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Understanding vector operations is fundamental in fields such as physics, engineering, and mathematics. Among these operations, the cross product stands out due to its unique properties and applications—particularly in calculating perpendicular vectors, torque, and rotational motion. In this comprehensive guide, we will explore how to find the cross products of two vectors, specifically U and V, given as:

- U = 2i + j + 2k
- V = i + 3j + 2k

Additionally, we will analyze the cross products Uv and Vu, simplifying and clarifying each step along the way. Whether you're a student seeking to grasp the concept or a professional brushing up on vector calculus, this article will provide detailed explanations, step-by-step calculations, and tips to master cross product computations.

Understanding Vectors and the Cross Product

What Are Vectors?

Vectors are mathematical objects that have both magnitude and direction. They are represented as directed line segments in space and are often expressed in component form relative to coordinate axes:

$$
\vec{U} = U_x \mathbf{i} + U_y \mathbf{j} + U_z \mathbf{k}
$$

$$
\vec{V} = V_x \mathbf{i} + V_y \mathbf{j} + V_z \mathbf{k}
$$

Where:
- $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the unit vectors along the x, y, and z axes, respectively.

- $U_x, U_y, U_z, V_x, V_y, V_z$ are scalar components.

In our case:
- $\vec{U} = 2 \mathbf{i} + 1 \mathbf{j} + 2 \mathbf{k}$
- $\vec{V} = 1 \mathbf{i} + 3 \mathbf{j} + 2 \mathbf{k}$

What Is the Cross Product?

The cross product (or vector product) of two vectors $\vec{U}$ and $\vec{V}$, denoted as $\vec{U} \times \vec{V}$, results in a vector that is perpendicular to both $\vec{U}$ and $\vec{V}$. Its magnitude corresponds to the area of the parallelogram formed by the two vectors:

$$
|\vec{U} \times \vec{V}| = |\vec{U}| |\vec{V}| \sin \theta
$$

Where θ is the angle between $\vec{U}$ and $\vec{V}$.

Key Properties of Cross Product:
- Anti-commutative: $\vec{U} \times \vec{V} = - (\vec{V} \times \vec{U})$
- Distributive: $\vec{U} \times (\vec{V} + \vec{W}) = \vec{U} \times \vec{V} + \vec{U} \times \vec{W}$
- Zero Vector: If $\vec{U}$ and $\vec{V}$ are parallel or one is zero, then $\vec{U} \times \vec{V} = \vec{0}$

Calculating the Cross Product of Vectors U and V

Step 1: Write the Vectors in Component Form

Given:
$$
\vec{U} = 2 \mathbf{i} + 1 \mathbf{j} + 2 \mathbf{k}
$$
$$
\vec{V} = 1 \mathbf{i} + 3 \mathbf{j} + 2 \mathbf{k}
$$

Components:
$$
U_x = 2, \quad U_y = 1, \quad U_z = 2
$$
$$

$$
V_x = 1, \quad V_y = 3, \quad V_z = 2
$$

Step 2: Set Up the Cross Product Formula

The cross product is computed using the determinant of a 3x3 matrix:

$$
\vec{U} \times \vec{V} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
U_x & U_y & U_z \\
V_x & V_y & V_z \\
\end{vmatrix}
$$

Expanding this determinant yields:

$$
\vec{U} \times \vec{V} = \mathbf{i}(U_y V_z - U_z V_y) - \mathbf{j}(U_x V_z - U_z V_x) + \mathbf{k}(U_x V_y - U_y V_x)
$$

Step 3: Calculate Each Component

- i-component:
$$
U_y V_z - U_z V_y = (1)(2) - (2)(3) = 2 - 6 = -4
$$

- j-component:
$$
U_x V_z - U_z V_x = (2)(2) - (2)(1) = 4 - 2 = 2
$$
Remember to negate this component because of the determinant expansion:
$$
-\mathbf{j}(2) = -2 \mathbf{j}
$$

- k-component:
$$
U_x V_y - U_y V_x = (2)(3) - (1)(1) = 6 - 1 = 5
$$

Step 4: Write the Final Cross Product

Combining all components:

\[
\boxed{
\vec{U} \times \vec{V} = -4 \mathbf{i} - 2 \mathbf{j} + 5 \mathbf{k}
}
\]

Result:

\[
\boxed{
\vec{U} \times \vec{V} = -4 \mathbf{i} - 2 \mathbf{j} + 5 \mathbf{k}
}
\]

This vector is perpendicular to both \(\vec{U}\) and \(\vec{V}\), with a magnitude of:

\[
|\vec{U} \times \vec{V}| = \sqrt{(-4)^2 + (-2)^2 + 5^2} = \sqrt{16 + 4 + 25}
= \sqrt{45} \approx 6.708
\]

Understanding and Calculating the Cross Product Vu

What Is Vu?

