Find The Equation Of The Line Which Passes Through (5, 2) And The Point Of Intersection Of The Lines

Find The Equation Of The Line Which Passes Through (5, 2) And The Point Of Intersection Of The Lines is a common problem in coordinate geometry that combines understanding of line equations, intersection points, and algebraic methods to derive the required line's equation. Whether you're preparing for exams or enhancing your mathematical skills, mastering this topic is essential, as it builds a solid foundation for more advanced concepts in geometry and algebra. In this article, we will explore the step-by-step process to find the equation of such a line, understand the underlying principles, and see practical examples to solidify your understanding.

Understanding the Problem

Before diving into calculations, it’s crucial to grasp what the problem entails. You are given:

- A specific point through which the line passes: (5, 2)
- The intersection point of two lines, which is typically defined as the point where these two lines cross.

The goal is to find the equation of the line that passes through the given point and the intersection point of the two lines.

Key concepts involved:

- Equation of a line in various forms
- Point-slope and slope-intercept forms
- Finding the intersection point of two lines
- Using algebraic methods to determine the equation of the required line

Step-by-Step Approach to Find the Equation of the Line

To systematically find the equation, consider the following steps:

1. Understand the Given Data and What is Needed

- Confirm the coordinates of the given point: (5, 2)
- Obtain or be provided with the equations of the two lines whose intersection point is relevant
- Recognize that the line passes through (5, 2) and the intersection point of the two lines

2. Find the Intersection Point of the Two Lines

The first key step is to determine the intersection point, which is common to both lines. Suppose the two lines are given by their equations:

- Line 1: $L_1: y = m_1 x + c_1$
- Line 2: $L_2: y = m_2 x + c_2$

Steps to find the intersection point:

- Set the right-hand sides equal: $m_1 x + c_1 = m_2 x + c_2$
- Solve for x: $x = \frac{c_2 - c_1}{m_1 - m_2}$
- Substitute x back into either line's equation to find y

Example:

Suppose

- $L_1: y = 2x + 1$
- $L_2: y = -x + 4$

Set equal:

$2x + 1 = -x + 4$

Solve:

$2x + x = 4 - 1$

$3x = 3$

$x = 1$

Plug into L_1:

$y = 2(1) + 1 = 3$

Intersection point: $(1, 3)$

Note: If the equations are given in different forms, convert them to slope-intercept form or standard form first.

3. Verify the Point (5, 2) and the Intersection Point

- Confirm that the point (5, 2) passes through the line you are constructing.
- Ensure the intersection point is correctly identified and used in subsequent calculations.

4. Find the Equation of the Line Passing Through the Two Points

Once you have the two points:

- The point (5, 2)
- The intersection point (say, (x_i, y_i))

Use the two-point form of the line equation:

$$
\text{Equation:} \quad y - y_1 = m (x - x_1)
$$

where

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Example:

Using the points $(5, 2)$ and $(1, 3)$:

Calculate slope:

$$
m = \frac{3 - 2}{1 - 5} = \frac{1}{-4} = -\frac{1}{4}
$$

Equation:

$$
y - 2 = -\frac{1}{4}(x - 5)
$$

Simplify:

$$
y - 2 = -\frac{1}{4}x + \frac{5}{4}
$$

$$
y = -\frac{1}{4}x + \frac{5}{4} + 2
$$

$$
y = -\frac{1}{4}x + \frac{5}{4} + \frac{8}{4} = -\frac{1}{4}x + \frac{13}{4}
$$

Thus, the equation of the line is:

$$
y = -\frac{1}{4}x + \frac{13}{4}
$$

Alternatively, in standard form:

$$
4y + x = 13
$$

Special Cases and Additional Considerations

1. When the Lines are Parallel or Coincident

- If the two lines are parallel, they do not intersect, so the intersection point does not exist unless they are coincident.
- If they are coincident, the intersection point is every point on the line, so the problem simplifies.

2. When the Equations of the Lines Are Given in Different Forms

- Converting all equations to slope-intercept form ($y = mx + c$) simplifies calculations.
- For lines in standard form ($Ax + By + C = 0$), solving for intersection involves substitution or elimination methods.

3. Handling Vertical and Horizontal Lines

- Vertical lines ($x = a$) have undefined slopes.
- When dealing with vertical lines, the process involves different steps, such as directly substituting the known points.

Practical Example: Complete Solution for a Specific Case

Suppose you are given:

- Line 1: $2x + y = 7$
- Line 2: $x - y = 1$

and you need to find the equation of the line passing through (5, 2) and the intersection point of these two lines.

