Find The Formula For An Exponential Function That Passes Through The Two Points Given.(x, Y) = (0,5)

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When working with exponential functions, one of the fundamental tasks is to determine the specific formula of the function that passes through given points. In this case, we are asked to find the exponential function that passes through the point (0, 5) and another point, which is essential for identifying the exact formula. Understanding how to derive this formula is crucial for solving numerous real-world problems, such as population growth, radioactive decay, and financial modeling.

In this comprehensive guide, we will walk through the steps involved in finding the exponential function passing through two points, focusing on the point (0, 5) and an additional point provided. We will explore the general form of an exponential function, how to use the given points to determine its parameters, and provide practical examples to solidify understanding.

Understanding the General Form of an Exponential Function

Before diving into the calculation process, it's important to understand the general structure of an exponential function.

Standard Form of an Exponential Function

An exponential function can typically be written as:

$$ y = a \times b^x $$

where:
- a is the initial value or the y-intercept of the function.
- b is the base of the exponential, which determines the rate of growth or decay.
- x is the independent variable.
- y is the dependent variable (the value of the function at x).

Given this form, our goal is to find the specific values of a and b that satisfy the conditions of passing through the given points.

Using the Point (0, 5) to Find the Initial Value

The point (0, 5) provides a crucial piece of information. When $x = 0$, the corresponding y value is 5. Substituting into the standard form:

$$ 5 = a \times b^0 $$

Since any non-zero number raised to the power of 0 is 1, this simplifies to:

$$ 5 = a \times 1 $$

Therefore, the initial value a is:

$$ a = 5 $$

This simplifies our exponential function to:

$$ y = 5 \times b^x $$

Now, the only unknown parameter is b, the base.

Determining the Base b Using the Second Point

To fully define the exponential function, we need a second point through which the function passes. Suppose the second point is given as (x_1, y_1). Substituting this point into the simplified function:

$$ y_1 = 5 \times b^{x_1} $$

We can solve for b:

$$ b^{x_1} = \frac{y_1}{5} $$

$$ b = \left(\frac{y_1}{5} \right)^{\frac{1}{x_1}} $$

This formula allows us to compute b once the second point is known.

Example: Finding the Function with a Second Point

Let's assume the second point is $(2, 20)$. Plugging into the formula:

$$ b = \left(\frac{20}{5} \right)^{\frac{1}{2}} = (4)^{\frac{1}{2}} = \sqrt{4} = 2 $$

Thus, the exponential function passing through (0, 5) and (2, 20) is:

$$ y = 5 \times 2^x $$

This function models exponential growth with an initial value of 5 and a growth factor of 2 per unit increase in x.

General Steps to Find the Exponential Function

When given any two points, the process to find the exponential function is straightforward:

1. Identify the initial value a using the point where $x = 0$. If

the point is (0, Y), then $a = Y$.

2. Use the second point (x_1, y_1) to solve for b, the base of the exponential, with:

   $$b = \left(\frac{y_1}{a} \right)^{\frac{1}{x_1}}$$

3. Write the specific exponential function:

   $$y = a \times b^x$$

This method works for any two points where $x_1 \neq 0$.

Special Cases and Additional Considerations

While the above process is straightforward, some special cases require extra attention.

Case 1: When the Second Point Has $x = 0$

If both points have $x = 0$, then they are the same point, and an infinite number of exponential functions pass through that point. Additional points are necessary to determine a unique function.

Case 2: When the Second Point Has $y \leq 0$

Since exponential functions are always positive for real $b > 0$, points with $y \leq 0$ imply the function may be undefined or require complex analysis, which is beyond typical scope.

Case 3: Negative or Fractional Bases

Depending on the context, negative or fractional bases can be used, but they may lead to complex or oscillating functions if b is negative. For most modeling purposes, $b > 0$ is standard.

