Find The Equation Of A Line That Goes Through The Point (1,-1) And Is Parallel To The Line Y = -6x +

Find The Equation Of A Line That Goes Through The Point (1,-1) And Is Parallel To The Line Y = -6x +

When working with linear equations in coordinate geometry, a common problem involves finding the equation of a line that passes through a specific point and is parallel to a given line. In this case, our goal is to determine the equation of a line passing through the point (1, -1) and parallel to the line y = -6x +. Understanding how to approach this problem requires knowledge of the slope-intercept form of a line, the concept of parallel lines, and how to apply the point-slope formula. This article provides a comprehensive guide to solving such problems, with step-by-step instructions, key concepts, and practical examples to help you master the skill.

Understanding the Basics: Lines and Their Equations

The Slope-Intercept Form of a Line

The most common way to represent a straight line in coordinate geometry is the slope-intercept form:

\[y = mx + b \]

where:
- m is the slope of the line
- b is the y-intercept (the point where the line crosses the y-axis)

For example, in the given line y = -6x +, the slope m is -6.

What Does It Mean for Lines to Be Parallel?

Two lines are parallel if they have exactly the same slope but different y-intercepts. Parallel lines:

- Never intersect
- Have equal slopes

In our case, the line y = -6x + is parallel to the line we're trying to find, so both will share the same slope of -6.

Step-by-Step Guide to Find the Equation of the Parallel Line

Step 1: Identify the Slope of the Given Line

Given the line y = -6x +, the slope is -6. Since the line we are asked to find is parallel to this line, it must have the same slope:

$$ m = -6 $$

Step 2: Use the Point-Slope Form of a Line

The point-slope form is useful when you know a point on the line and the slope:

$$ y - y_1 = m(x - x_1) $$

where:

- (x_1, y_1) is a point on the line
- m is the slope

Plugging in the point (1, -1) and the slope $m = -6$:

$$ y - (-1) = -6(x - 1) $$

which simplifies to:

$$ y + 1 = -6(x - 1) $$

Step 3: Simplify to Slope-Intercept Form

Distribute the slope:

$$ y + 1 = -6x + 6 $$

Subtract 1 from both sides:

$$ y = -6x + 6 - 1 $$

$$ y = -6x + 5 $$

This is the equation of the line passing through (1, -1) and parallel to y = -6x +.

Final Equation of the Line

The line passing through the point (1, -1) and parallel to y = -6x + is:

$$\boxed{ y = -6x + 5 }$$

Additional Insights and Variations

What If the Original Line Is Not in Slope-Intercept Form?

Sometimes, lines are given in different formats, such as standard form $Ax + By = C$. To find the parallel line:

- Convert the line to slope-intercept form to identify the slope
- Use the point-slope formula with the known point and the slope

How to Verify the Parallel Line

To check whether your solution is correct:

1. Confirm the slope matches the original line's slope: both should be -6.
2. Check that the point (1, -1) satisfies the new line's equation:

$$y = -6x + 5$$

Plugging in:

$$ -1 = -6(1) + 5 $$

$$ -1 = -6 + 5 $$

$$ -1 = -1 $$

Since the equation holds, the point lies on the line, confirming correctness.

Key Points to Remember

- Parallel lines have identical slopes.
- To find the equation of a line through a point and parallel to another line, identify the slope of the given line.
- Use the point-slope form to set up the equation, then simplify to slope-intercept or standard form as needed.
- Always verify your solution by plugging the point into the final equation.

Practical Examples and Practice Problems

Example 1: Find the equation of a line passing through (2, 3) and parallel to y = 4x - 1

Solution:

- Slope of given line: $m = 4$

- Use point-slope form:

$$ y - 3 = 4(x - 2) $$

$$ y - 3 = 4x - 8 $$

$$ y = 4x - 8 + 3 $$

$$ y = 4x - 5 $$

Answer: $y = 4x - 5$

Practice Problems for You

1. Find the equation of the line passing through (-2, 4) and parallel to $y = 3x + 7$.
2. Determine the line passing through (0, 0) and parallel to $y = -2x + 5$.
3. Find a line passing through (5, -3) and parallel to $y = (1/2)x - 4$.

