Find The Indicated Antiderivative. (a) Using Substitution, Findx1x2dx(b) Using Integrition By Parts,

Find The Indicated Antiderivative. (a) Using Substitution, Find x^1x^2 dx (b) Using Integration By Parts

Understanding how to compute antiderivatives, also known as indefinite integrals, is a fundamental skill in calculus. These integrals are essential for solving problems involving areas, volumes, and differential equations. When faced with a challenging integral, choosing the appropriate technique—such as substitution or integration by parts—can simplify the process significantly. This comprehensive guide will walk you through the methods to find the indicated antiderivatives, focusing on two common techniques: substitution and integration by parts. Whether you're a student new to calculus or looking to reinforce your skills, this detailed explanation will help you master these integral techniques.

Part A: Using Substitution to Find $\int x^{1} x^{2} dx$

Understanding the Integral

The integral to evaluate in this section is:

$$
\int x^{1} x^{2} \, dx
$$

At first glance, this appears straightforward, but it's important to simplify the integrand before applying substitution. Notice that $x^{1} x^{2} = x^{1+2} = x^{3}$. So, the integral simplifies to:

$$
\int x^{3} \, dx
$$

However, to illustrate the substitution process, we'll pretend we're not aware of that simplification and approach it as if the integrand is a product of functions.

Step 1: Recognize the Structure of the Integral

The integrand is a product:

$$
x^{1} \times x^{2}
$$

which can be written as:

$$
x^{1} \times x^{2} = x^{1} \times x^{2}
$$

Since both are powers of x, substitution can be used to simplify the integral.

Step 2: Choose the Substitution

In substitution, we look for a part of the integrand whose derivative also appears within the integral. Here, consider:

- Let $u = x^{2}$

Then,

$$
du = 2x \, dx
$$

But our differential dx is multiplied by x^{1}, which is x. To match the du expression, note that:

$$
x \, dx = \frac{1}{2} du
$$

Now, rewrite the original integral accordingly.

Step 3: Rewrite the Integral in Terms of u

Express the integral as:

$$
\int x^{1} x^{2} \, dx = \int x \times x^{2} \, dx
$$

Using $u = x^{2}$, then:

$$
x = \sqrt{u}
$$

But this complicates things; a better approach is to consider the substitution:

- $u = x^{3}$

Since:

$$
du = 3x^{2} \, dx
$$

and our integrand involves $x^{1} x^{2} = x^{3}$, which is perfect.

Thus:

- Let $u = x^{3}$

- Then, $du = 3x^{2} \, dx$

Rearranged:

$$
x^{2} \, dx = \frac{1}{3} du
$$

Note that in the original integral, we have $x^{1} x^{2} dx = x \times x^{2} dx$. But:

$$
x^{1} x^{2} = x^{3}
$$

and

$$
x^{2} dx = \frac{1}{3} du
$$

So, to include x in the substitution, express x in terms of u:

$$
u = x^{3} \Rightarrow x = u^{1/3}
$$

which complicates the substitution. Thus, the simplest substitution here is to recognize that:

$$
x^{1} x^{2} = x^{3}
$$

which makes the integral:

$$
\int x^{3} \, dx
$$

This is straightforward:

$$
\int x^{3} \, dx = \frac{x^{4}}{4} + C
$$

Conclusion: The integral evaluates to:

$$
\boxed{\frac{x^{4}}{4} + C}
$$

Part B: Using Integration By Parts to Find $\int x \sin x \, dx$

Understanding the Technique

Integration by parts is based on the product rule of differentiation:

$$
\frac{d}{dx} (uv) = u \frac{dv}{dx} + v \frac{du}{dx}
$$

Rearranged for integration:

$$
\int u \, dv = uv - \int v \, du
$$

This technique is especially useful when the integrand is a product of two functions, where differentiating one simplifies it, and integrating the other is straightforward.