Since the cross product is not commutative, \(\vec{V} \times \vec{U}\) (or Vu) is generally not equal to \(\vec{U} \times \vec{V}\). Instead, it is equal to the negative of the cross product:

\[
\vec{V} \times \vec{U} = - (\vec{U} \times \vec{V})
\]

Step-by-Step Calculation of Vu

Given the previous result:

\[

\vec{U} \times \vec{V} = -4 \mathbf{i} - 2 \mathbf{j} + 5 \mathbf{k}
\]

Therefore:

\[
\boxed{
\vec{V} \times \vec{U} = - (\vec{U} \times \vec{V}) = 4 \mathbf{i} + 2
\mathbf{j} - 5 \mathbf{k}
}
\]

This vector is perpendicular to both \(\vec{V}\) and \(\vec{U}\), but points in the opposite direction compared to \(\vec{U} \times \vec{V}\).

Summary of Key Points

1. **Vectors in component form:** Express vectors \(\vec{U}\) and \(\vec{V}\) with their i, j, k components for straightforward calculations.

2. **Cross product formula:** Use the determinant method for accurate computation:

\[
\vec{A} \times \vec{B} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
A_x & A_y & A_z \\
B_x & B_y & B_z \\
\end{vmatrix}
\]

3. **Properties of cross product:** Remember the anti-commutative property: \(\vec{U} \times \vec{V} = - (\vec{V} \times \vec{U})\).

4. **Magnitude of cross product:** Calculate using the square root of the sum of squares of components.

5. **Application:** Cross products are vital in physics for calculating torque, angular momentum, and normal vectors to surfaces.

Practical Applications and Importance of Cross Products

Physics and

Frequently Asked Questions

What is the vector U given as in the problem?

The vector U is given as 2i + 2j + 2k.

What is the vector V in the problem?

The vector V is given as i + 3j + 2k.

How do you compute the cross product U × V?

To compute U × V, use the determinant of a matrix with unit vectors i, j, k in the first row, the components of U in the second, and the components of V in the third.

What is the expression for the cross product V × U?

V × U is calculated similarly, by taking the

determinant with V's components first and U's components second.

What is the simplified form of the cross product U × V?

The simplified U × V is 0i - 4j + 7k.

What is the simplified form of the cross product V × U?

The simplified V × U is 0i + 4j - 7k, which is the negative of U × V.

Are the cross products U × V and V × U always negatives of each other?

Yes, in vector algebra, U × V = - (V × U).

Why is it important to verify the signs of the cross products?

Because the cross product is anti-commutative, meaning reversing the order changes the sign, which confirms the calculations are correct.

Additional Resources

Cross Products: An In-Depth Exploration of U×V and V×U in Vector Algebra

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Introduction to Cross Products: The Cornerstone of Vector Operations

In the realm of vector calculus, the cross product is a fundamental operation that finds extensive application across physics, engineering, computer graphics, and more. It provides a vector perpendicular to two given vectors in three-dimensional space, encapsulating both magnitude and directional information. The cross product not only helps in calculating areas of parallelograms, torque, and angular momentum but also plays a vital role in understanding the orientation of vector systems.

This article delves deep into the calculation and interpretation of the cross products U×V and V×U for specific vectors, illustrating the process with detailed explanations, step-by-step derivations, and practical insights.

Defining the Vectors: U and V

Before jumping into computations, let's clearly define our vectors:

- Vector U: \(\mathbf{U} = 2\mathbf{i} + 2\mathbf{j} + 2\mathbf{k} \)
This vector has components:
\[
\mathbf{U} = (2, 2, 2)
\]

- Vector V: \(\mathbf{V} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} \)
Its components are:
\[
\mathbf{V} = (1, 3, 2)
\]

Calculating the Cross Product: $U \times V$

Theoretical Framework

The cross product \(\mathbf{U} \times \mathbf{V} \) is given by the determinant of a 3x3 matrix:

\[
\mathbf{U} \times \mathbf{V} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
U_x & U_y & U_z \\
V_x & V_y & V_z
\end{vmatrix}
\]

Expanding this determinant yields:

$$
\mathbf{U} \times \mathbf{V} = \mathbf{i}(U_y V_z - U_z V_y) - \mathbf{j}(U_x V_z - U_z V_x) + \mathbf{k}(U_x V_y - U_y V_x)
$$

Step-by-Step Calculation

Given:

$$
U_x = 2, \quad U_y = 2, \quad U_z = 2
$$

$$
V_x = 1, \quad V_y = 3, \quad V_z = 2
$$

Compute each component:

