# Find The Equation Of The Tangent Plane To The Surface Defined Bythe Function Z = 2x^2 + Y^2 At Point

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Understanding how to find the tangent plane to a surface at a specific point is a fundamental concept in multivariable calculus. It provides insight into the local linear approximation of the surface, which is essential for various applications such as optimization, differential geometry, and computer graphics. In this article, we will explore the step-by-step process of deriving the tangent plane equation for the surface defined by the function $Z = 2x^2 + y^2$ at a given point.

## Understanding the Surface and Its Equation

Before diving into the calculations, it is crucial to understand the nature of the surface described by the function:

$$Z = 2x^2 + y^2$$

This is a quadratic surface, specifically an elliptic paraboloid opening upward along the z-axis. The surface curves upward in the x and y directions, with the steepness along x being controlled by the coefficient 2, and along y by the coefficient 1.

The key to finding the tangent plane is recognizing that it is the best linear approximation to the surface at a specific point. This approximation is based on the tangent vectors derived from the partial derivatives of the function with respect to x and y.

## Prerequisites: Partial Derivatives and Gradient Vector

To find the tangent plane, we need the gradient vector of the surface at the point of interest. The gradient vector points in the direction of the steepest ascent and is perpendicular to the tangent plane.

Given the function:

$$Z = f(x, y) = 2x^2 + y^2$$

the partial derivatives are:

$$
\frac{\partial f}{\partial x} = 4x \\
\frac{\partial f}{\partial y} = 2y
$$

The gradient vector of the surface at any point $(x, y)$ is:

$$
\nabla f(x, y) = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = (4x, 2y)
$$

Note that since the surface is given as $Z = f(x, y)$, the tangent plane at a point $(x_0, y_0, z_0)$ can be derived using the gradient of $f$ and the point's coordinates.

# Step-by-Step Process to Find the Tangent Plane Equation

Let's now detail the steps involved in deriving the tangent plane equation for the surface at a specific point.

## Step 1: Identify the Point of Tangency

Suppose the tangent plane is to be found at point $P = (x_0, y_0, z_0)$, where:

$$
z_0 = f(x_0, y_0) = 2x_0^2 + y_0^2
$$

The explicit coordinates are determined by substituting $x_0, y_0$ into the function.

Example: Let's choose a specific point for illustration:

$$
x_0 = 1, \quad y_0 = 2
$$

then,

$$
z_0 = 2(1)^2 + (2)^2 = 2 + 4 = 6
$$

\]

Thus, the point of tangency is \( P = (1, 2, 6) \).

## Step 2: Compute the Gradient Vector at the Point

Calculate the partial derivatives at \((x_0, y_0)\):

\[
\frac{\partial f}{\partial x} (x_0, y_0) = 4x_0 = 4 \times 1 = 4
\]
\[
\frac{\partial f}{\partial y} (x_0, y_0) = 2y_0 = 2 \times 2 = 4
\]

Therefore, the gradient vector at the point is:

\[
\nabla f(x_0, y_0) = (4, 4)
\]

Since the surface is expressed as \(z = f(x, y)\), the normal vector to the tangent plane is:

\[
\mathbf{n} = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, -1 \right) = (4, 4, -1)
\]

This vector is perpendicular to the tangent plane.

## Step 3: Write the Equation of the Tangent Plane

The general form of the tangent plane at point \((x_0, y_0, z_0)\) with normal vector \(\mathbf{n} = (A, B, C)\) is:

\[
A(x - x_0) + B(y - y_0) + C(z - z_0) = 0
\]

Using our specific normal vector:

$$
4(x - 1) + 4(y - 2) - 1(z - 6) = 0
$$

Simplify:

$$
4x - 4 + 4y - 8 - z + 6 = 0
$$

$$
4x + 4y - z - 6 = 0
$$

Rearranged, the equation of the tangent plane is:

$$
z = 4x + 4y - 6
$$

This is the linear approximation of the surface at the point $(1, 2, 6)$.

## Generalized Formula for the Tangent Plane

The process described can be generalized for any point $(x_0, y_0)$ on the surface:

$$
z = f(x, y) = 2x^2 + y^2
$$

with the point:

$$
(x_0, y_0, z_0) = \left( x_0, y_0, 2x_0^2 + y_0^2 \right)
$$

The normal vector at the point is:

$$
\mathbf{n} = (4x_0, 2y_0, -1)
$$

Therefore, the tangent plane equation becomes:

$$
4x_0 (x - x_0) + 2y_0 (y - y_0) - (z - z_0) = 0
$$

which simplifies to:

$$
4x_0 x + 2y_0 y - z = 4x_0^2 + 2y_0^2
$$

or equivalently:

$$
z = 4x_0 x + 2y_0 y - (4x_0^2 + 2y_0^2)
$$

This formula allows you to find the tangent plane at any point on the surface.