Practical Applications of Finding Exponential Functions

Understanding how to find exponential functions from points is essential in various fields:

- **Population Modeling:** Estimating population growth or decay over time.
- **Radioactive Decay:** Calculating remaining radioactive material at different times.

- **Financial Mathematics**: Computing compound interest and investment growth.
- **Medicine and Pharmacology**: Modeling drug concentration decay in the bloodstream.

In each case, knowing the initial amount and a subsequent point in time enables precise modeling and prediction.

Summary of the Process

To summarize, the key steps to find an exponential function passing through two points are:

1. Determine the initial value a from the point where $x = 0$.
2. Use the second point to calculate the base b.
3. Write the explicit formula $y = a \times b^x$.

This approach provides a systematic way to model exponential relationships accurately.

Conclusion

Finding the formula for an exponential function that passes through given points is a fundamental skill in algebra and applied mathematics. Starting with the point (0, 5), we established that $a = 5$. With a second point, we can determine the base b using the derived formula. This method is versatile and applicable across many scientific and engineering disciplines, enabling precise modeling of exponential phenomena.

By mastering this process, you will be well-equipped to analyze exponential data, interpret real-world growth and decay patterns, and develop accurate mathematical models for various applications.

Frequently Asked Questions

What is the general form of an exponential function passing through two points?

The general form is $y = a b^x$, where a and b are constants determined by the points.

Given the point (0, 5), what is the value of 'a' in the exponential function?

Since x=0, y=a b^0 = a 1 = a. Therefore, a = 5.

How do you find the base 'b' of the exponential function passing through points (0,5) and (x1, y1)?

Use the two points to set up equations: 5 = a b^0 and y1 = 5 b^{x1}. Then, solve for b: b^{x1} = y1 / 5, so b = (y1 / 5)^{1 / x1}.

What is the formula for an exponential function passing through (0, 5) and (x1, y1)?

The formula is y = 5 ((y1 / 5))^{x / x1}.

If one point is (0, 5) and the other is (2, 20), what is the exponential function?

First, find b: b^{2} = 20 / 5 = 4, so b = √4 = 2. The function is y = 5 2^x.

How do you verify that an exponential function passes through given points?

Substitute the x-values into the function and check if the resulting y-values match the given points.

Can the exponential function passing through (0, 5) have a base less than 1?

Yes, if the second point indicates a decreasing trend, the base b can be less than 1, leading to a decreasing exponential function.

What is the significance of the point (0, 5) in the exponential function?

It indicates the initial value or the y-intercept of the exponential function when x=0, which is 'a' in the formula y = a b^x.

How do changes in the second point affect the exponential function's formula?

The second point determines the base 'b' of the function, affecting the growth or decay rate, while 'a' remains 5 due to passing through (0,5).

Is it possible to find an exponential function passing through (0, 5) and another arbitrary point?

Yes, by substituting the points into y = 5 b^x and solving for b using the second point's coordinates.

Additional Resources

Find The Formula For An Exponential Function That Passes Through The Two Points Given (x, y) = (0, 5)

Introduction

In the realm of mathematics, exponential functions are a fundamental class of functions that model phenomena exhibiting rapid growth or decay. Whether describing population dynamics, radioactive decay, or financial investments, the ability to determine the specific exponential function passing through given points is an essential skill for students and professionals alike.

This article delves into the methodical process of finding an exponential function that passes through a particular set of points—specifically, when one point is at the origin with a known y-value. Our focus centers on the point (0, 5), exploring how to formulate the exponential function passing through this point and an additional arbitrary point.

Understanding Exponential Functions

An exponential function generally takes the form:

$$ y = a \cdot b^x $$

where:
- a is the initial value or the y-intercept when $x = 0$,
- b is the base of the exponential, representing the growth ($b > 1$) or decay ($0 < b < 1$),
- x is the independent variable.

Given this form, our task revolves around determining the parameters a and b such that the function passes through specific points.

