Find The 17th Term Of The Arithmetic Sequence Whose Common Difference Is D=3 And Whose First Term Is

Find The 17th Term Of The Arithmetic Sequence Whose Common Difference Is D=3 And Whose First Term Is an essential problem in understanding the fundamentals of arithmetic sequences. Whether you're a student preparing for exams, a math enthusiast, or someone looking to strengthen their problem-solving skills, grasping how to determine specific terms in an arithmetic sequence is crucial. In this article, we will explore the key concepts, step-by-step methods, and practical examples to find the 17th term of such a sequence, especially when given the common difference and the first term.

Understanding Arithmetic Sequences

What Is an Arithmetic Sequence?

An arithmetic sequence is a sequence of numbers in which each term after the first is obtained by adding a fixed, constant value called the common difference (D) to the previous term. The general form of an arithmetic sequence is:

$$ a_n = a_1 + (n - 1)d $$

where:
- $ a_n $ is the nth term,
- $ a_1 $ is the first term,
- $ d $ is the common difference,
- $ n $ is the term number.

Examples of Arithmetic Sequences

- 2, 5, 8, 11, 14, ... (common difference D=3)
- 10, 7, 4, 1, -2, ... (common difference D=-3)

Understanding the structure of an arithmetic sequence enables us to find any term when the first term and common difference are known.

Given Data and Problem Statement

Details Provided

- First term, a_1 (unspecified in the prompt, but assumed to be known or provided)
- Common difference, $D = 3$
- Term to find, the 17th term, a_{17}

Clarification

Since the first term is not explicitly provided, in most scenarios, either it is given or needs to be assumed for the purpose of the calculation. For this article, we will assume that the first term, a_1, is known or can be represented as a variable for generality.

Step-by-Step Method to Find the 17th Term

Step 1: Recall the Formula for nth Term

The general formula for the nth term of an arithmetic sequence is:

$$a_n = a_1 + (n - 1)d$$

In our case, for the 17th term:

$$a_{17} = a_1 + (17 - 1)d$$

which simplifies to:

$$a_{17} = a_1 + 16d$$

Step 2: Substitute Known Values

Given:
- $d = 3$
- a_1 (unknown, but represented as a_1)

The formula becomes:

$$a_{17} = a_1 + 16 \times 3$$
$$a_{17} = a_1 + 48$$

Therefore, the 17th term is 48 more than the first term.

Step 3: Final Expression

The complete expression for the 17th term is:

$$ a_{17} = a_1 + 48 $$

If the first term $ a_1 $ is known, simply substitute its value to obtain the specific 17th term.

Practical Examples

Example 1: First Term Is Known

Suppose the first term $ a_1 = 5 $. Then:

$$ a_{17} = 5 + 48 = 53 $$

So, the 17th term of the sequence is 53.

Example 2: First Term Is Zero

If $ a_1 = 0 $, then:

$$ a_{17} = 0 + 48 = 48 $$

Thus, the 17th term is 48.

Example 3: First Term Is Negative

Suppose $ a_1 = -10 $. Then:

$$ a_{17} = -10 + 48 = 38 $$

The 17th term in this case is 38.

Additional Considerations

What if the Common Difference Is Negative?

If the common difference D were negative, for example, D = -3, the formula adjusts accordingly:

$$ a_{17} = a_1 + (17 - 1)(-3) = a_1 - 48 $$

This means the 17th term decreases by 3 each time, starting from $ a_1 $.

Importance of Knowing the First Term

The calculation hinges on knowing the first term. If this value isn't provided, you must obtain it from additional context or data.

Summary of the Procedure

- Use the nth term formula.
- Plug in the term number (17) and the common difference (3).
- Add this to the first term to find the specific term.

Applications of Finding Specific Terms in Arithmetic Sequences

Real-Life Scenarios

- Calculating payments over time with fixed increments.
- Estimating population growth with consistent increases.
- Planning schedules or sequences with regular intervals.

Educational Importance

Understanding how to find specific terms enhances problem-solving skills and conceptual clarity in algebra and sequences.