## Applications of Tangent Planes in Various Fields

Understanding and calculating tangent planes have significant applications across multiple disciplines:

- **Mathematical Analysis:** Approximating complex surfaces with linear planes for easier analysis.
- **Optimization:** Using tangent planes to find local maxima, minima, or saddle points of functions.
- **Physics:** Analyzing the local behavior of potential fields and surfaces.
- **Computer Graphics:** Rendering surfaces and calculating reflections require tangent plane computations.
- **Engineering:** Stress analysis and surface modeling often rely on tangent planes for precise measurements.

## Visualizing the Tangent Plane

Visualizing the tangent plane alongside the surface helps in understanding the local behavior of the surface at a specific point. It provides an intuitive grasp of how the surface "touches" or "behaves" in the immediate vicinity of the point.

To visualize:

1. Plot the surface $Z = 2x^2 + y^2$.
2. Mark the point of tangency.
3. Draw the tangent plane based on the derived equation.
4. Observe how the plane approximates the surface near the point.

This visualization can be performed using graphing software like GeoGebra, MATLAB, or WolframAlpha for better comprehension.

## Conclusion

Finding the equation of the tangent plane to a surface like $Z = 2x^2 + y^2$ at a specific point is a vital skill in multivariable calculus. It involves calculating the partial derivatives to find the gradient vector, determining the normal vector to the surface at the point, and then applying the point-normal form of a plane equation. Whether for academic purposes, engineering applications, or computer graphics, mastering this technique enhances your understanding of surfaces and their local linear approximations.

Remember, the key steps are:

- Identify the point on the surface.
- Compute partial derivatives to find the gradient.
- Use the gradient as the normal vector.
- Write the tangent plane equation using the point-normal form.

With practice, you'll be able to find tangent planes for more complex surfaces, enabling deeper insights into their local behavior and properties.

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Additional Resources:

- "Multivariable Calculus" by James Stewart
- Online graphing tools: GeoGebra, Desmos, WolframAlpha

- Interactive tutorials on tangent planes and surface analysis

# Frequently Asked Questions

## How do you find the equation of the tangent plane to the surface z = 2x^2 + y^2 at a given point?

To find the tangent plane, first compute the partial derivatives $\partial z/\partial x$ and $\partial z/\partial y$ at the given point, then use the point-slope form: $z - z_0 = (\partial z/\partial x)(x - x_0) + (\partial z/\partial y)(y - y_0)$.

## What is the process for calculating the partial derivatives needed for the tangent plane of z = 2x^2 + y^2?

Calculate $\partial z/\partial x = 4x$ and $\partial z/\partial y = 2y$. Evaluate these derivatives at the point $(x_0, y_0)$ to determine the slopes of the tangent plane in the x and y directions.

## Given the point (1, 2, 8), how do you find the tangent plane to the surface z = 2x^2 + y^2?

First, find the partial derivatives at (1,2): $\partial z/\partial x = 4(1) = 4$ and $\partial z/\partial y = 2(2) = 4$. The tangent plane equation is $z - 8 = 4(x - 1) + 4(y - 2)$. Simplify to get the explicit form.

## Can the tangent plane be parallel to the surface at a point? Why or why not?

No, the tangent plane is always tangent precisely at the point on the surface; it cannot be parallel to the surface everywhere unless the surface is a plane itself. The tangent plane only approximates the surface locally.

## How does the gradient vector relate to the tangent plane of a surface z = f(x, y)?

The gradient vector $(\partial z/\partial x, \partial z/\partial y, -1)$ at a point is normal (perpendicular) to the tangent plane. It can be used directly to write the equation of the tangent plane.

## What is the general formula for the tangent plane to a surface defined by $z = f(x, y)$ at $(x_0, y_0, z_0)$?

The tangent plane is given by: $z - z_0 = (\partial f/\partial x)(x_0, y_0)(x - x_0) + (\partial f/\partial y)(x_0, y_0)(y - y_0)$.

# Additional Resources

Tangent Plane Equation

Understanding the geometry of surfaces in three-dimensional space is fundamental in fields ranging from engineering and physics to computer graphics and mathematics. One essential concept in this domain is the tangent plane—a flat surface that best approximates a surface at a specific point. This article provides a comprehensive guide to deriving the equation of the tangent plane to a surface defined by the function $Z = 2x^2 + y^2$, focusing on clarity, depth, and practical insights.

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# Introduction to the Surface and Its Significance

In the realm of multivariable calculus, surfaces are often described by functions $Z = f(x, y)$, where $Z$ is the height or value of the surface at each point $(x, y)$. The surface defined by

$$Z = 2x^2 + y^2$$

is a classic example of a paraboloid—a quadratic surface opening upward. Understanding the tangent plane at a particular point on this surface allows us to approximate the surface locally, analyze slopes, and investigate curvature. This is crucial in optimization problems, surface analysis, and 3D modeling.

