Find The General Solution Of The Differential Equation 9y" + 48y' + 64y = 0. Use C1, C2, ... For The

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Understanding how to find the general solution of a differential equation like 9y" + 48y' + 64y = 0 is fundamental in advanced calculus and differential equations. This type of equation is a second-order linear homogeneous differential equation with constant coefficients. Solving such equations provides insights into various scientific and engineering phenomena, including oscillations, electrical circuits, and mechanical vibrations. In this comprehensive guide, we will explore the step-by-step process to find the general solution of the differential equation 9y" + 48y' + 64y = 0, including the derivation of the characteristic equation, solving for roots, and expressing the solution in terms of arbitrary constants C1, C2, etc.

Understanding the Differential Equation 9y" + 48y' + 64y = 0

Before diving into the solution process, let's understand the structure and nature of the differential equation:

- It is a second-order differential equation, involving the second derivative y" and the first derivative y'.
- It is linear and homogeneous, meaning all terms involve the function y or its derivatives, and the right-hand side equals zero.
- The coefficients (9, 48, 64) are constants, which simplifies the solution process significantly.

This type of differential equation is often solved using the method of characteristic equations, which transforms the differential equation into an algebraic equation.

Step-by-Step Solution Process

The process of solving the differential equation involves several systematic steps:

1. Write the Differential Equation in Standard Form

The equation is already in standard form:

$$ 9y'' + 48y' + 64y = 0 $$

where:

- y'' denotes the second derivative of y with respect to x,
- y' denotes the first derivative,
- y is the unknown function.

2. Formulate the Characteristic Equation

For constant coefficient linear differential equations, the solution involves assuming a trial solution of the form:

$$ y = e^{rx} $$

where r is a constant to be determined.

Substituting into the differential equation yields the characteristic (or auxiliary) equation:

$$ 9r^2 + 48r + 64 = 0 $$

This quadratic equation in r encapsulates the behavior of the solutions.

3. Solve the Characteristic Equation

Solve the quadratic:

$$ 9r^2 + 48r + 64 = 0 $$

by using the quadratic formula:

$$ r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where:

- $a = 9$,
- $b = 48$,
- $c = 64$.

Calculating the discriminant:

$$ D = b^2 - 4ac = (48)^2 - 4 \times 9 \times 64 $$

$$ D = 2304 - 2304 = 0 $$

Since the discriminant is zero, the roots are real and equal:

$$ r = \frac{-48}{2 \times 9} = \frac{-48}{18} = -\frac{8}{3} $$

Thus, the roots:

$$ r_1 = r_2 = -\frac{8}{3} $$

indicate that the general solution involves repeated roots.

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4. Write the General Solution Based on Roots

For repeated roots r, the general solution to the differential equation is:

$$ y(x) = (C_1 + C_2 x) e^{r x} $$

where:

- C_1 and C_2 are arbitrary constants determined by initial conditions.

In our case:

$$ y(x) = (C_1 + C_2 x) e^{-\frac{8}{3} x} $$

This expression represents the general solution of the differential equation.

Summary of the General Solution

The comprehensive form of the solution is:

$$ \boxed{
\text{General Solution:} \quad y(x) = \left(C_1 + C_2 x \right) e^{-\frac{8}{3} x}
} $$

where:

- C_1 and C_2 are arbitrary constants,
- $e^{-\frac{8}{3} x}$ reflects exponential decay behavior due to the negative root.

Additional Details and Considerations

Understanding some additional points can deepen your grasp of solving such equations:

1. Nature of the Roots and Their Impact on the Solution

- Distinct real roots: The general solution involves two different exponential functions.
- Repeated roots: The solution includes a polynomial term multiplied by an

exponential.
- Complex roots: Would involve sinusoidal functions (sine and cosine) multiplied by an exponential decay or growth.

In our case, the repeated real roots result in the specific form with C_1 and C_2.

2. Initial Conditions and Particular Solutions

While the general solution contains arbitrary constants, specific solutions are obtained by applying initial conditions such as:

- $y(0) = y_0$,
- $y'(0) = y'_0$.

These allow solving for C_1 and C_2.

3. Graphical Interpretation

The solution:

$$y(x) = (C_1 + C_2 x) e^{-\frac{8}{3} x}$$

depicts a damped exponential behavior, which is common in physical systems like damping oscillators or RC circuits.

Practical Applications of the Differential Equation

This differential equation models various physical systems:

- Mechanical vibrations: Damped harmonic oscillators.
- Electrical circuits: RLC circuits with damping.
- Population dynamics: When considering decay processes.
- Control systems: Stability analysis.

Understanding how to derive the general solution helps engineers and scientists predict system behaviors accurately.

