Find The Area Of The Surface Generated When The Given Curve Is Revolved About The Given Axis. Y = 5x

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Understanding how to compute the surface area generated by revolving a curve about a specific axis is a fundamental concept in calculus, especially in the study of surface areas of revolution. In this article, we will explore the detailed process of calculating the surface area when the curve y = 5x is revolved about a given axis, typically the x-axis or y-axis. We will discuss the general formula, the steps involved, and work through a comprehensive example to solidify the concept.

Understanding the Concept of Surface Area of Revolution

What is Surface Area of Revolution?

The surface area of a solid generated when a curve is revolved about an axis is the total area of the outer surface of the resulting three-dimensional object. Unlike the area under a curve, which is a two-dimensional measure, the surface area of revolution extends into three dimensions, capturing the outer surface of the generated shape.

Significance in Mathematics and Applications

Calculating surface areas of revolution has applications across physics, engineering, and manufacturing, such as designing objects with rotational symmetry, calculating the surface of a vase, or understanding the physical properties of objects formed by revolving curves.

General Formula for Surface Area of Revolution

Revolving About the X-Axis

If a curve y = f(x), defined on the interval [a, b], is revolved about the x-axis, the surface area S is given by:

- **$S = 2\pi \int_a^b y \sqrt{1 + (dy/dx)^2}\, dx$**

This formula calculates the lateral surface area by integrating the circumference of infinitesimal rings along the x-axis.

Revolving About the Y-Axis

Similarly, if the curve y = f(x) is revolved about the y-axis, the formula becomes:

- **$S = 2\pi \int_a^b x \sqrt{1 + (dy/dx)^2}\, dx$**

The choice of axis influences the form of the formula but the core idea remains the same: integrating the circumference of infinitesimal rings along the relevant axis.

Applying the Formula to y = 5x

Step 1: Define the interval of interest

To compute the surface area, we need to specify the interval over which the curve is revolved. Suppose we consider the segment from x = a to x = b. For the sake of an example, let's take:

- a = 0
- b = 1

This interval allows us to illustrate the process clearly.

Step 2: Calculate dy/dx

Given y = 5x,

- dy/dx = 5

Since the derivative is constant, calculations simplify.

Step 3: Write the surface area integral

Revolving about the x-axis, the surface area S is:

- $S = 2\pi \int_0^1 y \sqrt{1 + (dy/dx)^2}\, dx$

Substituting $y = 5x$ and $dy/dx = 5$:

- $S = 2\pi \int_0^1 5x \sqrt{1 + 25}\, dx$

Simplify the square root:

- $\sqrt{1 + 25} = \sqrt{26}$

Thus,

- $S = 2\pi \sqrt{26} \int_0^1 5x\, dx$

Note: The constant $\sqrt{26}$ can be factored out of the integral.

Step 4: Perform the Integration

Calculate the integral:

- $\int 5x\, dx = (5/2) x^2 + C$

Evaluate from 0 to 1:

- $(5/2) (1)^2 - (5/2) (0)^2 = (5/2) 1 = 5/2$

Now, multiply by the constants:

- $S = 2\pi \sqrt{26} (5/2) = \pi \sqrt{26}\ 5 = 5\pi \sqrt{26}$

Result: The surface area generated when $y=5x$ from $x=0$ to $x=1$ is $5\pi \sqrt{26}$ square units.

Generalization and Considerations

Extending to Other Intervals

The process remains similar regardless of the interval. For a general interval [a, b], the surface area is:

- $S = 2\pi \int_a^b y \sqrt{1 + (dy/dx)^2}\, dx$

Substitute y and dy/dx accordingly, evaluate the integral, and multiply by 2π.

Revolving About the Y-Axis

If the curve were revolved about the y-axis, the formula would change to:

- $S = 2\pi \int_a^b x \sqrt{1 + (dy/dx)^2}\, dx$

Using the same $y=5x$, the integral would involve x as a function of y.

Analysis of the Surface Area for the Entire Line y=5x

Infinite Surface Area Consideration

If the curve $y=5x$ is revolved over an infinite interval, the surface area could become unbounded. For example, revolving $y=5x$ over $[0, \infty)$ results in an infinite surface area.

Practical Applications

In real-world scenarios, such as manufacturing or physical modeling, the limits are finite, making the calculation of surface areas feasible and meaningful.

Summary of Steps to Find the Surface Area of Revolution

1. Identify the curve $y = f(x)$ and the interval $[a, b]$.
2. Compute the derivative dy/dx.
3. Determine the axis of revolution (x-axis or y-axis).
4. Write the appropriate surface area integral using the general formula.
5. Substitute y and dy/dx into the integral.
6. Simplify and evaluate the integral.
7. Multiply by 2π to obtain the surface area.