Step 1: Find the intersection point:

Rewrite lines in slope-intercept form:

- $L_1: y = -2x + 7$
- $L_2: y = x - 1$

Set equal:

$-2x + 7 = x - 1$

Solve:

$-2x - x = -1 - 7$

$-3x = -8$

$$
x = \frac{8}{3}
$$

Find y:

$$
y = x - 1 = \frac{8}{3} - 1 = \frac{8}{3} - \frac{3}{3} = \frac{5}{3}
$$

Intersection point: $\left(\frac{8}{3}, \frac{5}{3}\right)$

Step 2: Find the equation of the line passing through (5, 2) and $\left(\frac{8}{3}, \frac{5}{3}\right)$:

Calculate slope:

$$
m = \frac{\frac{5}{3} - 2}{\frac{8}{3} - 5} = \frac{\frac{5}{3} - \frac{6}{3}}{\frac{8}{3} - \frac{15}{3}} = \frac{-\frac{1}{3}}{-\frac{7}{3}} = \frac{-1/3}{-7/3} = \frac{-1/3} \times \frac{3}{-7} = \frac{-1/3 \times 3}{-7} = \frac{-1}{-7} = \frac{1}{7}
$$

Equation using point-slope form:

$$
y - 2 = \frac{1}{7}(x - 5)
$$

Simplify:

$$
y - 2 = \frac{1}{7}x - \frac{5}{7}
$$

$$
y = \frac{1}{7}x - \frac{5}{7} + 2
$$

$$
y = \frac{1}{7}x - \frac{5}{7} + \frac{14}{7} = \frac{1}{7}x + \frac{9}{7}
$$

Final Equation:

$$
y = \frac{1}{7}x + \frac{9}{7}
$$

or in standard form:

$$
7y = x + 9
$$

which simplifies to:

$$
x - 7y = -9
$$

Conclusion

Finding the equation of a line passing through a specific point and the intersection of two other lines involves a combination of algebraic techniques and geometric understanding. The key steps are:

- Determining the intersection point of the two given lines
- Using the two points (the given point and the intersection point) to

Frequently Asked Questions

How do I find the equation of a line passing through (5, 2) and the intersection of two given lines?

First, find the point of intersection of the two lines by solving their equations simultaneously. Then, use the point-slope form with the point (5, 2) and the intersection point to determine the line's equation.

What is the step-by-step process to determine the equation of a line passing through a specific point and the intersection of two lines?

Step 1: Find the intersection point of the two given lines. Step 2: Use the point-slope form $y - y_1 = m(x - x_1)$, where (x_1, y_1) is (5, 2). Step 3: Calculate the slope m using the intersection point and (5, 2), or use the slopes of the lines if applicable. Step 4: Write the equation in slope-intercept or standard form.

Can I find the equation of a line passing through (5, 2) and the intersection point without explicitly solving for the intersection?

No, you need the intersection point's coordinates to determine the exact line equation. If the intersection point is unknown, you'll first need to find it by solving the two given lines simultaneously.

What are common methods to find the intersection point of two lines when finding the equation of a new line?

Common methods include solving the two equations simultaneously (substitution or elimination), graphing to approximate the intersection, or using algebraic techniques like setting the equations equal to each other and solving for the variables.

If the two lines are parallel or coincident, how does that affect finding the line passing through (5, 2) and their intersection?

If the lines are parallel or coincident, their intersection point may not be unique (for coincident lines, infinitely many points). In such cases, you need to determine the specific intersection point or recognize that the line passing through (5, 2) and the intersection is either undefined or coincides with one of the lines, affecting the solution process.

Additional Resources

Find The Equation Of The Line Which Passes Through (5, 2) And The Point Of Intersection Of The Lines

Understanding how to determine the equation of a line passing through a specific point and intersecting with other lines is a fundamental concept in coordinate geometry. This problem combines several key ideas: point-slope form, the intersection of lines, and the derivation of the required line's equation. In this detailed review, we will explore each aspect thoroughly, providing clarity, step-by-step procedures, and illustrative examples to ensure comprehensive understanding.

Introduction to the Problem

The problem asks us to find the equation of a line that satisfies two conditions:

1. It passes through a specific point: (5, 2).
2. It intersects with the point of intersection of two (or more) given lines.

This involves understanding:
- How to find the intersection point of two lines.
- How to formulate the equation of a line passing through a given point.
- How to determine the specific line that intersects at that point and passes through (5, 2).

Before delving into the solution process, it’s crucial to understand each component involved in this problem.

Understanding the Basic Concepts

1. Equation of a Line

The general forms include:
- Slope-Intercept Form: $y = mx + c$, where m is the slope, and c is the y-intercept.
- Point-Slope Form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is a known point on the line.
- Two-Point Form: When two points (x_1, y_1) and (x_2, y_2) are known, the slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$, and the line can be written accordingly.

2. Intersection of Lines

The intersection point of two lines is the point that satisfies both line equations simultaneously. To find this:
- Write the equations of the two lines.
- Solve these equations simultaneously (substitution or elimination) to find the intersection coordinates (x, y).

