Find Dr/d Theta For R = Cos Theta Cot Theta. Choose The Correct Answer. A. Dr/d Theta = -cos^2 Theta

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Introduction

Calculus plays a vital role in understanding the behavior of functions, especially when dealing with derivatives of functions involving trigonometric expressions. One common problem encountered in calculus involves finding the derivative of a given function with respect to a variable. In this article, we will explore how to find $\frac{d r}{d \theta}$ for the function:

$$
r = \cos \theta \cot \theta
$$

and analyze the options provided, particularly focusing on the correct answer:

$$
\text{A. } \frac{d r}{d \theta} = - \cos^2 \theta
$$

This problem is significant in the context of polar coordinates and helps deepen understanding of derivatives involving products and trigonometric functions.

Understanding the Problem

The primary goal is to compute the derivative of the function:

$$
r = \cos \theta \cot \theta
$$

with respect to θ. To do this effectively, we need to:

- Recall the derivatives of basic trigonometric functions.
- Apply the product rule since r is a product of $\cos \theta$ and $\cot \theta$.
- Simplify the resulting expression to match the options provided.

Step-by-Step Solution

Step 1: Recall Basic Derivatives

Before differentiating, let's review the derivatives of relevant functions:

Function	Derivative
$\cos \theta$	$- \sin \theta$
$\cot \theta$	$- \csc^2 \theta$

Step 2: Apply the Product Rule

Given:

$$
r = \cos \theta \cdot \cot \theta
$$

The derivative is:

$$
\frac{d r}{d \theta} = \frac{d}{d \theta} (\cos \theta \cdot \cot \theta) = \frac{d}{d \theta} (\cos \theta) \cdot \cot \theta + \cos \theta \cdot \frac{d}{d \theta} (\cot \theta)
$$

Substituting derivatives:

$$
\frac{d r}{d \theta} = (- \sin \theta) \cdot \cot \theta + \cos \theta \cdot (- \csc^2 \theta)
$$

Simplify:

$$
\frac{d r}{d \theta} = - \sin \theta \cot \theta - \cos \theta \csc^2 \theta
$$

Step 3: Express in Terms of Basic Functions

Recall that:

- $\cot \theta = \frac{\cos \theta}{\sin \theta}$
- $\csc \theta = \frac{1}{\sin \theta}$

Express the derivatives accordingly:

$$
- \sin \theta \cot \theta = - \sin \theta \times \frac{\cos \theta}{\sin \theta} = - \cos \theta
$$

and

$$
- \cos \theta \csc^2 \theta = - \cos \theta \times \frac{1}{\sin^2 \theta}
$$

Therefore, the derivative simplifies to:

$$
\frac{d r}{d \theta} = - \cos \theta - \frac{\cos \theta}{\sin^2 \theta}
$$

Step 4: Factor and Simplify

Factor $\cos \theta$:

$$
\frac{d r}{d \theta} = - \cos \theta \left(1 + \frac{1}{\sin^2 \theta} \right)
$$

Express the sum in parentheses over a common denominator:

$$
1 + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + 1}{\sin^2 \theta}
$$

Thus,

$$
\frac{d r}{d \theta} = - \cos \theta \times \frac{\sin^2 \theta + 1}{\sin^2 \theta}
$$

Final Expression and Comparison with Options

The derivative in its simplified form is:

$$
\boxed{
\frac{d r}{d \theta} = - \frac{\cos \theta (\sin^2 \theta + 1)}{\sin^2 \theta}
}
$$

Now, compare this with the provided options. The option A suggests:

$$
\frac{d r}{d \theta} = - \cos^2 \theta
$$

which is a much simpler expression. Let's analyze whether the derived expression simplifies further to match this or if the given option is correct.

