Find The Derivative Of F(x, Y) = X + Xy + Y2 At The Point (1, 2) In The Direction Towards The Point

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Understanding how to compute derivatives of multivariable functions is fundamental in advanced calculus, especially when analyzing the behavior of functions in specific directions at given points. In this article, we will thoroughly explore the process of finding the derivative (specifically, the directional derivative) of the function $F(x, y) = x + xy + y^2$ at the point $(1, 2)$, in the direction towards a specific point. This involves understanding concepts such as partial derivatives, gradients, and directional derivatives, as well as applying them step-by-step to arrive at the solution.

Understanding the Function and the Problem

Defining the Function

The function provided is:

$$
F(x, y) = x + xy + y^2
$$

This is a scalar-valued function of two variables x and y. It combines linear, product, and quadratic terms.

The Point of Interest

The point at which we analyze the function is:

$$
(1, 2)
$$

This point is crucial because derivatives in multivariable calculus depend on the point of evaluation.

Direction Towards a Point

The problem states "In the direction towards the point." Usually, this implies we are given or choosing another point (x_0, y_0) towards which we want to find the directional derivative at $(1, 2)$.

Suppose the point towards which the direction is taken is (x_1, y_1). Without a specified point, a common approach is to consider the direction towards a specific point, say (x_1, y_1). For the purpose of this explanation, let's assume the target point is (x_1, y_1).

Example:
Let’s assume the point towards which the direction is given is $(x_1, y_1) = (3, 4)$.

The goal:
- Calculate the directional derivative of F at $(1, 2)$ in the direction of the vector pointing from $(1, 2)$ to $(3, 4)$.

Step-by-Step Solution Approach

1. Understanding the Concept of Directional Derivative

The directional derivative of a function $F(x, y)$ at a point (x_0, y_0) in the direction of a unit vector $\mathbf{u} = (u_x, u_y)$ is defined as:

$$
D_{\mathbf{u}}F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

where:
- $\nabla F(x_0, y_0)$ is the gradient vector of F at (x_0, y_0),
- $\mathbf{u}$ is a unit vector indicating the direction.

The gradient vector is composed of partial derivatives:

$$
\nabla F(x, y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right)
$$

---

2. Calculating the Gradient of $F(x, y)$

Given:

$$
F(x, y) = x + xy + y^2
$$

Compute the partial derivatives:

- Partial derivative with respect to x:

$$
\frac{\partial F}{\partial x} = 1 + y
$$

(since derivative of x w.r.t. x is 1, and derivative of xy w.r.t. x is y, derivative of y^2 w.r.t. x is 0).

- Partial derivative with respect to y:

$$
\frac{\partial F}{\partial y} = x + 2y
$$

---

3. Evaluating the Gradient at $(1, 2)$

Plugging in $x=1$, $y=2$:

$$
\frac{\partial F}{\partial x} (1, 2) = 1 + 2 = 3
$$

$$
\frac{\partial F}{\partial y} (1, 2) = 1 + 2 \times 2 = 1 + 4 = 5
$$

Thus, the gradient vector at $(1, 2)$:

$$
\nabla F(1, 2) = (3, 5)
$$

---

4. Determining the Direction Vector $\mathbf{u}$

Suppose the point towards which the direction is taken is $(x_1, y_1) = (3, 4)$.

The vector pointing from $(1, 2)$ to $(3, 4)$:

$$
\mathbf{v} = (3 - 1, 4 - 2) = (2, 2)
$$

To get the unit vector in this direction:

$$
\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{(2, 2)}{\sqrt{2^2 + 2^2}} = \frac{(2, 2)}{\sqrt{4 + 4}} = \frac{(2, 2)}{\sqrt{8}} = \frac{(2, 2)}{2 \sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

---

5. Computing the Directional Derivative

Using the formula:

$$
D_{\mathbf{u}}F(1, 2) = \nabla F(1, 2) \cdot \mathbf{u}
$$

Calculate:

$$
D_{\mathbf{u}}F(1, 2) = (3, 5) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = 3 \times \frac{1}{\sqrt{2}} + 5 \times \frac{1}{\sqrt{2}} = \frac{3 + 5}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 8 \times \frac{\sqrt{2}}{2} = 4 \sqrt{2}
$$

Result:

$$
\boxed{
D_{\mathbf{u}}F(1, 2) = 4 \sqrt{2}
}
$$

This value represents the rate of change of the function F at $(1, 2)$ in the direction towards $(3, 4)$.

Additional Considerations and Variations

Choosing Different Points

If the point towards which the direction is taken is different, simply repeat the process:

- Calculate the vector from $(1, 2)$ to the new point.
- Normalize it to get the unit vector.
- Dot with the gradient at $(1, 2)$.

Understanding the Geometric Interpretation

The directional derivative indicates how rapidly the function increases or decreases as we move from $(1, 2)$ in the specified direction. A positive value implies an increase, negative implies a decrease, and zero indicates no change in that direction at the point.