Step 1: Identify u and dv

For the integral:

$$
\int x \sin x \, dx
$$

Choose:

- $u = x \Rightarrow du = dx$
- $dv = \sin x \, dx \Rightarrow v = -\cos x$

Step 2: Apply the Integration by Parts Formula

Using:

$$
\int u \, dv = uv - \int v \, du
$$

we get:

$$
\int x \sin x \, dx = -x \cos x - \int -\cos x \, dx
$$

Simplify:

$$
= -x \cos x + \int \cos x \, dx
$$

The integral of $\cos x$ is $\sin x$:

$$
= -x \cos x + \sin x + C
$$

Final answer:

$$
\boxed{-x \cos x + \sin x + C}
$$

Summary and Key Takeaways

- **Substitution Technique:** Best suited for integrals where a composite function or a function multiplied by its derivative appears. Recognize patterns, choose an appropriate substitution, and simplify accordingly.
- **Integration by Parts:** Ideal for integrals involving the product of functions where differentiating one simplifies the integral. Select u and dv strategically to reduce the problem.

Additional Tips for Mastering Integration Techniques

1. Always look for common patterns or structures in the integrand before choosing your method.
2. For substitution, identify inner functions or powers that can be replaced to simplify the integral.
3. In integration by parts, select u as the function that simplifies upon differentiation, and dv as the part that is easily integrable.
4. Practice with different types of integrals to develop intuition for which technique to use.
5. Remember that some integrals may require multiple steps or combination of techniques.

Conclusion

Mastering substitution and integration by parts equips you with powerful tools to evaluate a wide range of indefinite integrals. Remember to analyze the integrand carefully, choose the most suitable method, and execute each step systematically. With practice, recognizing patterns and applying these techniques will become second nature, greatly enhancing your problem-solving skills in calculus.

Feel free to explore further examples and practice problems to solidify your understanding of these essential integration methods!

Frequently Asked Questions

How do you approach finding the antiderivative of x^2 using substitution?

To find the antiderivative of x^2 using substitution, set u = x, then du = dx. The integral becomes ⍰ u^2 du, which is straightforward to evaluate as (u^3)/3 + C. Substituting back u = x gives (x^3)/3 + C.

What is the step-by-step process for applying substitution to find ⍰ x^1x^2 dx?

First, identify a substitution such as u = x^2, then compute du = 2x dx. Rewrite the integral in terms of u: ⍰ x u du / (2x). Simplify to get (1/2) ∫ u du, which integrates to (1/4) u^2 + C. Substitute back u = x^2 to get (1/4) x^4 + C.

When using integration by parts, what are the typical choices for u and dv in the integral ∫ x e^x dx?

For ∫ x e^x dx, choose u = x (since its derivative simplifies) and dv = e^x dx (which is straightforward to integrate). Then, du = dx and v = e^x. Applying integration by parts formula gives the antiderivative.

Can you demonstrate the use of integration by parts to find ∫ x e^x dx?

Yes. Let u = x, so du = dx; dv = e^x dx, so v = e^x. Applying integration by parts: ∫ x e^x dx = x e^x - ∫ e^x dx = x e^x - e^x + C = e^x (x - 1) + C.

What are common mistakes to avoid when using substitution for antiderivatives?

Common mistakes include choosing an incorrect substitution that doesn't simplify the integral, forgetting to change bounds if it's a definite integral, and not substituting back to the original variable after integration. Always verify that substitution simplifies the integral and include the constant of integration.

How do you decide whether to use substitution or integration by parts for a given integral?

Use substitution when the integral contains a composite function or a function and its derivative. Use integration by parts when the integrand is a product of functions where one simplifies upon differentiation and the other is easily integrable, such as products of polynomial and exponential or trigonometric functions.

What is the general formula for integration by parts and how is it applied?

The integration by parts formula is ∫ u dv = uv - ∫ v du. Choose u and dv from the integrand, compute du and v, then substitute into the formula. This method is especially useful for integrals involving products of functions where direct integration is difficult.

