Find The Equation For The Tangent Line To The Curve Y = F(x) At The Given X-value. F(x) = (2x + 5)^4

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Understanding how to find the equation of a tangent line to a curve at a specific point is a fundamental skill in calculus. The tangent line provides the best linear approximation of the function near that point, and its slope is given by the derivative of the function evaluated at the given x-value. In this article, we will explore the process step-by-step to determine the equation of the tangent line to the curve $y = F(x) = (2x + 5)^4$ at a specified x-value.

Understanding the Problem

What is a Tangent Line?

A tangent line to a curve at a particular point is a straight line that just "touches" the curve at that point, sharing the same slope as the curve at that point. It does not cross the curve at that point (except in special cases like points of inflection) but approximates the curve locally.

Why Find the Equation of a Tangent Line?

Knowing the tangent line's equation allows us to:
- Approximate the behavior of the function near a specific point.
- Analyze the slope and rate of change of the function.
- Apply this understanding to solve real-world problems involving optimization or modeling.

Step 1: Find the Derivative \(F'(x) \)

The first step in finding the tangent line at a specific x-value is to compute the derivative of the function \(F(x) \), which gives the slope of the tangent line at any point.

Given Function:

\[
F(x) = (2x + 5)^4
\]

Applying the Chain Rule

Since \(F(x) \) is a composite function, we apply the chain rule:

\[
\frac{d}{dx} [g(h(x))] = g'(h(x)) \cdot h'(x)
\]

Here, \(g(u) = u^4 \) and \(h(x) = 2x + 5 \).

- Derivative of \(g(u) = u^4 \):

\[
g'(u) = 4u^3
\]

- Derivative of \(h(x) = 2x + 5 \):

\[
h'(x) = 2
\]

Now, apply the chain rule:

\[
F'(x) = 4(2x + 5)^3 \times 2 = 8(2x + 5)^3
\]

Thus, the derivative of the function is:

$$
F'(x) = 8(2x + 5)^3
$$

Step 2: Evaluate the Derivative at the Given X-value

To find the slope of the tangent line at a specific $x = a$, substitute a into $F'(x)$.

Suppose the problem provides a specific x-value; for example, $x = a$. (If no specific value is given, we can keep the process general or choose a sample value for illustration.)

Example: Let's find the tangent line at $x = 0$.

- Compute $F'(0)$:

$$
F'(0) = 8(2 \times 0 + 5)^3 = 8(5)^3 = 8 \times 125 = 1000
$$

This means the slope of the tangent at $x=0$ is 1000.

Step 3: Find the Corresponding Point on the Curve

Next, find the point $(a, F(a))$ on the curve where the tangent line touches.

- For $x = 0$:

$$
F(0) = (2 \times 0 + 5)^4 = 5^4 = 625
$$

So, the point of tangency is $(0, 625)$.

Step 4: Write the Equation of the Tangent Line

The general form of the tangent line at $x = a$ is:

$$
y = F(a) + F'(a)(x - a)
$$

Using our example with $a = 0$:

$$
y = 625 + 1000 (x - 0) = 625 + 1000x
$$

This is the equation of the tangent line to the curve at $x=0$.

Generalizing for Any X-value

To find the tangent line at any arbitrary $x = a$, follow these steps:

1. Compute $F(a) = (2a + 5)^4$

2. Compute $F'(a) = 8(2a + 5)^3$

3. Plug into the tangent line formula:

   $$
   y = F(a) + F'(a)(x - a)
   $$

This formula allows you to find the tangent line at any specified x-value.

Example: Find the Tangent Line at $x = 1$

Let's illustrate the process with a different example:

1. Calculate $F(1)$:

$$
F(1) = (2 \times 1 + 5)^4 = (2 + 5)^4 = 7^4 = 2401
$$

2. Calculate $F'(1)$:

$$
F'(1) = 8(2 \times 1 + 5)^3 = 8(7)^3 = 8 \times 343 = 2744
$$

3. Equation of the tangent line:

$$
y = 2401 + 2744 (x - 1)
$$

Simplify:

$$
y = 2401 + 2744x - 2744
$$

$$
y = 2744x - 343
$$

Thus, the tangent line at $x=1$ is:

$$
\boxed{ y = 2744x - 343 }
$$

Conclusion

Finding the equation of the tangent line to a curve at a given x-value involves three main steps: differentiating the function to find the derivative, evaluating the derivative at the specific x-value to get the slope, and then applying the point-slope form of a line. For the function \(F(x) = (2x + 5)^4 \), the derivative \(F'(x) = 8(2x + 5)^3 \) captures the rate of change at any point.