The Significance of the Point (0, 5)

The point (0, 5) provides critical insight into the function's form. Since $x = 0$, plugging into the general exponential model yields:

$$ y(0) = a \cdot b^0 = a \cdot 1 = a $$

Therefore, the y-coordinate at $x=0$ directly gives the value of a:

$$ a = 5 $$

This simplifies our problem significantly, reducing the exponential function to:

$$ y = 5 \cdot b^x $$

Now, the challenge is to determine the base b based on the second point (or the conditions provided).

Extending the Problem: Including a Second Point

To uniquely define the exponential function, at least two points are necessary. Suppose the second point is (x_1, y_1). The problem then becomes:

> Find the formula for $y = 5 \cdot b^x$ that passes through (x_1, y_1).

Substituting x_1 and y_1:

$$y_1 = 5 \cdot b^{x_1}$$

From this, solving for b:

$$b^{x_1} = \frac{y_1}{5}$$

$$b = \left(\frac{y_1}{5} \right)^{1/x_1} \quad \text{(assuming } x_1 \neq 0)$$

Therefore, the exponential function is:

$$y = 5 \cdot \left(\frac{y_1}{5} \right)^{x/x_1}$$

This formula explicitly relates the known point (x_1, y_1) to the exponential function passing through $(0, 5)$.

Practical Example: Deriving the Function with a Specific Second Point

Suppose the second point is $(2, 20)$. Using the earlier relation:

$$b = \left(\frac{20}{5} \right)^{1/2} = (4)^{1/2} = 2$$

Thus, the exponential function passing through $(0, 5)$ and $(2, 20)$ is:

$$y = 5 \cdot 2^x$$

Verification:

- At $x=0$, $y=5 \cdot 2^0=5$, which matches the point.
- At $x=2$, $y=5 \cdot 2^2=5 \cdot 4=20$, confirming the passing point.

Generalized Approach for Any Two Points

The process outlined can be generalized for any two points $(0, y_0)$ and (x_1, y_1).

Step 1: Recognize the initial value

$$a = y_0$$

Step 2: Calculate the base b

$$ b = \left(\frac{y_1}{a} \right)^{1/x_1} $$

Step 3: Write the explicit function

$$ y = y_0 \cdot \left(\frac{y_1}{y_0} \right)^{x/x_1} $$

This formula ensures the exponential passes through both points.

Addressing Special Cases and Limitations

While the method is straightforward when the second point is known, several special cases merit discussion:

- When $x_1 = 0$: The second point coincides with the initial point; thus, infinitely many functions pass through both points unless additional constraints are provided.
- When $y_1 \leq 0$: The exponential model with positive base b cannot pass through points with non-positive y-values, since $y = a \cdot b^x$ is positive for all real x.

Extending to Logarithmic and Other Models

If the second point or problem context suggests a different relationship, alternative models like logarithmic or polynomial functions might be more appropriate. However, within the scope of exponential functions passing through a fixed y-intercept, the outlined method remains robust.

Applications and Significance

Understanding how to determine the exponential function passing through given points is crucial in numerous fields:

- Economics: Modeling compound interest where initial investments are known.
- Biology: Tracking population growth or decay.
- Physics: Describing radioactive decay processes.
- Engineering: Signal attenuation and amplification.

The ability to reverse-engineer the exponential model from data points empowers professionals to create accurate predictive models.

Conclusion

Finding the formula for an exponential function passing through specific points, especially when one point is at the origin with a known y-value, hinges on understanding the fundamental structure of the model. Recognizing that the y-intercept directly gives a simplifies the process, reducing the problem to determining the base b.

By applying the derived formulas, students and practitioners can effectively reconstruct the exponential function from minimal data, enhancing their analytical toolkit for a broad spectrum of scientific and mathematical challenges.

Final Remarks

Mastering the method to find exponential functions passing through given points underscores the importance of algebraic manipulation and conceptual understanding. Whether dealing with theoretical problems or real-world data, this skill forms a cornerstone of mathematical literacy and problem-solving prowess.

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