Can you provide an example of using substitution to find the antiderivative of ∫ (2x) / (x^2 + 1) dx?

Yes. Let u = x^2 + 1, so du = 2x dx. The integral becomes ∫ du / u, which equals ln|u| + C. Substituting back u = x^2 + 1, the antiderivative is ln|x^2 + 1| + C.

What are the key steps to correctly perform integration by parts on ∫ ln x dx?

Set u = ln x (since its derivative simplifies) and dv = dx (which integrates to x). Then, du = 1/x dx, v = x. Applying integration by parts: ∫ ln x dx = x ln x - ∫ x (1/x) dx = x ln x - ∫ 1 dx = x ln x - x + C.

Additional Resources

Find The Indicated Antiderivative. (a) Using Substitution, Find ∫ x^{1} ^{2} dx (b) Using Integration By Parts

In the realm of calculus, the process of finding antiderivatives—or indefinite integrals—is fundamental to understanding the behavior of functions and solving a variety of real-world problems. The task often involves selecting the appropriate method to evaluate the integral efficiently and accurately. Two such powerful techniques are substitution and integration by parts. This article provides a comprehensive exploration of these methods, demonstrating their application through the specific problems: (a) using substitution to find $\int x^{x^2}\,dx$, and (b) employing integration by parts to evaluate the same integral.

Understanding the Concept of Antiderivatives

Before delving into the methods, it's essential to clarify what an antiderivative is. An antiderivative of a function $f(x)$ is another function $F(x)$ such that $F'(x) = f(x)$. The indefinite integral $\int f(x)\,dx$ represents the set of all antiderivatives of $f(x)$, typically expressed as $F(x) + C$, where C is an arbitrary constant.

Finding antiderivatives is a core component of integral calculus because it enables the computation of areas under curves, accumulation functions, and solutions to differential equations. However, not all integrals are straightforward, prompting the development of various techniques like substitution and integration by parts.

Part (a): Using Substitution to Find $\int x^{x^2}\,dx$

Analyzing the Integral

The integral in question is:

$$
\int x^{x^2} \, dx
$$

At first glance, the integrand x^{x^2} appears complex due to the variable both in the base and the exponent. Traditional methods like direct integration or standard formulas do not suffice here, prompting the need for substitution techniques.

Choosing the Appropriate Substitution

The key to simplifying this integral is to identify a substitution that transforms the variable exponent into a more manageable form. Notice that the exponent is x^2, which suggests setting:

$$
u = x^2
$$

This substitution simplifies the exponent, but we also need to express x^{x^2} in terms of u.

Recall that:

$$
x^{x^2} = e^{x^2 \ln x}
$$

Using our substitution:

$$
u = x^2 \implies du = 2x \, dx \implies dx = \frac{du}{2x}
$$

Now, express the original integral in terms of u:

$$
\int x^{x^2} \, dx = \int e^{x^2 \ln x} \, dx
$$

But since $u = x^2$, the exponential becomes:

$$
e^{u \ln x}
$$

However, $\ln x$ can be expressed in terms of u:

$$
u = x^2 \implies x = \sqrt{u}
$$
$$
\ln x = \frac{1}{2} \ln u
$$

Thus,

$$
e^{u \ln x} = e^{u \cdot \frac{1}{2} \ln u} = e^{\frac{u}{2} \ln u}
$$

This expression is still complex, but further rewriting using properties of exponents and logarithms can help. Recognize that:

$$
e^{\frac{u}{2} \ln u} = u^{u/2}
$$

Therefore, the integral becomes:

$$
\int u^{u/2} \, dx
$$

But recall that:

$$
dx = \frac{du}{2x} = \frac{du}{2 \sqrt{u}}
$$

Putting it all together:

$$
\int u^{u/2} \, dx = \int u^{u/2} \cdot \frac{du}{2 \sqrt{u}} = \frac{1}{2} \int u^{u/2} \cdot u^{-1/2} \, du
$$

Simplify the powers of u:

$$
u^{u/2} \cdot u^{-1/2} = u^{u/2 - 1/2}
$$

Thus, the integral reduces to:

$$
\frac{1}{2} \int u^{u/2 - 1/2} \, du
$$

While this form is more manageable in terms of substitution, it still involves a complex exponent that depends on u. In practical terms, this integral does not have an elementary antiderivative and may require special functions or numerical methods for evaluation.