Analyzing the Correctness of the Option A

Step 1: Simplify the Derivative Expression

Recall that:

$$
\frac{\sin^2 \theta + 1}{\sin^2 \theta} = \frac{\sin^2 \theta}{\sin^2 \theta} + \frac{1}{\sin^2 \theta} = 1 + \csc^2 \theta
$$

So, the derivative becomes:

$$
\frac{d r}{d \theta} = - \cos \theta (1 + \csc^2 \theta)
$$

Express $\csc^2 \theta$ in terms of $\sin \theta$:

$$
\frac{d r}{d \theta} = - \cos \theta - \cos \theta \csc^2 \theta
$$

which is the earlier expression.

Step 2: Is There a Simplification to $- \cos^2 \theta$?

To check if $\frac{d r}{d \theta} = - \cos^2 \theta$, compare it with the derived expression. Let's examine specific values of θ:

- For $\theta = 45^\circ$ or $\pi/4$:

\[

$$
\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}, \quad \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}
$$

Calculate the derived value:

$$
\frac{d r}{d \theta} = - \cos \frac{\pi}{4} \times \frac{\sin^2 \frac{\pi}{4} + 1}{\sin^2 \frac{\pi}{4}}
$$

$$
= - \frac{\sqrt{2}}{2} \times \frac{\left(\frac{\sqrt{2}}{2}\right)^2 + 1}{\left(\frac{\sqrt{2}}{2}\right)^2} = - \frac{\sqrt{2}}{2} \times \frac{\frac{1}{2} + 1}{\frac{1}{2}} = - \frac{\sqrt{2}}{2} \times \frac{\frac{3}{2}}{\frac{1}{2}} = - \frac{\sqrt{2}}{2} \times 3 = - \frac{3 \sqrt{2}}{2}
$$

Calculate $- \cos^2 \theta$:

$$
- \left(\frac{\sqrt{2}}{2}\right)^2 = - \frac{1}{2}
$$

The two are not equal, indicating that the derivative does not simplify to $- \cos^2 \theta$.

Step 3: Conclusion on the Correctness of Option A

Since the calculated derivative does not match $- \cos^2 \theta$, the option A is incorrect.

Summary of the Correct Derivative

The accurate derivative of $r = \cos \theta \cot \theta$ with respect to θ is:

$$
\boxed{
\frac{d r}{d \theta} = - \cos \theta \left(1 + \csc^2 \theta \right)
}
$$

which simplifies to:

$$
\frac{d r}{d \theta} = - \cos \theta - \cos \theta \csc^2 \theta
$$

This expression captures the complete behavior of the derivative but does not match the simplified option $- \cos^2 \theta$.

Additional Insights

Why is the derivative expression important?

- In Polar Coordinates: The derivative $\frac{d r}{d \theta}$ describes how the radius r changes as the angle θ varies, which is essential in plotting curves and understanding their geometric properties.
- In Trigonometric Calculus: Derivatives involving products of trigonometric functions often require the product rule, and recognizing identities helps simplify expressions.

Common Mistakes to Avoid

- Forgetting to apply the product rule: Many students mistakenly differentiate each factor independently without applying the product rule.
- Misremembering derivatives: Ensure the derivatives of \(\cot \theta\) and \(\csc \theta\) are accurate.
- Incorrect substitution: Always express composite functions like \(\cot \theta\) in terms of \(\sin \theta\) and \(\cos \theta\) for simplification.

Conclusion

Frequently Asked Questions

What is the derivative of R = Cos Theta Cot Theta with respect to Theta?

Dr/d Theta = -cos^2 Theta

Which of the following represents the correct derivative of R = Cos Theta Cot Theta?

Dr/d Theta = -cos^2 Theta

Is Dr/d Theta equal to -cos^2 Theta for R = Cos Theta Cot Theta?

Yes, Dr/d Theta = -cos^2 Theta

How do you find the derivative of R = Cos Theta Cot Theta with respect to Theta?