By choosing a specific x-value, calculating the corresponding point on the curve, and evaluating the derivative to find the slope, you can confidently write the tangent line's equation. This process is fundamental in calculus and has applications in physics, engineering, and anywhere modeling of rates of change is essential.

Remember, always verify the point on the curve and the derivative's value before writing the tangent line's equation to ensure accuracy.

Frequently Asked Questions

How do I find the equation of the tangent line to the curve y = (2x + 5)^4 at a specific x-value?

To find the tangent line, first compute the derivative f'(x) to get the slope at the given x-value, then evaluate f(x) and f'(x) at that x, and finally use the point-slope form: y - f(a) = f'(a)(x - a).

What is the derivative of the function y = (2x + 5)^4?

Using the chain rule, the derivative is y' = 4(2x + 5)^3 2 = 8(2x + 5)^3.

How do I find the slope of the tangent line at x = 1 for y = (2x + 5)^4?

First, find f'(x) = 8(2x + 5)^3. Then, substitute x = 1: slope = 8(21 + 5)^3 = 8(7)^3 = 8 343 = 2744.

What point should I use to write the tangent line equation at x = 1?

Calculate f(1): f(1) = (21 + 5)^4 = 7^4 = 2401. So, the point is (1, 2401).

What is the equation of the tangent line to y = (2x + 5)^4 at x = 1?

Using point-slope form: y - 2401 = 2744(x - 1). Simplified, the tangent line is y = 2744x - 3443.

Additional Resources

Finding the Equation of the Tangent Line to the Curve $y = F(x)$ at a Given x-Value

When analyzing curves in calculus, one fundamental task is to find the tangent line at a specific point. This process involves understanding the function's behavior at that point, calculating its derivative, and then using that derivative to determine the slope of the tangent line. In this comprehensive guide, we will explore how to find the equation of the tangent line to the curve $y = F(x) = (2x + 5)^4$ at a specified x-value. We will delve into the concepts involved, step-by-step procedures, and provide detailed examples to clarify each stage.

Understanding the Problem

Before diving into calculations, it's essential to understand what it means to find a tangent line and why it's important.

What Is a Tangent Line?

A tangent line to a curve at a point is a straight line that just "touches" the curve at that point. It has two key properties:

- Passes through the point of tangency: The line intersects the curve at exactly one point in the immediate vicinity.
- Shares the same slope as the curve at that point: The slope of the tangent line is equal to the derivative of the function evaluated at that point.

Why Find the Equation of a Tangent Line?

The tangent line provides a linear approximation of the function near the point of tangency. It's particularly useful in:

- Estimating the function's value for nearby x-values.
- Analyzing the function's behavior (increasing/decreasing, concavity).
- Solving problems involving instantaneous rates of change.

Analyzing the Given Function $F(x) = (2x + 5)^4$

Our focus function is:

$$
F(x) = (2x + 5)^4
$$

This is a composite function involving a linear inner function $u = 2x + 5$, raised to the fourth power.

Key Characteristics

- Type of function: Polynomial (specifically, a quartic function composed via a chain rule).
- Domain: All real numbers, since polynomials are defined everywhere.
- Range: $y \geq 0$, as the quartic of any real number is non-negative.

Step-by-Step Approach to Find the Tangent Line

The process involves several stages:

1. Identify the point of tangency: Determine the y-coordinate corresponding to the given x-value.
2. Compute the derivative $F'(x)$: Find the slope of the tangent line at the point.
3. Evaluate the derivative at the given x-value: Obtain the slope m.
4. Write the equation of the tangent line: Use the point-slope form with the point and slope.

Let’s explore each step in detail.

---

1. Choosing the x-Value: The Point of Tangency

Suppose you are asked to find the tangent line at a specific x-value, say $x = a$. The first step is to find the corresponding point $P(a, F(a))$:

$$
P(a, F(a)) = \left(a, (2a + 5)^4 \right)
$$

This point lies on the curve, and the tangent line will pass through it.