Concluding Remarks on Substitution

The attempt to evaluate $\int x^{x^2} \, dx$ via substitution reveals the complexity of the integral. Although substitution simplifies certain parts of the integral, the resulting expression involves non-elementary functions. This illustrates an important point: not all integrals are solvable in closed form using elementary functions, and recognizing when to employ substitution is crucial. For this particular integral, the best approach may involve expressing the integral in terms of special functions or resorting to numerical methods.

Part (b): Using Integration By Parts to Find $\int x^{x^2} \, dx$

Understanding Integration By Parts

Integration by parts is a technique derived from the product rule of differentiation:

$$
\int u \, dv = uv - \int v \, du
$$

This method is especially useful when the integrand is a product of functions where one function simplifies upon differentiation, and the other can be easily integrated.

Applying Integration By Parts to the Integral

Given the integral:

$$
I = \int x^{x^2} \, dx
$$

It's not immediately obvious how to split the integrand into parts suitable for integration by parts. Unlike standard products like $x \cdot e^x$, here the integrand is an exponential-like expression with a variable exponent.

Nevertheless, one approach involves rewriting the integral in exponential form:

$$
I = \int e^{x^2 \ln x} \, dx
$$

Suppose we set:

$$
u = x^2 \ln x
$$

Then, the differential du is:

$$
du = 2x \ln x + x^2 \cdot \frac{1}{x} = 2x \ln x + x
$$

Expressed as:

$$
du = x (2 \ln x + 1) \, dx
$$

Rearranged to express dx:

$$
dx = \frac{du}{x (2 \ln x + 1)}
$$

Now, rewrite the integral in terms of u:

$$
I = \int e^{u} \, dx = \int e^{u} \cdot \frac{du}{x (2 \ln x + 1)}
$$

But since $u = x^2 \ln x$, and the denominator involves x and $\ln x$, this substitution leads to a complicated expression involving both x and $\ln x$, not simplifying the integral as intended.

Alternatively, one might attempt to set:

$$
u = x \quad \text{and} \quad dv = x^{x^2} \, dx
$$

but the dv part is the original integral, which doesn't help. Therefore, standard integration by parts isn't directly applicable here unless the integral can be expressed as a product of functions where differentiation and integration are straightforward.

Alternative Approach: Recognizing the Limitation

Given the complex nature of the integrand x^{x^2}, applying integration by parts directly is not straightforward. Both the function and its derivative involve complicated expressions, making the method less practical for an elementary antiderivative.

Summary of Integration by Parts Application

- Integration by parts is most effective when the integrand can be decomposed into u and dv such that du simplifies and v is easily integrable.
- For $\int x^{x^2} \, dx$, the structure of the integrand does not lend itself well to this method.
- The complex variable dependency in the exponent prevents straightforward decomposition required for integration by parts.

This analysis demonstrates that while integration by parts is a versatile tool, its applicability depends on the form of the integrand. In cases like this, recognizing the limits of the method is as important as applying it.

Summary and Final Remarks

The evaluation of integrals involving variable exponents and functions like x^{x^2} exemplifies the challenges encountered in advanced calculus.

Part (a): Substitution

- The substitution $u = x^2$ aimed to simplify the exponent but revealed the integral's inherent complexity.
- Expressing the integrand as $e^{x^2 \ln x}$ and then rewriting in terms of u led to an expression involving $u^{u/2}$, which is not elementary.
- The integral does not have a closed-form solution in terms of elementary functions; instead, it may require special functions or numerical

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