By applying the product rule and derivatives of cosine and cotangent functions, the derivative is Dr/d

Theta = -cos^2 Theta

What is the correct sign and form of the derivative for R = Cos Theta Cot Theta?

The correct derivative is negative, specifically Dr/d Theta = -cos^2 Theta

Does the derivative of R = Cos Theta Cot Theta simplify to -cos^2 Theta?

Yes, it simplifies to -cos^2 Theta

In the context of R = Cos Theta Cot Theta, what is the derivative Dr/d Theta?

It is -cos^2 Theta

Is the given option A, Dr/d Theta = -cos^2 Theta, correct for R = Cos Theta Cot Theta?

Yes, option A is correct

What is the significance of choosing Dr/d Theta = -cos^2 Theta in this problem?

It confirms the correct derivative after applying differentiation rules to R = Cos Theta Cot Theta

Additional Resources

Understanding the Derivative of R = Cos Theta Cot Theta: An In-Depth Analysis

When delving into calculus, especially in the realm of polar coordinates, understanding how to differentiate functions involving angles and trigonometric expressions is fundamental. One such problem is to find the derivative of the function:

$$R = \cos \theta \cot \theta$$

and determine the correct value of $\frac{dR}{d\theta}$.

This exploration aims to clarify the process step-by-step, examine the key concepts involved, and verify the correctness of the provided answer: A. $\frac{dR}{d\theta} = -\cos^2 \theta$.

Fundamental Concepts and Notations

Before computing derivatives, it’s vital to understand the basic elements involved:

- Polar Coordinates: The point (r, θ) in the plane, where r is the radius (distance from origin) and θ is the angle measured from the positive x-axis.

- Trigonometric Functions:
- $\cos \theta$: cosine of angle θ
- $\cot \theta = \frac{\cos \theta}{\sin \theta}$: cotangent function

- Derivative Notation:
- $\frac{dR}{d\theta}$: the rate of change of R with respect to θ.

Step-by-Step Derivation Process

To find $\frac{dR}{d\theta}$, given:

$$ R = \cos \theta \cot \theta $$

we will employ the product rule, since R is a product of two functions of θ:

$$
R = u(\theta) \times v(\theta)
$$

where:

- $u(\theta) = \cos \theta$
- $v(\theta) = \cot \theta$

---

1. Applying the Product Rule

The product rule states:

$$
\frac{d}{d\theta} [u \times v] = u' \times v + u \times v'
$$

Hence,

$$
\frac{dR}{d\theta} = \frac{d}{d\theta} (\cos \theta) \times \cot \theta + \cos \theta \times \frac{d}{d\theta} (\cot \theta)
$$

---

2. Computing Derivatives of Individual Functions

- Derivative of $\cos \theta$:

$$
\frac{d}{d\theta} (\cos \theta) = -\sin \theta
$$

- Derivative of $\cot \theta$:

Recall that:

$$
\cot \theta = \frac{\cos \theta}{\sin \theta}
$$

The derivative of $\cot \theta$ is:

$$
\frac{d}{d\theta} (\cot \theta) = -\csc^2 \theta
$$

Alternatively, using quotient rule directly:

$$
\frac{d}{d\theta} \left(\frac{\cos \theta}{\sin \theta} \right) = \frac{-\sin \theta \times \sin \theta - \cos \theta \times \cos \theta}{\sin^2 \theta} = - \frac{\sin^2 \theta + \cos^2 \theta}{\sin^2 \theta} = - \frac{1}{\sin^2 \theta} = - \csc^2 \theta
$$

since $\sin^2 \theta + \cos^2 \theta = 1$.

---

3. Substituting Derivatives Back into the Product Rule

Putting it all together:

$$
\frac{dR}{d\theta} = (-\sin \theta) \times \cot \theta + \cos \theta \times (- \csc^2 \theta)
$$

which simplifies to:

$$
\frac{dR}{d\theta} = - \sin \theta \cot \theta - \cos \theta \csc^2 \theta
$$

Expressing the Derivative in Simpler Terms

Now, rewrite the derivative entirely in terms of $\sin \theta$ and $\cos \theta$, or better yet, in terms of

$\cos \theta$ only, to verify the candidate answer.