Connection to Total Differential

The directional derivative also relates to the total differential:

$$
dF = \frac{\partial F}{\partial x} dx + \frac{\partial F}{\partial y} dy
$$

which, in vector form, aligns with the gradient dot product with the displacement vector.

Summary and Final Notes

- To find the derivative of a multivariable function in a specific direction, compute the gradient at the point of interest.
- Normalize the direction vector to a unit vector.
- Compute the dot product between the gradient and the unit vector to get the directional derivative.
- This process provides insights into the function’s behavior in particular

directions, essential in optimization, differential geometry, and applied mathematics.

Conclusion

In conclusion, finding the derivative of $F(x, y) = x + xy + y^2$ at the point $(1, 2)$ in the direction towards another point involves calculating the gradient, identifying the direction vector, normalizing it, and then computing their dot product. This comprehensive process enables a precise understanding of how the function behaves locally in any specified direction, which is a cornerstone concept in multivariable calculus and mathematical analysis.

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Frequently Asked Questions

How do you find the derivative of the function F(x, y) = x + xy + y^2 at the point (1, 2)?

To find the derivative, compute the total differential or gradient at (1, 2), which involves calculating partial derivatives with respect to x and y, then evaluate at the given point.

What is the gradient vector of F(x, y) = x + xy + y^2 at the point (1, 2)?

The gradient vector is $(\partial F/\partial x, \partial F/\partial y)$. Calculating, $\partial F/\partial x = 1 + y$, and $\partial F/\partial y = x + 2y$. At (1, 2), this becomes $(1 + 2, 1 + 4) = (3, 5)$.

How do you determine the derivative of F in the direction towards a specific point?

You find the unit vector pointing from the current point to the target point, then take the dot product of the gradient vector at the current point with this unit vector. This gives the directional derivative in that direction.

Given the point (1, 2) and a target point (x_t, y_t), how do you compute the direction vector?

The direction vector is ($x_t - 1$, $y_t - 2$). To get the unit vector, divide this vector by its magnitude.

What is the formula for the directional derivative of F at a point in a given direction?

The directional derivative $D_vF = \nabla F \cdot \hat{v}$, where ∇F is the gradient at the point, and $\hat{v}$ is the unit vector in the desired direction.

How do you evaluate the derivative at (1, 2) when moving towards a specific point, say (x_t, y_t)?

First, find the gradient at (1, 2), then determine the unit vector towards (x_t, y_t), and compute the dot product of these vectors to get the derivative in that direction.

What are common applications of finding derivatives in specific directions for functions like F(x, y)?

This technique is used in optimization, gradient descent, and in analyzing how functions change in particular directions, which is essential in machine learning, physics, and engineering.

Can you explain how to find the rate of change of F(x, y) in the direction towards the point (x_t, y_t)?

Yes, by calculating the gradient at the current point and projecting it onto the unit vector pointing towards (x_t, y_t), you obtain the rate of change in that direction.

What steps are involved in calculating the derivative of $F(x, y) = x + xy + y^2$ at (1, 2) towards an arbitrary point (x_t, y_t)?

Step 1: Find the gradient at (1, 2). Step 2: Determine the vector from (1, 2) to (x_t, y_t). Step 3: Normalize this vector to get the unit direction vector. Step 4: Compute the dot product of the gradient with this unit vector to find the directional derivative.

Additional Resources

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Introduction

Mathematics, particularly calculus, plays a vital role in understanding the behavior of functions and their rates of change. The concept of derivatives extends beyond simple single-variable functions to multivariable functions, offering nuanced insights into how a function varies with respect to multiple inputs. In applied fields such as physics, engineering, and computer science, understanding directional derivatives—how a function changes as we move in a specific direction—is essential for optimization, modeling, and analysis.

In this comprehensive review, we explore the process of finding the derivative of the function $F(x, y) = x + xy + y^2$ at the point $(1, 2)$, specifically in the direction towards a given point. This involves calculating the directional derivative, which provides the rate of change of the function along a specified vector. The focus will be on meticulous derivation, conceptual understanding, and practical implications, making this article suitable for academic review or publication.

Fundamental Concepts

Multivariable Functions and Partial Derivatives

A function like $F(x, y) = x + xy + y^2$ maps points in a two-dimensional plane to real numbers. To analyze how F changes at a specific point, we compute its partial derivatives:

- $\frac{\partial F}{\partial x}$
- $\frac{\partial F}{\partial y}$

These derivatives measure the rate of change of F along the coordinate axes.

Gradient Vector

The gradient vector ∇F at any point (x, y) encapsulates the direction and rate of fastest increase of the function:

$$
\nabla F(x, y) = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right)
$$

Evaluating the gradient at a specific point provides a vector that points in the direction of greatest increase.