---

2. Computing the Derivative $F'(x)$

The derivative tells us the slope of the tangent line at any point x. Since $F(x) = (2x + 5)^4$, we use the chain rule:

$$
F'(x) = \frac{d}{dx} \left[(2x + 5)^4 \right]
$$

Applying the chain rule:

$$
F'(x) = 4 \cdot (2x + 5)^3 \cdot \frac{d}{dx}(2x + 5)
$$

Since $\frac{d}{dx}(2x + 5) = 2$:

$$
F'(x) = 4 \cdot (2x + 5)^3 \times 2 = 8 \cdot (2x + 5)^3
$$

Summary of derivative:

$$
\boxed{
F'(x) = 8 (2x + 5)^3
}
$$

This derivative formula provides the slope of the tangent line at any x.

---

3. Evaluating the Derivative at the Given x-Value

Suppose the specific x-value is a. Then, the slope at that point is:

$$
m = F'(a) = 8 \cdot (2a + 5)^3
$$

The actual value depends on the a you are given, but the process remains consistent for any a.

---

4. Writing the Equation of the Tangent Line

Using the point-slope form:

$$
y - y_0 = m (x - x_0)
$$

where $(x_0, y_0) = (a, F(a))$ and $m = F'(a)$.

Substituting:

$$
y - (2a + 5)^4 = 8 (2a + 5)^3 (x - a)
$$

This is the equation of the tangent line to $y = (2x + 5)^4$ at $x = a$.

Worked Example: Find the Tangent Line at $x = 0$

Let's apply the procedure for a specific x-value, $a = 0$.

Step 1: Find the point of tangency

$$
F(0) = (2 \times 0 + 5)^4 = 5^4 = 625
$$

Point:

$$
P(0, 625)
$$

Step 2: Compute the slope

$$
F'(0) = 8 \cdot (2 \times 0 + 5)^3 = 8 \times 5^3 = 8 \times 125 = 1000
$$

Step 3: Write the tangent line

Using point-slope form:

$$
y - 625 = 1000 (x - 0)
$$

Simplify:

$$
\boxed{
y = 1000 x + 625
}
$$

This is the equation of the tangent line to the curve at $x = 0$.

Key Insights and Additional Considerations

Generalizing for Any x-Value

The process outlined is applicable for any x-value:

- Find $F(a) = (2a + 5)^4$.
- Compute $F'(a) = 8 (2a + 5)^3$.
- Write the tangent line as:

\[

$$
y = F(a) + F'(a) (x - a)
$$

which simplifies to the point-slope form.

Visualizing the Tangent Line

Graphically, the tangent line will touch the curve exactly at the point $(a, F(a))$, sharing the same slope. For the function $(2x + 5)^4$, which exhibits a quartic growth, the tangent line can vary significantly depending on the x-value.

Special Cases

- When $2a + 5 = 0 \Rightarrow a = -\frac{5}{2}$, the derivative becomes zero:

$$
F'(-\frac{5}{2}) = 8 \times 0^3 = 0
$$

The tangent line in this case is horizontal:

$$
y = F\left(-\frac{5}{2}\right) = (2 \times -\frac{5}{2} + 5)^4 = 0^4 = 0
$$

The tangent line is:

$$
y = 0
$$

which is a horizontal line tangent at the minimum point of the curve.

Summary and Final Remarks

Finding the equation of the tangent line to a curve at a specific x-value involves a systematic process rooted in differential calculus:

- Identify the point: Calculate $y = F(x)$ at the given x.

- Differentiate: Find \(F'(x) \) using appropriate rules (chain rule here).
- Evaluate: Determine the slope \(m = F'(a) \) at the specific point.
- Formulate: Use the point-slope form to write the line’s equation.

Applying these steps to \(F(x) = (2x + 5)^4 \), we derive a general formula for the tangent line at any \(x = a \):

\[
\boxed{
\text{Tangent line at } x=a: \quad y = (2a + 5)^4 + 8 (2a + 5)^3 (x - a)
}
\]

This method is versatile and can be adapted to various functions, highlighting the power of derivatives in geometric interpretations of functions.

Remember: Understanding the concepts underlying the tangent line

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