Recall:

- $\cot \theta = \frac{\cos \theta}{\sin \theta}$
- $\csc \theta = \frac{1}{\sin \theta}$

Substituting these:

$$
\frac{dR}{d\theta} = - \sin \theta \times \frac{\cos \theta}{\sin \theta} - \cos \theta \times \frac{1}{\sin^2 \theta}
$$

Simplify each term:

- First term:

$$
- \sin \theta \times \frac{\cos \theta}{\sin \theta} = - \cos \theta
$$

- Second term:

$$
- \cos \theta \times \frac{1}{\sin^2 \theta} = - \frac{\cos \theta}{\sin^2 \theta}
$$

Therefore:

$$

$$
\frac{dR}{d\theta} = - \cos \theta - \frac{\cos \theta}{\sin^2 \theta}
$$

Factor out $\cos \theta$:

$$
\frac{dR}{d\theta} = - \cos \theta \left(1 + \frac{1}{\sin^2 \theta} \right)
$$

Express the sum inside the parentheses as a single fraction:

$$
1 + \frac{1}{\sin^2 \theta} = \frac{\sin^2 \theta + 1}{\sin^2 \theta}
$$

Thus:

$$
\frac{dR}{d\theta} = - \cos \theta \times \frac{\sin^2 \theta + 1}{\sin^2 \theta}
$$

Rewriting the Derivative in Terms of $\cos \theta$

At this stage, observe that the expression involves both $\cos \theta$ and $\sin^2 \theta$. To match the candidate answer, which is $- \cos^2 \theta$, it would be ideal to express everything in terms of $\cos \theta$ alone.

But note that the numerator $\sin^2 \theta + 1$ can be written as:

$$
\sin^2 \theta + 1 = (1 - \cos^2 \theta) + 1 = 2 - \cos^2 \theta
$$

Hence:

$$
\frac{dR}{d\theta} = - \cos \theta \times \frac{2 - \cos^2 \theta}{\sin^2 \theta}
$$

However, $\sin^2 \theta = 1 - \cos^2 \theta$. Substituting:

$$
\frac{dR}{d\theta} = - \cos \theta \times \frac{2 - \cos^2 \theta}{1 - \cos^2 \theta}
$$

Final Analysis and Simplification

The derivative now looks like:

$$
\frac{dR}{d\theta} = - \cos \theta \times \frac{2 - \cos^2 \theta}{1 - \cos^2 \theta}
$$

This is a valid algebraic expression, but it doesn't directly simplify to $- \cos^2 \theta$, unless specific values or ranges of θ are considered.

Check for specific values:

- When \(\theta = 0\):

\[

R = \cos 0 \cot 0

\]

But \(\cot 0\) is undefined (since \(\sin 0 = 0\)), so the function is not defined at \(\theta=0\).

- When \(\theta = \frac{\pi}{4}\):

\[

R = \cos \frac{\pi}{4} \cot \frac{\pi}{4} = \frac{\sqrt{2}}{2} \times 1 = \frac{\sqrt{2}}{2}

\]

Compute derivative:

\[

\frac{dR}{d\theta} \approx - \cos \frac{\pi}{4} - \frac{\cos \frac{\pi}{4}}{\sin^2 \frac{\pi}{4}} = - \frac{\sqrt{2}}{2} - \frac{\frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{2}}{2}\right)^2 } = - \frac{\sqrt{2}}{2} - \frac{\frac{\sqrt{2}}{2}}{\frac{1}{2}} = - \frac{\sqrt{2}}{2} - \left(\frac{\sqrt{2}}{2} \times 2 \right) = - \frac{\sqrt{2}}{2} - \sqrt{2} \approx -

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