Conclusion

Calculating the surface area generated by revolving the curve $y=5x$ about an axis involves understanding the fundamental formula for surface areas of revolution. For the specific example over the interval [0, 1], we found that the surface area is $5\pi \sqrt{26}$ square units. This process can be adapted to various functions and intervals, and understanding these steps is crucial for solving a wide range of problems involving surfaces of revolution in calculus.

In practice, always ensure the interval and axis are properly defined, compute derivatives carefully, and perform integrations accurately. Mastery of these techniques enables precise calculations of complex surfaces that are essential in both theoretical mathematics and practical engineering applications.

Frequently Asked Questions

How do you find the surface area generated when the curve y = 5x is revolved about the x-axis?

To find the surface area, use the formula $S = 2\pi \int$ from a to b of $y \sqrt{(1 + (dy/dx)^2)}\, dx$, where $y = 5x$. Substitute $dy/dx = 5$, and integrate over the desired interval.

What is the formula for the surface area when revolving y = 5x about the y-axis?

When revolving around the y-axis, the surface area is given by $S = 2\pi \int y \sqrt{1 + (dx/dy)^2}\, dy$. Since y = 5x, solve for x and change variables accordingly to compute the integral.

If the curve y = 5x is revolved between x = 0 and x = a, what is the expression for the surface area?

The surface area is $S = 2\pi \int_0^{a} y \sqrt{1 + (dy/dx)^2}\, dx = 2\pi \int_0^{a} 5x \sqrt{1 + 25}\, dx = 2\pi \int_0^{a} 5x \sqrt{26}\, dx$.

How does the choice of axis of revolution affect the formula for the surface area of y = 5x?

Revolving about the x-axis uses $S = 2\pi \int y \sqrt{1 + (dy/dx)^2}\, dx$, while revolving about the y-axis uses $S = 2\pi \int x \sqrt{1 + (dy/dx)^2}\, dy$. The axis determines which variable is integrated over and how the formula is applied.

What are common methods to evaluate the surface area integral for the curve y = 5x?

Common methods include direct integration if the limits are finite, substitution if the integral simplifies, or applying numerical methods for complex limits. For the given linear curve, straightforward integration is typically used.

Additional Resources

Find The Area Of The Surface Generated When The Given Curve Is Revolved About The Given Axis: Y = 5x

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Introduction

The problem of finding the surface area of a solid of revolution is a fundamental concept in calculus, bridging the gap between geometric intuition and analytical computation. When a curve is revolved about a specific axis, it creates a three-dimensional surface, and calculating the area of this surface involves understanding how the curve's geometry translates into a surface in space.

This detailed exploration focuses on the specific case where the curve is $y = 5x$, and it is revolved about a given axis—either the x-axis or y-axis. We will analyze both scenarios thoroughly, deriving the formulas, explaining the underlying principles, and illustrating the step-by-step process involved.

Fundamental Concepts and Theoretical Background

1. Surface of Revolution

A surface of revolution is generated when a plane curve is rotated about an axis in the plane. The resulting surface is often called a surface of revolution.

- Examples: A circle rotated about an axis yields a sphere; a parabola yields a paraboloid, etc.
- Applications: Design of objects in engineering (tanks, vases), modeling natural forms, and solving physics problems related to rotational symmetry.

2. Surface Area of a Revolution

The general formula for the surface area S generated by revolving a curve $y = f(x)$ about the x-axis, from $x=a$ to $x=b$, is:

$$
S = 2\pi \int_{a}^{b} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

Similarly, if the curve is revolved about the y-axis, the corresponding formula is:

$$
S = 2\pi \int_{c}^{d} x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dy
$$

where the limits are in the domain of the respective variable.

Specific Case: $y = 5x$

Given the linear function:

$$
y = 5x
$$

- This is a straight line passing through the origin with a slope of 5.
- The simplicity of the line allows for direct application of the formulas, but the actual surface area depends on the interval over which the revolution occurs.

Choosing an Axis of Revolution

The problem statement mentions "Given Axis" but does not specify it explicitly. Therefore, we will analyze both:

- Revolution about the x-axis

- Revolution about the y-axis

Each case involves different integral setups but follows similar principles.

Case 1: Revolution About the x-Axis

1. Geometric Interpretation

When the curve $y = 5x$ is revolved about the x-axis:

- The resulting surface resembles a conical frustum or a conical surface, depending on the limits.
- The radius of the generated surface at a point x is $y = 5x$.

