Find The Derivative And Simplify. Y (9x + 6)(x - 4x). Step 1 We Want To Find Y' If Y= (9x + -Select-

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When working with derivatives in calculus, a common task is to find the derivative of a function that involves products of algebraic expressions. In this article, we will walk through the process of finding the derivative of the function $Y = (9x + 6)(x - 4x)$, simplifying the expression, and applying the rules of differentiation. Whether you're a student preparing for exams or someone brushing up on calculus concepts, understanding this step-by-step approach will help clarify how to handle similar problems efficiently.

Understanding the Function and Its Components

Before diving into differentiation, it’s essential to understand the structure of the given function and simplify it where possible. The function provided is:

$$ Y = (9x + 6)(x - 4x) $$

Notice that the second factor can be simplified:

$$ x - 4x = -3x $$

So, the function simplifies to:

$$ Y = (9x + 6)(-3x) $$

which can be rewritten as:

$$ Y = -3x(9x + 6) $$

This simplification makes it easier to differentiate the function since it reduces to a single product involving a polynomial and a linear term.

Step 1: Simplify the Function

Distribute or Expand the Expression

To make differentiation straightforward, expand the expression:

$$ Y = -3x(9x + 6) $$

Applying distributive property:

$$
Y = -3x \times 9x + -3x \times 6
$$

$$
Y = -27x^2 - 18x
$$

Now, the function is expressed as a polynomial:

$$ Y = -27x^2 - 18x $$

This form is easier to differentiate because it involves basic power rules.

Step 2: Find the Derivative Y'

Using the power rule for derivatives:

$$
\frac{d}{dx} [ax^n] = n \times a x^{n-1}
$$

we differentiate each term:

- For $-27x^2$:

$$
\frac{d}{dx} (-27x^2) = -27 \times 2 x^{2-1} = -54x
$$

- For $-18x$:

$$
\frac{d}{dx} (-18x) = -18 \times 1 x^{1-1} = -18
$$

Thus, the derivative of the entire function is:

$$
Y' = -54x - 18
$$

This is the simplified form of the derivative.

Step 3: Final Expression and Interpretation

The derivative Y' gives the rate of change of Y with respect to x. The expression:

$$
Y' = -54x - 18
$$

indicates that for each unit increase in x, the value of Y changes proportionally according to this linear function.

Additional notes:

- The negative coefficients suggest the function is decreasing as x increases.
- The derivative is linear, which is typical for derivatives of quadratic functions.

Alternative Approach: Using the Product Rule Directly

While simplifying the function first is often easier, understanding how to differentiate directly using the product rule is valuable, especially in more complex cases.

Applying the Product Rule

Recall the product rule:

$$
\frac{d}{dx} [u(x) v(x)] = u'(x) v(x) + u(x) v'(x)
$$

Let:

$$
u(x) = 9x + 6
$$
$$
v(x) = x - 4x = -3x
$$

Compute derivatives:

$$
u'(x) = 9
$$
$$
v'(x) = -3
$$

Applying the product rule:

$$
Y' = u'(x) v(x) + u(x) v'(x) = 9 \times (-3x) + (9x + 6) \times (-3)
$$

Simplify:

$$
Y' = -27x - 3(9x + 6) = -27x - 27x - 18 = -54x - 18
$$

which confirms our previous result.

Key Takeaways

- Always simplify algebraic expressions before differentiating when possible to streamline calculations.
- The power rule is fundamental for differentiating polynomial terms.
- The product rule is essential for differentiating products of functions, especially when they cannot be easily simplified.
- Understanding both approaches—simplification first or applying the product rule directly—enhances flexibility in solving calculus problems.

Practice Problems

To solidify your understanding, try differentiating these functions:

1. $Y = (5x + 2)(x^2 - 3)$
2. $Y = (2x^3 + x)(x^2 + 4x)$
3. $Y = (7x - 5)(x + 3)$

Remember to first simplify the expression where possible, then differentiate using the rules discussed.

Conclusion

Finding the derivative and simplifying a function like \(Y = (9x + 6)(x - 4x)\) involves understanding the structure of the expression, simplifying it to a polynomial, and then applying basic differentiation rules. Whether you choose to expand first or differentiate directly using the product rule, the key is to be systematic and clear in your approach. Mastering these techniques will make tackling more complex calculus problems more manageable and help you develop a strong foundation in differential calculus.

Frequently Asked Questions

How do I find the derivative of Y = (9x + 6)(x - 4x)?

First, simplify the expression inside the parentheses: (x - 4x) = -3x. So, Y = (9x + 6)(-3x). Then, apply the product rule or distribute and differentiate term-by-term.

What is the simplified form of Y before finding the derivative?

Simplify (9x + 6)(x - 4x) to (9x + 6)(-3x) which expands to -27x^2 - 18x.

What is the derivative of Y = -27x^2 - 18x?

Using power rule, Y' = -54x - 18.

Why do we need to simplify the expression before differentiating?

Simplifying makes differentiation easier and reduces the chance of errors, especially with complex expressions.

Can I apply the product rule directly to (9x + 6)(x - 4x)?

Yes, but since (x - 4x) simplifies to -3x, it's easier to work with the simplified form before differentiating.

What is the value of Y' when x = 2?

Substitute x=2 into Y' = -54x - 18, so Y' = -54(2) - 18 = -108 - 18 = -126.

How do I verify my derivative calculation?

You can verify by differentiating the original expression using the product rule or by differentiating the simplified form and confirming the same result.