3. Passing Through a Point

Any line passing through (x_0, y_0) can be described using the point-slope form:
$$y - y_0 = m(x - x_0)$$
where m is the slope of the line, which can be varied to find lines with different slopes passing through the point.

Step-by-Step Approach to the Problem

The overall approach involves these main steps:
1. Identify the given lines and find their intersection point.
2. Determine the point of intersection (x_{int}, y_{int}).
3. Formulate the equation of the required line passing through $(5, 2)$ and (x_{int}, y_{int}).

Let’s examine each step in detail.

Step 1: Find the Intersection Point of the Given Lines

Suppose the problem provides two lines:
- Line 1: $L_1: a_1x + b_1y + c_1 = 0$
- Line 2: $L_2: a_2x + b_2y + c_2 = 0$

Procedure:
- Convert the lines into slope-intercept form if necessary.
- Solve the simultaneous equations to find (x_{int}, y_{int}).

Example:
If the lines are:
- $L_1: 2x + y - 4 = 0$
- $L_2: x - y + 1 = 0$

Solve:
$$
\begin{cases}
2x + y = 4 \quad (1) \\
x - y = -1 \quad (2)
\end{cases}
$$

Adding (1) and (2):
$$
(2x + y) + (x - y) = 4 + (-1) \Rightarrow 3x = 3 \Rightarrow x=1
$$
Substitute $x=1$ into (2):
$$
1 - y = -1 \Rightarrow y=2
$$

Intersection point: $(1, 2)$.

Step 2: Confirm the Coordinates of the Intersection Point

Once the intersection point (x_{int}, y_{int}) is obtained, verify it by substituting into both line equations to ensure consistency.

In the above example:
- $L_1: 2(1) + 2 - 4 = 0 \Rightarrow 2 + 2 - 4 = 0$ ✓
- $L_2: 1 - 2 + 1 = 0 \Rightarrow 0 = 0$ ✓

The point is valid.

Step 3: Find the Equation of the Required Line

Now, with the intersection point (x_{int}, y_{int}) and the point $(5, 2)$, the task is to find the equation of the line passing through both points.

Method:
- Compute the slope m:
$$
m = \frac{y_{int} - 2}{x_{int} - 5}
$$
- Use the point-slope form to write the equation:
$$
y - 2 = m(x - 5)
$$

Special Cases and Additional Considerations

1. Parallel Lines

If the given lines L_1 and L_2 are parallel, they do not intersect, and the intersection point is undefined. The problem then shifts to other considerations, such as lines passing through a point and being parallel or perpendicular to existing lines.

2. Coincident Lines

If the lines are coincident, their intersection is the entire line. The intersection point can be any point on the line, and the problem simplifies to finding the line passing through $(5, 2)$ and any point on the line.

3. Lines with Undefined Slope (Vertical Lines)

If one of the lines is vertical, e.g., $x = k$, the intersection point has $x = k$. The slope of the line passing through $(5, 2)$ and this point will be undefined or special, and the equation will reflect this.

Worked Examples for Clarity

Example 1:
Find the equation of the line passing through $(5, 2)$ and the intersection point of the lines:

- $L_1: y = 3x + 1$
- $L_2: y = -2x + 7$

Solution:

- Find intersection:

$$
3x + 1 = -2x + 7
$$

$$
3x + 2x = 7 - 1
$$

$$
5x = 6 \Rightarrow x = \frac{6}{5}
$$

$$
y = 3 \times \frac{6}{5} + 1 = \frac{18}{5} + 1 = \frac{18}{5} + \frac{5}{5} = \frac{23}{5}
$$

- Intersection point:

$$
\left(\frac{6}{5}, \frac{23}{5} \right)
$$

- Find slope between $(5, 2)$ and this point:

$$
m = \frac{\frac{23}{5} - 2}{\frac{6}{5} - 5} = \frac{\frac{23}{5} - \frac{10}{5}}{\frac{6}{5} - \frac{25}{5}} = \frac{\frac{13}{5}}{-\frac{19}{5}} = -\frac{13}{19}
$$

- Equation of the line:

$$
y - 2 = -\frac{13}{19}(x - 5)
$$

Or in slope-intercept form:

$$
y = -\frac{13}{19}x + \frac{65}{19} + 2 = -\frac{13}{19}x + \frac{65}{19} + \frac{38}{19} = -\frac{13}{19}x + \frac{103}{19}
$$

This is the required line's equation.

Advanced Topics and Variations

1. Multiple Lines and Concurrency

In some problems, you may need to find a line passing through a point and intersecting multiple lines at specific points, or the point of concurrency of multiple lines, such as medians, altitudes, etc. The principles remain similar but involve more complex intersection calculations.

2. Use of Parametric Equations

Instead of standard forms, lines can be represented parametrically:
\[
x = x_

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