Summary and Conclusion

To find the 17th term of an arithmetic sequence with a known common difference $ D=3 $, the key is to know the first term $ a_1 $. The formula:

$$ a_{17} = a_1 + 16 \times 3 = a_1 + 48 $$

provides a straightforward method to determine the desired term. Whether the first term is given or needs to be inferred, the process remains consistent, emphasizing the importance of understanding the structure of arithmetic sequences for effective mathematical problem-solving.

Final Tips for Solving Similar Problems

- Always verify whether the first term is given.
- Pay attention to the sign of the common difference.
- Substitute accurately into the formula.
- Practice with different values to build confidence.

By mastering these steps, you'll be well-equipped to handle various arithmetic sequence problems confidently and efficiently.

Frequently Asked Questions

How do I find the 17th term of an arithmetic sequence with a common difference of D=3 and a given first term?

To find the 17th term, use the formula: $a_n = a_1 + (n - 1)d$. Plug in n=17, d=3, and the given first term a_1 to calculate the term.

What information do I need to determine the 17th term of an arithmetic sequence?

You need the first term (a_1) and the common difference (D=3). With these, you can find the 17th term using the formula for the nth term.

If the first term of an arithmetic sequence is 5 and the common difference is 3, what is the 17th term?

Using the formula $a_n = a_1 + (n - 1)d$: a_1=5, d=3, n=17. So, $a_1 + (17-1)\times3 = 5 + 16\times3 = 5 + 48 = 53$. The 17th term is 53.

Can you provide the general formula to find the 17th term of any arithmetic sequence?

Yes. The formula is $a_{17} = a_1 + (17 - 1) \times D$, where a_1 is the first term and D is the common difference.

What should I do if I know the 17th term and the common difference but not the first term?

You can rearrange the formula: $a_1 = a_{17} - (17 - 1) \times D$. Substitute the known 17th term and common difference to find the first term.

Additional Resources

Find The 17th Term Of The Arithmetic Sequence Whose Common Difference Is D=3 And Whose First Term Is

Understanding arithmetic sequences is fundamental in many areas of mathematics and real-world applications, from calculating interest rates to analyzing patterns in data. In this comprehensive review, we will explore the concept of arithmetic sequences, focusing specifically on how to find the 17th term of such a sequence when given the common difference and the first term. We will delve into the foundational principles, step-by-step methods, and practical examples to ensure a thorough grasp of the topic.

Introduction to Arithmetic Sequences

What Is an Arithmetic Sequence?

An arithmetic sequence is a sequence of numbers in which each term after the first is obtained by adding a fixed constant, known as the common difference (D), to the previous term. This predictable pattern makes arithmetic sequences one of the simplest types of sequences to analyze and understand.

Formal Definition:
- An arithmetic sequence is denoted as: $a_1, a_2, a_3, \ldots$
- where $a_{n} = a_1 + (n - 1)d$
- Here, a_1 is the first term, and d is the common difference.

Key Characteristics:
- The difference between any two consecutive terms is constant.
- The sequence can be increasing, decreasing, or constant depending on the sign and value of d.

Examples of Arithmetic Sequences

- 2, 5, 8, 11, 14, ... (common difference $d=3$)
- 10, 7, 4, 1, -2, ... (common difference $d=-3$)
- 5, 5, 5, 5, ... (common difference $d=0$)

Understanding the Problem: Find the 17th Term

The problem states:
- Common difference $D=3$
- First term a_1 is given (though unspecified in the prompt, we will analyze the general case and then apply a specific value)

Our goal:
- To find the 17th term of this sequence, i.e., a_{17}.

Mathematical Formula for the nth Term

The crux of solving such problems lies in the nth term formula of an arithmetic sequence:

$$
a_{n} = a_1 + (n - 1)d
$$

Where:
- a_{n} is the nth term we want to find.
- a_1 is the first term.
- d is the common difference.
- n is the position of the term in the sequence.

Applying the formula:
- To find the 17th term, plug in $n=17$:

$$
a_{17} = a_1 + (17 - 1)d = a_1 + 16d
$$

This formula provides a direct computation method once the first term a_1 and common difference d are known.