Additional Resources

Find The Derivative And Simplify. Y (9x + 6)(x - 4x). Step 1 We Want To Find Y' If Y= (9x + -Select-, written in a journalistic or review-style tone. The content is over 1000 words and structured with detailed explanations each section. Begin with a paragraph that starts with the keyword in bold. Use

and

headings, with each heading explaining subtopics thoroughly. Include numbered or bulleted lists if relevant. Exclude

and full HTML wrappers like or .>

Introduction: The Significance of Derivatives in Mathematics

Find The Derivative And Simplify. Y (9x + 6)(x - 4x) is not merely an exercise in algebra; it embodies the fundamental concept of derivatives, a cornerstone of calculus that underpins much of modern science and engineering. Derivatives measure how a function changes at any given point, providing insights into slopes, rates of change, and the behavior of complex

systems. Whether you're analyzing the speed of a moving object or optimizing a business process, understanding how to find and simplify derivatives is essential.

This article aims to dissect this particular problem step-by-step, elucidating the process of derivative calculation, algebraic simplification, and interpretation. Beyond mere computation, we'll explore the underlying principles, common pitfalls, and strategies for tackling similar problems efficiently. Our goal is to offer a comprehensive, detailed guide that bridges conceptual understanding with practical application.

Understanding the Problem: Clarifying the Function

Deciphering the Given Expression

At the heart of the problem lies the function:

$$ Y = (9x + 6)(x - 4x) $$

Initially, this might seem straightforward, but a closer look reveals an important detail: the second factor, $x - 4x$, simplifies immediately, and understanding this simplification is crucial before moving forward with derivative calculations.

Let's analyze the components:

- The first factor: $9x + 6$, a linear expression in x.
- The second factor: $x - 4x$, which simplifies algebraically.

Simplifying the Expression

The key step in simplifying the function begins with the second factor:

$$ x - 4x = -3x $$

This simplification reduces the original function to:

$$ Y = (9x + 6)(-3x) $$

which can be rewritten as:

$$ Y = -3x(9x + 6) $$

Expanding this product yields a more manageable form for differentiation.

Step-by-Step Derivation and Simplification

Step 1: Expand the Expression

Applying the distributive property (also known as the FOIL method for binomials), we expand:

$$ Y = -3x \times 9x + (-3x) \times 6 $$

Calculating each term:

- $-3x \times 9x = -27x^2$
- $-3x \times 6 = -18x$

Thus, the expanded form of Y is:

$$ Y = -27x^2 - 18x $$

This quadratic expression is now ready for differentiation.

Step 2: Find the Derivative Y'

Differentiation involves applying the power rule, which states:

$$ \frac{d}{dx} [ax^n] = a n x^{n-1} $$

Applying the rule term-by-term:

- For $-27x^2$:

$$\frac{d}{dx} (-27x^2) = -27 \times 2 x^{2-1} = -54x$$

- For $-18x$:

$$\frac{d}{dx} (-18x) = -18 \times 1 x^{1-1} = -18 \times 1 = -18$$

Therefore, the derivative of the function Y is:

$$Y' = -54x - 18$$

Interpreting and Simplifying the Derivative

Final Simplified Derivative

The derivative, expressed in its simplest form, is:

$$ Y' = -54x - 18 $$

This linear function describes the rate of change of Y with respect to x. A negative coefficient indicates that Y decreases as x increases, which aligns with the original function's behavior—since the quadratic term is negative, the parabola opens downward.

Analyzing the Derivative's Behavior

To understand the significance of the derivative:

- Critical points occur where $Y' = 0$:

$$ -54x - 18 = 0 \Rightarrow -54x = 18 \Rightarrow x = -\frac{18}{54} = -\frac{1}{3} $$

- Interpretation: At $x = -\frac{1}{3}$, the function Y has a critical point, which, given the negative leading coefficient, is likely a maximum.

- Rate of change: For $x > -\frac{1}{3}$, Y' is negative, indicating Y decreases. For $x < -\frac{1}{3}$, Y' is positive, indicating Y increases.

This analysis provides critical insight into the function's behavior, useful in optimization problems.

Broader Context: The Role of Derivatives in Mathematical Analysis

Why Is Derivative Simplification Important?

Simplifying derivatives is not just a matter of algebraic neatness; it has practical implications:

- Efficiency: Simplified derivatives are easier to interpret and utilize in further calculations.
- Accuracy: Reducing complex expressions minimizes the risk of algebraic errors.
- Application: Simplified derivatives can be directly plugged into formulas for optimization, related rates, and graph analysis.

Common Challenges and How to Overcome Them

- Complex expressions: Break down functions into manageable parts and simplify before differentiation.
- Chain rule application: When functions are composed, identify inner and outer functions.
- Product and quotient rules: Remember the formulas:

- Product rule: $(uv)' = u'v + uv'$
- Quotient rule: $\left(\frac{u}{v}\right)' =$

\frac{u'v - uv'}{v^2} \)

In this problem, the product rule was effectively applied after recognizing the need for initial simplification.

Conclusion: Mastering Derivative Computation and Simplification

The process of finding and simplifying derivatives, as exemplified by the problem \(Y = (9x + 6)(x - 4x) \), underscores the importance of methodical analysis in calculus. Starting with algebraic simplification ensures clarity and reduces computational complexity. Expanding the expression allows straightforward application of differentiation rules, leading to an interpretable derivative that reveals the function's behavior.

Such exercises form the backbone of advanced mathematics, underpinning numerous applications from physics to economics. As learners and practitioners develop proficiency in these techniques, they enable more sophisticated analyses, optimization tasks, and problem-solving strategies.

In summary, mastering the steps—simplification, differentiation, and interpretation—not only enhances mathematical fluency but also empowers users to apply calculus principles effectively across diverse disciplines. Whether tackling academic problems or real-world challenges, these foundational skills are indispensable tools in the modern analytical toolkit.

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