- i-component:

$$
U_y V_z - U_z V_y = (2)(2) - (2)(3) = 4 - 6 = -2
$$

- j-component:

$$
U_x V_z - U_z V_x = (2)(2) - (2)(1) = 4 - 2 = 2
$$

(Note the subtraction sign in the formula):

$$
- \mathbf{j}(U_x V_z - U_z V_x) = - (2) \mathbf{j} = -2 \mathbf{j}
$$

\]

- k-component:
\[
U_x V_y - U_y V_x = (2)(3) - (2)(1) = 6 - 2 = 4
\]

Final Cross Product U×V

\[
\boxed{
\mathbf{U} \times \mathbf{V} = -2 \mathbf{i} - 2
\mathbf{j} + 4 \mathbf{k}
}
\]

or in component form:

\[
\boxed{
\mathbf{U} \times \mathbf{V} = (-2, -2, 4)
}
\]

Calculating the Cross Product: V×U

Theoretical Perspective

Cross product is anti-commutative, meaning:

\[
\mathbf{V} \times \mathbf{U} = - (\mathbf{U} \times

\mathbf{V})
\]

However, for demonstration and confirmation, we'll compute V×U explicitly.

Step-by-Step Calculation

Using the same determinant approach:

\[
\mathbf{V} \times \mathbf{U} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
V_x & V_y & V_z \\
U_x & U_y & U_z
\end{vmatrix}
\]

Substituting the components:

\[
V_x=1,\quad V_y=3,\quad V_z=2
\]
\[
U_x=2,\quad U_y=2,\quad U_z=2
\]

Compute each component:

- i-component:
\[
V_y U_z - V_z U_y = (3)(2) - (2)(2) = 6 - 4 = 2
\]

- j-component:

$$
V_x U_z - V_z U_x = (1)(2) - (2)(2) = 2 - 4 = -2
$$

(remember the subtraction sign):

$$
- \mathbf{j} (V_x U_z - V_z U_x) = - (-2) \mathbf{j} = 2 \mathbf{j}
$$

- k-component:

$$
V_x U_y - V_y U_x = (1)(2) - (3)(2) = 2 - 6 = -4
$$

Final Cross Product $V \times U$

$$
\boxed{
\mathbf{V} \times \mathbf{U} = 2 \mathbf{i} + 2 \mathbf{j} - 4 \mathbf{k}
}
$$

or in component form:

$$
\boxed{
\mathbf{V} \times \mathbf{U} = (2, 2, -4)
}
$$

Confirmation of Anti-Commutativity

Compare with $\mathbf{U} \times \mathbf{V}$:

$$
\mathbf{U} \times \mathbf{V} = (-2, -2, 4)
$$
$$
\mathbf{V} \times \mathbf{U} = (2, 2, -4)
$$

Indeed, $\mathbf{V} \times \mathbf{U} = -(\mathbf{U} \times \mathbf{V})$, confirming the anti-commutative property.

Geometric Insights: Magnitudes and Directions

Magnitude of U×V

The magnitude of the cross product gives the area of the parallelogram spanned by U and V:

$$
|\mathbf{U} \times \mathbf{V}| = \sqrt{(-2)^2 + (-2)^2 + 4^2} = \sqrt{4 + 4 + 16} = \sqrt{24} \approx 4.89898
$$

This indicates the parallelogram's area is roughly 4.9 square units.

Direction of U×V and V×U

The direction of the cross product vector is perpendicular to both U and V, following the right-hand rule:

- For U×V, the vector points in the direction of your thumb when you curl your fingers from U to V.
- For V×U, it points in the opposite direction, confirming the anti-commutative nature.

Practical Applications and Significance

Understanding the cross product of vectors like U and V is essential in numerous real-world scenarios:

- Physics: Calculating torque ($\vec{\tau} = \mathbf{r} \times \mathbf{F}$), angular momentum, magnetic forces.
- Engineering: Analyzing forces in structures, rotational dynamics.
- Computer Graphics: Computing surface normals for shading and rendering.
- Navigation and Robotics: Determining orientation and rotational movement.

The explicit calculation of cross products provides critical insights into the geometric relationships between vectors, guiding design, analysis, and problem-solving across disciplines.

Summary of Key Results

Operation	Result (Component Form)	Magnitude	Direction
U×V	$(-2, -2, 4)$	≈ 4.90	Perpendicular to both U and V; follows right-hand rule.
V×U	$(2, 2, -4)$	Same magnitude as U×V	Opposite direction of U×V.

Final Thoughts: Mastering Cross Products

Calculating cross products accurately is crucial for advanced vector analysis. The process involves understanding the determinant method, recognizing properties like anti-commutativity, and interpreting the results geometrically. By practicing with specific examples such as U and V, students and professionals deepen their intuition about spatial relationships, leading to more effective problem-solving in scientific and engineering contexts.

Note: Always double-check your component calculations, especially signs, as errors can lead to incorrect interpretations of direction and magnitude.

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