Step-by-Step Calculation Process

Let's consider the general scenario and then proceed with specific examples to illustrate the process:

Step 1: Identify the first term a_1
- The problem states “whose first term is” but does not specify a value. For the sake of clarity, we'll analyze the problem in two ways:
- Case A: When the first term a_1 is known (say, $a_1 = A$)
- Case B: The sequence is purely abstract, and the first term is left as a variable.

Step 2: Use the formula for the nth term
- Substitute the known values into the nth term formula:

$$
a_{17} = a_1 + 16d
$$

- Since $d=3$, the formula simplifies to:

$$
a_{17} = a_1 + 16 \times 3 = a_1 + 48
$$

Step 3: Calculate the 17th term
- For specific first term values:
- If $a_1=2$:

$$
a_{17} = 2 + 48 = 50
$$

- If $a_1=5$:

$$
a_{17} = 5 + 48 = 53
$$

- For an unknown first term, the answer remains expressed as:

$$
a_{17} = a_1 + 48
$$

Key takeaway: The 17th term depends linearly on the first term when the common difference is known.

Practical Examples and Applications

Example 1: First term $a_1=10$, common difference $d=3$

- Calculate a_{17}:

$$
a_{17} = 10 + 16 \times 3 = 10 + 48 = 58
$$

- Interpretation: The 17th term in this sequence is 58.

Example 2: First term $a_1=0$, common difference $d=3$

- Calculate a_{17}:

$$
a_{17} = 0 + 16 \times 3 = 48
$$

Example 3: First term $a_1=-5$, common difference $d=3$

- Calculate a_{17}:

$$
a_{17} = -5 + 16 \times 3 = -5 + 48 = 43
$$

Exploring Variations: What If the Common Difference Changes?

While the primary problem specifies $d=3$, it's instructive to understand how changing d affects the sequence and the 17th term:

- If $d=0$: The sequence is constant, and every term equals the first term, so $a_{17} = a_1$.
- If $d>0$: The sequence increases by 3 each time, leading to larger terms as n increases.
- If $d<0$: The sequence decreases, and terms become smaller as n increases.

Implication: The formula $a_{17} = a_1 + 16d$ remains valid regardless of the sign or magnitude of d.

Real-World Applications of Arithmetic Sequences

Understanding how to find specific terms in an arithmetic sequence is vital beyond pure mathematics. Here are some real-world contexts where this knowledge applies:

- Financial Planning: Calculating the total amount after a certain number of regular payments or interest accruals.
- Construction and Engineering: Determining the position of evenly spaced components or materials.
- Scheduling: Planning events occurring at fixed intervals.
- Population Studies: Modeling populations with consistent growth or decline per time period.

Additional Considerations and Tips

Tip 1: Clarify the First Term
- Always ensure to identify the value of a_1 before calculations.
- If the problem does not specify a_1, check the context or assumptions.

Tip 2: Use the Formula Correctly
- Remember the formula:

$$
a_{n} = a_1 + (n - 1)d
$$

- Carefully substitute the correct values for a_1, d, and n.

Tip 3: Handle Negative or Zero Differences
- Be cautious when d is negative; the sequence decreases.
- For $d=0$, the sequence is constant, and the nth term equals the first term.

Tip 4: Verify Units and Context
- Confirm units if dealing with real-world data (e.g., meters, dollars, days).
- Cross-check calculations for consistency.

Conclusion

Finding the 17th term of an arithmetic sequence with a known common difference and first term is straightforward when using the fundamental formula:

$$
a_{n} = a_1 + (n - 1)d
$$

In this specific case, where $d=3$, the formula simplifies to:

$$
a_{17} = a_1 + 16 \times 3 = a_1 + 48
$$

The key steps involve:
- Identifying or being given the first term a_1,
- Applying the formula correctly,
- Plugging in the values to compute the desired term.

Understanding and applying this process enhances problem-solving skills in mathematics and provides a foundation for exploring more complex sequences and series. Whether you're tackling a homework problem, preparing for exams, or applying these principles practically, mastery of arithmetic sequences and their formulas is an invaluable tool in your mathematical toolkit.

Remember: The power of arithmetic sequences lies in their simplicity and predictability. Mastery of this topic enables you to analyze patterns, forecast future values, and develop a strong foundation for more advanced mathematical concepts.

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