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# Fundamentals of the Tangent Plane

## What Is a Tangent Plane?

Geometrically, the tangent plane at a point on a surface is the plane that "just touches" the surface at that point, sharing the same first-order behavior. Think of it as a flat sheet that best approximates the surface locally; it provides a linear approximation of the surface near the point of tangency.

## Mathematical Representation

For a surface given by $Z = f(x, y)$, the equation of the tangent plane at a point $(x_0, y_0, z_0)$ on the surface is:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

where:

- $f_x$ and $f_y$ are the partial derivatives of $f$ with respect to $x$ and $y$, respectively.

This linear approximation captures the surface's slope behavior at the point, providing essential insights into the surface's local geometry.

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# Step-by-Step Derivation of the Tangent Plane Equation

To determine the tangent plane to the surface $Z = 2x^2 + y^2$ at a specific point, we follow a systematic process.

## 1. Identify the Point of Tangency ($x_0, y_0, z_0$)

Suppose we are interested in the tangent plane at a point $P$ on the surface. The point must satisfy the surface's equation:

$$
z_0 = 2x_0^2 + y_0^2
$$

For illustration, let's choose a specific point, such as:

$$
x_0 = 1, \quad y_0 = 2
$$

then

$$
z_0 = 2(1)^2 + (2)^2 = 2 + 4 = 6
$$

So, the point of tangency is $(1, 2, 6)$.

## 2. Compute the Partial Derivatives $f_x$ and $f_y$

Given the function:

$$
f(x, y) = 2x^2 + y^2
$$

the partial derivatives are:

- $f_x(x, y) = \frac{\partial}{\partial x} (2x^2 + y^2) = 4x$

- $f_y(x, y) = \frac{\partial}{\partial y} (2x^2 + y^2) = 2y$

Evaluating at the point $(x_0, y_0) = (1, 2)$:

$$
f_x(1, 2) = 4 \times 1 = 4
$$

$$
f_y(1, 2) = 2 \times 2 = 4
$$

## 3. Write the Equation of the Tangent Plane

Using the general formula:

$$
z - z_0 = f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

we substitute:

$$
z - 6 = 4(x - 1) + 4(y - 2)
$$

Expanding:

$$
z - 6 = 4x - 4 + 4y - 8
$$

Simplify:

$$
z = 4x + 4y - 6
$$

Thus, the equation of the tangent plane at $(1, 2, 6)$ is:

$$
z = 4x + 4y - 6
$$

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# Generalized Formula and Application

The process demonstrated can be generalized for any point $(x_0, y_0)$ on the surface:

$$
z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)
$$

which simplifies to:

$$
z = (2x_0^2 + y_0^2) + 4x_0(x - x_0) + 2y_0(y - y_0)
$$

This form allows for straightforward calculation of the tangent plane at any point on the surface.

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# Practical Insights and Applications

## Why Is the Tangent Plane Important?

- Local Approximation: It provides an accurate first-order approximation of the surface near the point, essential for numerical methods and simulations.

- Optimization: Helps identify critical points, slopes, and directions of steepest ascent or descent.

- Surface Analysis: Facilitates the study of curvature and other differential properties.

- Engineering and Design: Used in CAD modeling, where tangent planes guide smooth surface transitions.

## Additional Considerations

- Curvature Analysis: The second derivatives inform about the surface's convexity or concavity at the point.

- Extensions to Implicit Surfaces: For surfaces defined implicitly, similar methods involving gradients apply.

- Higher Dimensions: The concept extends to hypersurfaces in higher-dimensional spaces.

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# Visualizing the Tangent Plane

Using 3D plotting tools or graphing calculators, visualize the surface $Z = 2x^2 + y^2$ along with its tangent plane at the chosen point. This visualization reinforces understanding, illustrating how the plane "touches" the surface exactly at the point, sharing the same slope, and approximating the surface locally.

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## Conclusion and Final Thoughts

Deriving the equation of the tangent plane to the surface \( Z = 2x^2 + y^2 \) is a foundational skill in multivariable calculus. It combines calculating partial derivatives, evaluating at specific points, and applying the tangent plane formula to produce a linear approximation that encapsulates the surface's local behavior.

This method is versatile, applicable across various functions and contexts, and essential in both theoretical and applied sciences. Whether you're analyzing the slope of a paraboloid, optimizing functions, or designing complex surfaces, mastering the tangent plane equation provides a powerful tool in your mathematical toolkit.

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In essence, by systematically applying calculus principles—derivatives, point evaluation, and algebra—you can confidently derive the tangent plane equation for any surface, unlocking deeper insights into the geometry and behavior of three-dimensional objects.

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