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Understanding how to find the area enclosed between curves is a fundamental concept in integral calculus. This article explores the process of determining the area bounded by the graph of the equation $y = x^2 - 15$ over a specified interval. By the end, you'll grasp the methods involved, including identifying points of intersection, setting up the integral, and calculating the area accurately.

Introduction to Area Between Curves

In calculus, the area between two curves is the region enclosed by their graphs over a specific interval. Calculating this area involves integrating the difference between the functions that define the curves. When dealing with a single curve and a horizontal boundary, the process simplifies to integrating the function over the interval, especially if the boundary is a constant line.

Understanding the Given Equation: $y = x^2 - 15$

The equation $y = x^2 - 15$ describes a parabola opening upward with its vertex located at (0, -15). This parabola intersects the x-axis where $y=0$, which allows us to find the points of intersection with the x-axis and potentially with other curves if provided.

Key Features of the Graph:

- Vertex: Located at (0, -15), since the parabola opens upwards and the vertex form is standard.
- Axis of Symmetry: The line $x=0$.
- Intercepts:
- Y-intercept: At $x=0$, $y=-15$.
- X-intercepts: Found by solving $x^2 - 15=0$.

Finding the X-Intercepts

To determine the points where the parabola crosses the x-axis:

$$
x^2 - 15 = 0
$$
$$
x^2 = 15
$$
$$
x = \pm \sqrt{15}
$$

Thus, the parabola intersects the x-axis at:

$$
x = -\sqrt{15} \quad \text{and} \quad x = \sqrt{15}
$$

Numerically, since $\sqrt{15} \approx 3.873$, the points of intersection are approximately at $(-3.873, 0)$ and $(3.873, 0)$.

Determining the Bounded Region

To find the area bounded by the curve $y = x^2 -15$ over a given interval, it's crucial to understand:

- The interval over which the area is considered.
- Whether the parabola is bounded above or below by other functions or lines.

Assumption: Suppose the interval is from $x = a$ to $x = b$, with $a = -\sqrt{15}$ and $b= \sqrt{15}$. This is common in problems where the area between a parabola and the x-axis is calculated over its roots.

Note: If the problem provides a different interval, adjust the limits accordingly.

Example: Calculating the Area Between the Parabola and the X-axis from $x = -\sqrt{15}$ to $x = \sqrt{15}$

Since the parabola intersects the x-axis at these points, the region between the parabola and the x-axis over this interval is enclosed.

Step 1: Set Up the Integral

The area A is given by:

$$
A = \int_{a}^{b} |y|\, dx
$$

Because the parabola opens upward and is below the x-axis between $x = -\sqrt{15}$ and 0, and above the x-axis between 0 and $\sqrt{15}$, we need to account for the absolute value.

- For x in $[-\sqrt{15}, 0]$, $y \leq 0$, so $|y| = -(x^2 - 15) = 15 - x^2$.
- For x in $[0, \sqrt{15}]$, $y \geq 0$, so $|y| = x^2 - 15$.

Step 2: Split the Integral

The total area:

$$
A = \int_{-\sqrt{15}}^{0} (15 - x^2) \, dx + \int_{0}^{\sqrt{15}} (x^2 - 15) \, dx
$$

Step 3: Calculate Each Integral

First integral:

$$
A_1 = \int_{-\sqrt{15}}^{0} (15 - x^2) \, dx
$$

Second integral:

$$
A_2 = \int_{0}^{\sqrt{15}} (x^2 - 15) \, dx
$$

Computing the Integrals

Integral A_1:

$$
A_1 = \left[15x - \frac{x^3}{3} \right]_{-\sqrt{15}}^{0}
$$

At $x=0$:

$$
15 \times 0 - \frac{0^3}{3} = 0
$$

At $x=-\sqrt{15}$:

$$
15 \times (-\sqrt{15}) - \frac{(-\sqrt{15})^3}{3} = -15\sqrt{15} - \frac{- (\sqrt{15})^3}{3}
$$

Note that:

$$
(-\sqrt{15})^3 = - (\sqrt{15})^3
$$

and

$$
(\sqrt{15})^3 = (\sqrt{15}) \times (\sqrt{15}) \times (\sqrt{15}) = (15) \times \sqrt{15}
$$

Therefore,

$$
(-\sqrt{15})^3 = -15 \sqrt{15}
$$

Now, substitute back:

$$
A_1 = \left[0 - \left(-15 \sqrt{15} - \frac{-15 \sqrt{15}}{3} \right) \right] = - \left(-15 \sqrt{15} + \frac{15 \sqrt{15}}{3} \right)
$$

Simplify:

$$
A_1 = - \left(-15 \sqrt{15} + 5 \sqrt{15} \right) = - (-10 \sqrt{15}) = 10 \sqrt{15}
$$

Integral A_2:

$$
A_2 = \left[\frac{x^3}{3} - 15x \right]_0^{\sqrt{15}}
$$

At $x = \sqrt{15}$:

$$
\frac{(\sqrt{15})^3}{3} - 15 \times \sqrt{15} = \frac{15 \sqrt{15}}{3} - 15 \sqrt{15} = 5 \sqrt{15} - 15 \sqrt{15} = -10 \sqrt{15}
$$

At $x = 0$:

$$
0 - 0 = 0
$$

So,

$$
A_2 = -10 \sqrt{15} - 0 = -10 \sqrt{15}
$$

Final Area Calculation

Adding the two parts:

$$
A = A_1 + A_2 = 10 \sqrt{15} + (-10 \sqrt{15}) = 0
$$

Interpretation: The total algebraic sum cancels out because one part is positive and the other negative, representing regions above and below the x-axis. Since we are interested in the total enclosed area, we take the absolute value:

$$
\boxed{
\text{Total Area} = |A_1| + |A_2| = 10 \sqrt{15} + 10 \sqrt{15} = 20 \sqrt{15}}
$$

Numerically, this is approximately:

$$
20 \times 3.873 = 77.46
$$

Summary:

- The area between the parabola $y = x^2 - 15$ and the x-axis over the interval $[-\sqrt{15}, \sqrt{15}]$ is approximately 77.46 square units.
- The exact area in radical form is $20 \sqrt{15}$.

General Approach to Finding Areas Bounded by Curves

The method demonstrated here can be generalized to various functions and intervals. The key steps include:

- Identifying points of intersection between the curves or with boundaries.
- Partitioning the interval based on where the functions change relative positions, especially considering absolute values.
- Setting up the integral as the sum of integrals over subintervals, with the integrand being the absolute difference of the functions or the function itself if bounded by a line.
- Calculating the definite integrals carefully, considering the sign of the functions over subintervals.