Conclusion

In this detailed guide, we have systematically derived the general solution of the differential equation:

$$9y'' + 48y' + 64y = 0$$

by:

- Formulating the characteristic equation,
- Solving for roots,
- Recognizing the repeated roots,
- Writing the general solution in terms of arbitrary constants C_1 and C_2.

The final form:

$$y(x) = (C_1 + C_2 x) e^{-\frac{8}{3} x}$$

serves as the foundation for solving initial value problems and analyzing the behavior of related physical systems. Mastery of these techniques is essential for anyone studying differential equations, mathematical modeling, or engineering disciplines.

Keywords: differential equations, second-order homogeneous equations, characteristic equation, repeated roots, exponential solutions, general solution, constants C_1, C_2, damping, oscillations, engineering applications

Frequently Asked Questions

What is the characteristic equation associated with the differential equation 9y'' + 48y' + 64y = 0?

The characteristic equation is $9r^2 + 48r + 64 = 0$.

How do you find the roots of the characteristic equation for this differential equation?

Solve $9r^2 + 48r + 64 = 0$ using the quadratic formula: $r = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$, where a=9, b=48, c=64.

What are the roots of the characteristic equation for the differential equation 9y'' + 48y' + 64y = 0?

The roots are real and equal: $r = -8/3$, $r = -8/3$, since the discriminant is zero.

What is the general solution of the differential equation 9y'' + 48y' + 64y = 0 based on the roots?

The general solution is $y(t) = (C1 + C2\, t) e^{-8t/3}$, where C1 and C2 are arbitrary constants.

Why does the general solution include a term with t

multiplied by the exponential function in this case?

Because the characteristic roots are repeated, the solution includes a term with t to account for the multiplicity, resulting in $(C1 + C2\, t)\, e^{r t}$.

How can you verify that the obtained general solution satisfies the original differential equation?

Differentiate the general solution twice, substitute y, y', y'' into the original equation, and confirm that the equation simplifies to zero for all t.

Additional Resources

Find The General Solution Of The Differential Equation 9y'' + 48y' + 64y = 0

Understanding how to find the general solution of differential equations is a fundamental skill in calculus and differential equations, with applications spanning physics, engineering, economics, and beyond. In this review, we focus on the differential equation:

$$ 9 y'' + 48 y' + 64 y = 0 $$

This particular second-order linear homogeneous differential equation with constant coefficients offers a classic example for illustrating solution techniques. We will systematically explore the steps involved, including deriving the characteristic equation, solving for roots, and constructing the general solution, along with insights into the underlying methods, their advantages, and limitations.

Understanding the Differential Equation Structure

The differential equation provided is a second-order linear homogeneous differential equation with constant coefficients. Its general form can be expressed as:

$$ a y'' + b y' + c y = 0 $$

where a, b, and c are constants. Such equations are well-understood and have standard solution methods involving characteristic equations.

Key features:
- The coefficients are constants.
- The equation is homogeneous (equal to zero).
- The order of the equation is 2, indicating that the general solution will involve two arbitrary constants, commonly denoted as C_1 and C_2.

Formulating the Characteristic Equation

The cornerstone of solving linear constant-coefficient differential equations is formulating and solving the characteristic (or auxiliary) equation. For the given differential equation:

$$ 9 y'' + 48 y' + 64 y = 0 $$

we replace derivatives with algebraic terms involving a parameter r:

$$ 9 r^2 + 48 r + 64 = 0 $$

This quadratic equation is called the characteristic equation, and solving it yields the roots that determine the form of the general solution.

Derivation:
- Substituting $ y = e^{r x} $, derivatives become $ y' = r e^{r x} $, $ y'' = r^2 e^{r x} $.
- Factoring out $ e^{r x} $ (which is never zero), leads to the characteristic polynomial:

$$ 9 r^2 + 48 r + 64 = 0 $$

Solving the Characteristic Equation

The quadratic characteristic equation:

$$ 9 r^2 + 48 r + 64 = 0 $$

can be solved using the quadratic formula:

$$ r = \frac{-b \pm \sqrt{b^2 - 4 a c}}{2 a} $$

where:
- $ a = 9 $
- $ b = 48 $
- $ c = 64 $

Calculations:

$$
\begin{aligned}
r &= \frac{-48 \pm \sqrt{48^2 - 4 \times 9 \times 64}}{2 \times 9} \\
&= \frac{-48 \pm \sqrt{2304 - 2304}}{18} \\
&= \frac{-48 \pm \sqrt{0}}{18} \\
&= \frac{-48 \pm 0}{18} \\
&= \frac{-48}{18} = -\frac{8}{3}
\end{aligned}
$$

Since the discriminant is zero, the roots are real and repeated:

$$ r_1 = r_2 = -\frac{8}{3} $$

Constructing the General Solution

When the characteristic roots are real and repeated, the general solution to the differential equation is:

$$ y(x) = (C_1 + C_2 x) e^{r x} $$

Applying this to our roots:

$$ y(x) = (C_1 + C_2 x) e^{-\frac{8}{3} x} $$

where:
- C_1 and C_2 are arbitrary constants, determined by initial conditions.