Directional Derivative

The directional derivative $D_{\mathbf{u}}F$ of F at a point (x_0, y_0) in the direction of a unit vector $\mathbf{u}$ is given by:

$$
D_{\mathbf{u}}F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

where $\cdot$ denotes the dot product. This measures how F changes as we move from (x_0, y_0) in the specified direction.

Step-by-Step Calculation

1. Define the Function and Point

Given:

$$
F(x, y) = x + xy + y^2
$$

At point:

$$
(x_0, y_0) = (1, 2)
$$

---

2. Compute Partial Derivatives

- Partial derivative with respect to x:

$$
\frac{\partial F}{\partial x} = 1 + y
$$

- Partial derivative with respect to y:

$$
\frac{\partial F}{\partial y} = x + 2y
$$

---

3. Evaluate the Gradient at $(1, 2)$

Plugging in the point:

$$
\nabla F(1, 2) = \left(1 + 2,\, 1 + 2 \times 2 \right) = (3, 5)
$$

This vector indicates the direction of steepest ascent and the magnitude of change at the point.

---

4. Determine the Direction Vector Towards a Point

Suppose the target point is (x_1, y_1). The direction vector $\mathbf{v}$ from $(1, 2)$ towards (x_1, y_1) is:

$$
\mathbf{v} = (x_1 - 1,\, y_1 - 2)
$$

To compute the directional derivative, we must normalize $\mathbf{v}$ to obtain a unit vector:

$$
\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{(x_1 - 1, y_1 - 2)}{\sqrt{(x_1 - 1)^2 + (y_1 - 2)^2}}
$$

Note: The specific point towards which the derivative is taken must be defined. For this example, let's assume the point towards which the function is evaluated is $(x_1, y_1) = (2, 3)$.

Practical Example: Direction Towards (2, 3)

1. Calculate the Direction Vector

$$
\mathbf{v} = (2 - 1, 3 - 2) = (1, 1)
$$

2. Normalize the Vector

$$
\|\mathbf{v}\| = \sqrt{1^2 + 1^2} = \sqrt{2}
$$

\[

$$
\mathbf{u} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)
$$

---

3. Compute the Directional Derivative

Using the gradient $\nabla F(1, 2) = (3, 5)$:

$$
D_{\mathbf{u}}F(1, 2) = \nabla F(1, 2) \cdot \mathbf{u} = (3, 5) \cdot \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) = \frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}} = \frac{8}{\sqrt{2}} = 4 \sqrt{2}
$$

This value, $4 \sqrt{2}$, indicates the rate of change of F at $(1, 2)$ in the direction towards $(2, 3)$.

Broader Applications and Significance

Optimization and Gradient Descent

The gradient vector is fundamental in optimization algorithms such as gradient descent, where it guides the search for minima or maxima of functions. In multivariable calculus, understanding the directional derivative informs how to move within the domain to optimize a given function.

Sensitivity Analysis

In engineering and physical sciences, the directional derivative assesses how sensitive a system or model is to changes in specific directions, aiding in robust design and analysis.

Geometric Interpretation

Visualizing the gradient as an arrow pointing towards the steepest ascent provides intuition about the local landscape of the function. Moving in the opposite direction indicates descent, which is critical in minimization problems.

Advanced Considerations

Directional Derivative in Arbitrary Directions

The general formula for the directional derivative in an arbitrary direction $\mathbf{u}$ (not necessarily normalized) is:

$$
D_{\mathbf{u}}F(x_0, y_0) = \nabla F(x_0, y_0) \cdot \mathbf{u}
$$

where $\mathbf{u}$ must be a unit vector for the derivative to represent the rate of change per unit movement.

Higher-Order Derivatives and Hessian Matrix

Beyond first derivatives, the Hessian matrix provides second-order information, essential for analyzing curvature and the nature of critical points.

Concluding Remarks

The process of finding the derivative of $F(x, y) = x + xy + y^2$ at a specific point in a particular direction exemplifies the power and elegance of multivariable calculus. By meticulously computing the gradient and normalizing the direction vector, we obtain the directional derivative, which quantifies how the function behaves as we move along a chosen path.

This technique extends beyond academic exercises, underpinning advanced fields like machine learning, physics, and engineering design. Understanding how functions change in various directions enables practitioners to optimize, control, and predict complex systems effectively.

In summary, the derivative of F at $(1, 2)$ in the direction towards $(2, 3)$ is $4 \sqrt{2}$, revealing a significant rate of increase along that path. Such insights are invaluable in both theoretical investigations and practical applications, illustrating the profound utility of calculus in multidimensional analysis.

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About the Author

This long-form article was prepared by a mathematics enthusiast dedicated to elucidating complex concepts in calculus and multivariable analysis. Its aim is to bridge theoretical understanding with practical application, fostering deeper appreciation for the elegance and utility of mathematical analysis.

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