Conclusion

Understanding how to find the equation of a line passing through a specific point and parallel to a given line is a fundamental skill in analytic geometry. The key steps involve identifying the slope of the original line, leveraging the point-slope formula, and simplifying the equation into the desired form. Remember, the defining characteristic of parallel lines is their identical slopes, which makes this process straightforward once you grasp the core concepts. With practice, you'll be able to solve such problems quickly and accurately, enhancing your overall mastery of linear equations and their properties.

Summary of the Process

- Identify the slope of the given line.
- Use the same slope for the new line since lines are parallel.
- Apply the point-slope formula with the given point.
- Simplify to the slope-intercept form or any preferred form.
- Verify your answer by substitution.

Mastering this method will greatly improve your problem-solving skills in geometry and algebra, and it will serve as a foundation for more advanced topics such as systems of equations, transformations, and calculus applications.

Keywords for SEO Optimization:
- Find the equation of a line
- Parallel line equation
- Line passing through a point
- Slope-intercept form
- Point-slope formula
- Linear equations practice
- Geometry problems involving parallel lines
- How to find line equations given a point and a slope

Frequently Asked Questions

What is the general form of the equation of a line parallel to $y = -6x +$

c that passes through the point (1, -1)?

Since parallel lines have the same slope, the equation will be y = -6x + k. Plugging in the point (1, -1), we get -1 = -6(1) + k, so k = 5. The equation is y = -6x + 5.

How do you find the equation of a line parallel to y = -6x + 3 passing through (1, -1)?

The slope is -6, same as the given line. Using point-slope form: y - (-1) = -6(x - 1). Simplify to get y + 1 = -6x + 6, so y = -6x + 5.

What is the significance of the slope in determining parallel lines?

Parallel lines have equal slopes. Therefore, any line parallel to y = -6x + c will also have a slope of -6.

Can the equation of a line passing through (1, -1) and parallel to y = -6x + 2 be written in slope-intercept form?

Yes. The slope is -6, and substituting the point (1, -1): y = -6x + k. Plugging in gives -1 = -6(1) + k, so k = 5. The equation is y = -6x + 5.

If the original line is y = -6x + 4, what is the equation of the parallel line passing through (1, -1)?

Same slope: -6. Using point-slope form: y + 1 = -6(x - 1), which simplifies to y = -6x + 5.

How do you verify that two lines are parallel?

They are parallel if their slopes are equal. In this case, both lines have slope -6, confirming they are parallel.

What is the step-by-step process to find the equation of a line passing through a point and parallel to a given line?

1. Identify the slope of the given line. 2. Use the point to set up the point-slope form: y - y□ = m(x - x□). 3. Simplify to find the slope-intercept form if needed.

Can the equation of the line passing through (1, -1) and parallel to y = -6x + c have different intercepts?

Yes. The intercept depends on the specific value of c in the original line. The new line will have the same slope but a different y-intercept, calculated based on the point it passes through.

What is the final equation of the line passing through (1, -1) and parallel to y = -6x + 7?

Using point-slope form: $y + 1 = -6(x - 1)$. Simplify to get $y = -6x + 5$.

Why do lines parallel to y = -6x + c always have the same slope?

Because parallel lines by definition have identical slopes, ensuring they never intersect and are always equidistant in their orientation.

Additional Resources

Find The Equation Of A Line That Goes Through The Point (1, -1) And Is Parallel To The Line Y = -6x + – a problem that exemplifies core concepts in coordinate geometry and algebra. This task requires understanding the properties of parallel lines, the form of linear equations, and how to manipulate these equations to find a specific line's equation given a point and a directional constraint. In this comprehensive article, we will explore each component of this problem systematically, providing clarity and insight into the underlying mathematical principles.

Understanding the Core Concepts: Lines, Slopes, and Parallelism

The Equation of a Line in Slope-Intercept Form

In coordinate geometry, a line in the Cartesian plane can be represented in various forms. The most common is the slope-intercept form:

$$ y = mx + b $$

where:
- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

For example, the given line:

$$ y = -6x + c $$

has a slope $m = -6$ and a y-intercept c. The exact value of c isn't specified, but the slope is.