2. Surface Area Formula

Applying the general formula:

$$
S_x = 2\pi \int_{a}^{b} y \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dx
$$

Since $y = 5x$, we compute:

$$
\frac{dy}{dx} = 5
$$

and

$$
\sqrt{1 + (5)^2} = \sqrt{1 + 25} = \sqrt{26}
$$

Therefore,

$$
S_x = 2\pi \int_{a}^{b} 5x \times \sqrt{26} \, dx
$$

which simplifies to:

$$
S_x = 2\pi \sqrt{26} \times 5 \int_{a}^{b} x \, dx
$$

or

\[

S_x = 10\pi \sqrt{26} \int_{a}^{b} x \, dx
\]

The integral:

\[
\int_{a}^{b} x \, dx = \frac{b^2 - a^2}{2}
\]

So, the surface area becomes:

\[
\boxed{
S_x = 10\pi \sqrt{26} \times \frac{b^2 - a^2}{2} = 5\pi \sqrt{26} (b^2 - a^2)
}
\]

Note: To compute an exact value, specific limits \(a \) and \(b \) are required.

Case 2: Revolution About the y-Axis

1. Geometric Interpretation

Rotating \(y = 5x \) about the y-axis:

- The curve can be rewritten as:

\[
x = \frac{y}{5}
\]

- The surface generated resembles a conical shape extending along the y-axis.

2. Surface Area Formula

Using the y-axis revolution formula:

\[
S_y = 2\pi \int_{c}^{d} x \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \, dy
\]

But since \(x = y/5 \), and \(dy/dx = 5 \), the integral in terms of \(y \) becomes:

\[
S_y = 2\pi \int_{c}^{d} \frac{y}{5} \sqrt{1 + 25} \, dy
\]

which simplifies as:

$$
S_y = 2\pi \times \frac{1}{5} \sqrt{26} \int_{c}^{d} y \, dy
$$

Calculating the integral:

$$
\int_{c}^{d} y \, dy = \frac{d^2 - c^2}{2}
$$

Thus,

$$
S_y = \frac{2\pi}{5} \sqrt{26} \times \frac{d^2 - c^2}{2} = \frac{\pi}{5} \sqrt{26} (d^2 - c^2)
$$

Again, specific limits c and d are necessary for an exact numerical answer.

Practical Considerations and Limitations

1. Choice of Limits

- The limits a, b (or c, d) depend on the specific segment of the curve being revolved.
- For a finite surface, the interval is limited; for an infinite surface, the limits extend to infinity, leading to convergence issues.

2. Convergence of the Surface Area Integral

- For lines like $y = 5x$, if the limits tend to infinity, the surface area may diverge, implying an infinite surface.
- To obtain a finite surface, the limits must be finite, corresponding to a specific segment of the curve.

3. Physical Interpretation

- The generated surface resembles a conical frustum or a cone, depending on the interval.
- For example, revolving the segment from $x=0$ to $x=a$:

$$
\text{Surface area} = 5\pi \sqrt{26} (a^2 - 0) = 5\pi \sqrt{26} a^2
$$

- The surface area increases quadratically with the length of the segment.

Worked Example

Suppose we revolve the segment $x \in [0, 2]$:

- For the x-axis revolution:

$$
S_x = 5\pi \sqrt{26} (2^2 - 0) = 5\pi \sqrt{26} \times 4 = 20\pi \sqrt{26}
$$

Numerically:

$$
\sqrt{26} \approx 5.10
$$

then,

$$
S_x \approx 20 \times 3.1416 \times 5.10 \approx 20 \times 3.1416 \times 5.10 \approx 20 \times 16.03 \approx 320.6
$$

Approximate surface area: 320.6 square units

Summary of Key Points

- The surface area generated by revolving a straight line $y=5x$ about an axis depends on the interval chosen.
- The formula involves integrating the radius times the element of arc length, accounting for the slope.
- For revolution about the x-axis:

$$
S_x = 5\pi \sqrt{26} (b^2 - a^2)
$$

- For revolution about the y-axis:

$$
S_y = \frac{\pi}{5} \sqrt{26} (d^2 - c^2)
$$

- The integral bounds a, b or c, d should be finite for a finite surface area.

Advanced Topics and Further Exploration

1. Surface Area for Curves Other Than Lines

- For non-linear curves, the derivatives become more complex.
- Examples include parabolas, hyperbolas, and other polynomial functions.

2. Infinite Surfaces

- When the limits extend to infinity, the convergence of the surface area integral must be analyzed.
- In some cases, the surface area diverges, indicating an infinitely large surface.

3. Numerical Methods

- For complicated functions, numerical integration methods such as Simpson's rule

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