Features of this solution:
- The exponential term $e^{-\frac{8}{3} x}$ indicates a decaying or growing behavior depending on the sign of the root.
- The presence of the x term multiplying C_2 accounts for the repeated root, ensuring the solution's linear independence.

Verification of the Solution

To ensure correctness, substitute the general solution back into the original differential equation.

Step 1: Compute derivatives

$$
\begin{aligned}
y &= (C_1 + C_2 x) e^{-\frac{8}{3} x} \\
y' &= C_2 e^{-\frac{8}{3} x} + (C_1 + C_2 x) \left(-\frac{8}{3}\right) e^{-\frac{8}{3} x} \\
&= e^{-\frac{8}{3} x} \left[C_2 - \frac{8}{3}(C_1 + C_2 x) \right]
\end{aligned}
$$

$$
\begin{aligned}
y'' &= \frac{d}{dx} y' \\
&= \frac{d}{dx} \left[e^{-\frac{8}{3} x} \left(C_2 - \frac{8}{3}(C_1 + C_2 x) \right) \right]
\end{aligned}
$$

Applying product rule:

$$
\begin{aligned}
y'' &= e^{-\frac{8}{3} x} \left[-\frac{8}{3} \left(C_2 - \frac{8}{3}(C_1 +
$$

C_2 x) \right) + \left(0 - \frac{8}{3} C_2 \right) \right] \\
&= e^{-\frac{8}{3} x} \left[-\frac{8}{3} C_2 + \frac{64}{9} (C_1 + C_2 x) - \frac{8}{3} C_2 \right] \\
&= e^{-\frac{8}{3} x} \left[-\frac{8}{3} C_2 - \frac{8}{3} C_2 + \frac{64}{9} C_1 + \frac{64}{9} C_2 x \right]
\end{aligned}
\]

Substituting \(y \), \(y' \), and \(y'' \) into the original equation:

\[9 y'' + 48 y' + 64 y = 0 \]

and simplifying confirms that the solution satisfies the differential equation identically, verifying correctness.

Alternative Methods and Considerations

While the characteristic equation approach is straightforward for constant-coefficient linear equations, alternative or supplementary methods include:

- Eigenvalue methods for systems of differential equations.
- Laplace transforms especially when initial conditions are involved.
- Reduction of order if one solution is known.

Pros of the characteristic equation method:
- Direct and systematic.
- Well-suited for constant coefficients.
- Leads to explicit solutions readily.

Cons or limitations:
- Not applicable if coefficients are variable.
- Roots may be complex, requiring interpretation.
- Repeated roots necessitate careful handling.

Features and Applications of the Solution

The general solution:

\[y(x) = (C_1 + C_2 x) e^{-\frac{8}{3} x} \]

has several features:

- Exponential decay/growth: The exponential term dominates the behavior, indicating damping or amplification.
- Linearly independent solutions: The two arbitrary constants, \(C_1 \) and \(C_2 \), allow for the modeling of various initial conditions.
- Applicability: Such solutions are relevant in mechanical vibrations (damped oscillations), electrical circuits, and dynamic systems with damping.

Conclusion and Summary

Finding the general solution of the differential equation $9 y'' + 48 y' + 64 y = 0$ involves a methodical process:

1. Formulate the characteristic equation: $9 r^2 + 48 r + 64 = 0$.
2. Solve for roots: Discriminant zero indicates repeated roots, $r = -\frac{8}{3}$.
3. Construct the general solution: For repeated roots, the form is $y(x) = (C_1 + C_2 x) e^{r x}$.

This systematic approach demonstrates the power of algebraic techniques in solving differential equations. The resulting solutions are fundamental to modeling various physical phenomena, especially those involving damping or decay processes. Mastery of these methods enhances one's ability to analyze complex systems and develop solutions for diverse scientific and engineering challenges.

In summary:

- The differential equation's roots are real and repeated.
- The general solution incorporates a polynomial factor for repeated roots.
- Verification confirms the solution's correctness.
- Alternative methods exist but are often more complex for such equations.
- Understanding the structure and solution form is crucial for applying these techniques in practical contexts.

By mastering this process, students and practitioners can confidently approach similar differential equations,

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