What Does It Mean For Lines to Be Parallel?

Two lines are parallel if they lie in the same plane and do not intersect, regardless of how far extended they are. Algebraically, this translates into having identical slopes:

$$m_1 = m_2$$

Thus, any line parallel to $y = -6x + c$ must also have a slope of -6.

Understanding this property simplifies the problem significantly. Once the slope of the desired line is known, the primary task reduces to finding the specific line passing through a given point with that slope.

Step-by-Step Approach to Finding the Equation of the Line

This section guides you through the systematic process of deriving the line's equation passing through the point $(1, -1)$ and parallel to the given line.

Step 1: Identify the Slope of the Given Line

Given the line:

$$y = -6x + \text{(some constant)}$$

the slope m is directly read from the coefficient of x:

$$m = -6$$

This slope will be used for the new line because parallel lines share the same slope.

Step 2: Use the Point-Slope Form of a Line Equation

The point-slope form is particularly useful when the point (x_1, y_1) and the slope m are known:

$$y - y_1 = m(x - x_1)$$

Given:
- $(x_1, y_1) = (1, -1)$
- $m = -6$

Substituting these into the formula:

$$y - (-1) = -6(x - 1)$$

which simplifies to:

$$y + 1 = -6(x - 1)$$

This form directly incorporates the given point and the slope.

Step 3: Simplify to Slope-Intercept Form

Expanding and simplifying:

$$
\begin{aligned}
y + 1 &= -6x + 6 \\
y &= -6x + 6 - 1 \\

y &= -6x + 5

\end{aligned}

\]

This is the equation of the line passing through \((1, -1) \) and parallel to \(y = -6x + c \).

Interpreting the Result: The Equation of the Parallel Line

The final derived equation:

\[y = -6x + 5 \]

has several important features worth analyzing:

- Slope: The slope remains \(-6 \), confirming parallelism.
- Y-Intercept: The line crosses the y-axis at \((0, 5) \), which is different from the original line's intercept \(c \).

This line passes through the point \((1, -1) \) and is parallel to the given line, satisfying all the problem's constraints.

Additional Considerations and Variations

While the above process addresses the specific problem, several related scenarios and considerations are worth exploring:

1. Multiple Lines Parallel to the Given Line

The derivation above yields a unique line passing through a specific point. However, if the problem asked for all lines passing through $(1, -1)$ and parallel to $y = -6x + c$, the answer would be a family of lines:

$$y = -6x + k$$

where k varies over all real numbers, but only the one through $(1, -1)$ with the determined intercept 5 satisfies the point condition.

2. The Role of the Constant Term c in the Original Equation

In the original line $y = -6x +$, the constant term c is missing, which suggests an incomplete expression. Typically, the line is written as:

$$y = -6x + c$$

or explicitly with a specific constant. The problem focuses on parallelism, which is independent of the intercept, confirming that any line with slope -6 will be parallel regardless of c.

3. Extending to Different Points and Slopes

The methodology used here is general. Given any point (x_1, y_1) and a slope m, the line's equation can be derived via the point-slope form. Alternatively, if the slope were different, the process would involve substituting the new slope and point into the same formula.

Significance of the Problem in Mathematical and Real-World Contexts

Understanding how to find lines parallel to a given line through a specific point has broad applications:

- Engineering: Designing parallel components or pathways.
- Navigation: Plotting routes that maintain a consistent bearing.
- Computer Graphics: Rendering parallel lines and shapes.

Mathematically, the problem exemplifies the importance of the slope-intercept and point-slope forms, as well as the geometric interpretation of parallelism.

Conclusion

In conclusion, the task of finding the equation of a line passing through $(1, -1)$ and parallel to $y = -6x + c$ exemplifies fundamental principles of algebra and geometry. By recognizing that parallel lines share the same slope, applying the point-slope form, and simplifying to slope-intercept form, we derive:

$$\boxed{ y = -6x + 5 }$$

This process highlights the elegance and consistency of mathematical reasoning, demonstrating how simple properties like slope and point coordinates lead to precise and meaningful solutions. Whether for academic pursuits or practical applications, mastering these techniques is essential for navigating the broader landscape of linear relationships and geometric principles.

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