrices?

The null space reveals the set of solutions to the homogeneous equation. It helps determine the dimension of the solution space, find particular solutions, and analyze the rank and invertibility of the matrix.

Additional Resources

Understanding How to Explicitly Describe the Null Space of a Matrix: A Comprehensive Guide

When studying linear algebra, one of the fundamental concepts is the null space (or kernel) of a matrix. It offers critical insights into the solutions of homogeneous systems, the rank-nullity theorem, and the structure of linear transformations. Specifically, given a matrix A, finding an explicit description of its null space involves identifying all vectors x such that $A x = 0$. In this detailed guide, we will analyze the process of describing the null space for the matrix:

$$
A = \begin{bmatrix}
1 & 0 & 0 & -3 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

and how to list vectors that span this null space.

Introduction to Null Space and Its Significance

The null space of a matrix A is defined as:

$$
\text{Null}(A) = \{ x \in \mathbb{R}^n \mid A x = 0 \}
$$

where A is an $m \times n$ matrix. The null space is a subspace of $\mathbb{R}^n$, and understanding it is crucial because:

- It describes all solutions to the homogeneous system $A x = 0$.
- Its dimension is called the nullity of A.
- Reveals the degree of freedom in solutions, indicating dependencies among columns.
- Plays a vital role in the rank-nullity theorem: $\text{rank}(A) + \text{nullity}(A) = n$.

Explicitly describing the null space involves finding a basis for it—i.e., a set of vectors that span the null space and are linearly independent.

Analyzing the Given Matrix A

The matrix provided is a row vector:

$$
A = \begin{bmatrix}
1 & 0 & 0 & -3 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

which is a 1×8 matrix. This matrix represents a linear functional on $\mathbb{R}^8$. The null space consists of all vectors $x \in \mathbb{R}^8$ such that:

$$
A x = 0
$$

or explicitly:

$$
1 \cdot x_1 + 0 \cdot x_2 + 0 \cdot x_3 - 3 x_4 + 0 \cdot x_5 + 0 \cdot x_6 + 0 \cdot x_7 + 1 \cdot x_8 = 0
$$

which simplifies to:

$$
x_1 - 3 x_4 + x_8 = 0
$$

This single linear equation constrains the vectors in the null space, but leaves many free variables.

Step-by-Step Process to Find the Null Space

Step 1: Set up the homogeneous equation

$$
x_1 - 3 x_4 + x_8 = 0
$$

Express x_1 in terms of the free variables:

$$
x_1 = 3 x_4 - x_8
$$

Step 2: Identify the free variables

The variables x_2, x_3, x_5, x_6, x_7 do not appear in the equation and are thus free. The variables x_4, x_8 are also free, as they appear in the equation but are not constrained beyond the relation for x_1.

Hence:

- Free variables: $x_2, x_3, x_4, x_5, x_6, x_7, x_8$
- Basic variable: x_1, which can be expressed as above.

Step 3: Express the solution set

The general solution vector x is:

$$
x = \begin{bmatrix}
x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8
\end{bmatrix}
= \begin{bmatrix}
3 x_4 - x_8 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8
\end{bmatrix}
$$

Now, express x as a linear combination of free variables:

$$
x = x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_3 \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_4 \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_5 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ x_6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}
+ x_7 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}
+ x_8 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}
$$

Step 4: Write the span of the null space

The null space is spanned by the following vectors:

$$
\boxed{
\begin{aligned}
v_1 &= \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_2) \\
v_2 &= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_3) \\
v_3 &= \begin{bmatrix} 3 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_4) \\
v_4 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_5) \\
v_5 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_6) \\
v_6 &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \quad \text{(corresponding to } x_7) \\
v_7 &= \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{(corresponding to } x_8)
\end{aligned}
}
$$

Explicit Description of the Null Space

The null space of A is thus:

$$
\text{Null}(A) = \text{span} \left\{ v_1, v_2, v_3, v_4, v_5, v_6, v_7 \right\}
$$

which explicitly includes all vectors of the form:

$$
x = c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4 + c_5 v_5 + c_6 v_6 + c_7 v_7
$$

for arbitrary scalars $c_1, c_2, c_3, c_4, c_5, c_6, c_7 \in \mathbb{R}$.

Implications and Insights

1. Dimension of the Null Space (Nullity):

Since the null space is spanned by 7 vectors, its dimension (nullity) is 7. This corresponds to the total number of free variables (excluding the one basic variable x_1 which is expressed in terms of others).

2. Relationship with the Matrix Rank:

Given that A is a 1×8 matrix and has rank 1 (since the row is non-zero), the null

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