

Find The Indicated Probability.A Machine Has 10 identical Components Which Function Independently. The

Find The Indicated Probability.A Machine Has 10 Identical Components Which Function Independently. The problem of calculating probabilities in systems with multiple components is fundamental in reliability engineering, quality assurance, and risk management. When dealing with machines composed of several independent parts, understanding the likelihood of overall system performance based on individual component probabilities is crucial. This article provides a comprehensive guide to determining the indicated probability for such systems, emphasizing clarity, practical examples, and effective strategies for calculation.

Understanding the Basics of System Reliability and Probability

Before delving into specific probability calculations, it is essential to grasp the core concepts involved in system reliability analysis.

What Is System Reliability?

System reliability refers to the probability that a machine or system performs its intended function without failure for a specified period under normal operating conditions. For systems with multiple components, overall reliability depends on how these components work together.

Components and Their Probabilities

Each component in a machine has an associated probability of functioning correctly, often denoted as ' p '. Conversely, the probability of failure is represented as ' $q = 1 - p$ '. When components are identical, this simplifies the analysis.

Independence of Components

In our scenario, the components function independently, meaning the performance of one component does not influence others. This assumption simplifies probability calculations significantly, allowing the use of basic probability rules.

Calculating the Probability of System Functioning

The core task is to find the probability that the entire system functions correctly, given the individual component probabilities and the system's configuration.

Types of System Configurations

Systems can be configured in different ways, primarily:

1. **Series Configuration:** The system functions only if all components work.
2. **Parallel Configuration:** The system functions if at least one component works.
3. **Mixed Configuration:** A combination of series and parallel arrangements.

Understanding the configuration helps determine the appropriate probability calculations.

Probability Calculations in Different System Configurations

Let's explore how to compute the probability of system success in various configurations involving 10 identical components.

Series System

In a series system, the failure of any component causes system failure.

- Probability that a single component functions: p
- Probability that all 10 components function: p^{10}

Formula:

Probability that the system functions (P_{series}) = p^{10}

Example:

If each component has a 0.9 chance of functioning correctly, then:

$P_{\text{series}} = 0.9^{10} \approx 0.3487$

This indicates a roughly 34.87% chance that the entire machine works under these conditions.

Parallel System

In a parallel system, the system functions if at least one component functions.

- Probability that a component fails: $q = 1 - p$

- Probability all components fail: q^{10}
- Probability that at least one works: $1 - q^{10}$

Formula:

$$P_{\text{parallel}} = 1 - (1 - p)^{10}$$

Example:

If $p = 0.9$, then $q = 0.1$

$$P_{\text{parallel}} = 1 - 0.1^{10} \approx 1 - 1e-10 \approx 0.9999999999$$

This demonstrates a very high probability of system success in a parallel configuration when component reliability is high.

Mixed Systems

Many real-world systems have a combination of series and parallel arrangements. Calculating probabilities in these systems involves breaking down the system into sub-systems and applying the rules recursively.

Steps to Find the Indicated Probability in Complex Systems

When dealing with complex configurations, follow these systematic steps:

1. **Identify the system configuration:** Determine which components are in series, parallel, or a combination.
2. **Break down the system into sub-systems:** Simplify the analysis by considering smaller sections.
3. **Calculate the reliability of each sub-system:** Use the formulas for series and parallel arrangements.
4. **Combine sub-system reliabilities:** Use the rules for combining systems to find the overall probability.
5. **Interpret the result:** Understand what the probability signifies in terms of system performance.

Practical Examples of Probability Calculations

Let's examine a practical example involving a machine with 10 identical components arranged in a mixed configuration.

Example Scenario

Suppose the machine consists of:

- Components 1-4 in series
- Components 5-7 in parallel
- Components 8-10 in series

The overall system functions if:

- The series group 1-4 functions
- The parallel group 5-7 functions
- The series group 8-10 functions

and all these groups operate successfully.

Calculating Each Sub-System

- **Series group 1-4:**

$$P = p^4$$

- **Parallel group 5-7:**

$$P = 1 - (1 - p)^3$$

- **Series group 8-10:**

$$P = p^3$$

Suppose each component has a 0.95 probability of functioning correctly ($p=0.95$).

Calculations:

- Series group 1-4: $0.95^4 \approx 0.8145$
- Parallel group 5-7: $1 - (0.05)^3 = 1 - 0.000125 = 0.999875$
- Series group 8-10: $0.95^3 \approx 0.8574$

Overall system probability:

$$P_{\text{total}} = 0.8145 \cdot 0.999875 \cdot 0.8574 \approx 0.698$$

This indicates approximately a 69.8% chance that the entire machine functions as intended.

Implications and Applications of Probability Calculations

Understanding the probability of system success has multiple applications:

1. **Reliability Engineering:** Designing systems with desired reliability thresholds.
2. **Maintenance Planning:** Identifying components that significantly impact overall reliability for targeted maintenance.
3. **Risk Assessment:** Quantifying the likelihood of system failures to inform safety protocols.
4. **Quality Control:** Ensuring components meet reliability standards to achieve system-level goals.

These calculations enable engineers and managers to make informed decisions about system design, maintenance schedules, and operational strategies.

Advanced Topics in Probability and Reliability

For more complex systems, additional factors may influence calculations:

Dependent Components

In real-world systems, components may not operate independently, leading to correlated failures. Such cases require advanced probabilistic models like Bayesian networks or Markov chains.

Time-Dependent Reliability

Component reliability can decline over time, and lifetime distributions (e.g., exponential, Weibull) may be used for dynamic probability assessments.

Redundancy and Fault Tolerance

Designing systems with redundancy can significantly improve overall reliability, and calculations involve considering replacement mechanisms and failover strategies.

Conclusion

Finding the indicated probability of a machine with 10 identical components functioning independently hinges on understanding the system's configuration and applying fundamental probability principles. Whether components are arranged in series, parallel, or a combination, the key lies in breaking down the system, calculating sub-system reliabilities, and combining these results systematically.

By mastering these concepts, engineers and analysts can effectively evaluate system reliability, enhance design robustness, and optimize maintenance and safety protocols. Accurate probability calculations serve as a foundation for resilient system design and risk management, ultimately leading to improved operational efficiency and safety.

Remember: The core formulas— p^n for series, $1 - (1 - p)^n$ for parallel—are invaluable tools in reliability analysis, and understanding how to apply them in various configurations is essential for effective system assessment.

Frequently Asked Questions

What is the probability that all 10 components in the machine function correctly?

Assuming each component functions independently with a probability p , the probability that all 10 components function correctly is p^{10} .

If each component has a 0.9 probability of functioning, what is the probability that exactly 8 components work?

Using the binomial probability formula: $P = C(10,8) (0.9)^8 (0.1)^2$, which calculates to approximately 0.193.

How do you find the probability that at most 2 components fail?

Calculate the probability that 8, 9, or 10 components function correctly: $P = C(10,8)(p)^8(1-p)^2 + C(10,9)p^9(1-p) + p^{10}$.

If the probability of a component failing is 0.05, what is the probability that exactly 2 components fail?

Using the binomial formula: $P = C(10,2)(0.95)^8(0.05)^2$, which equals approximately 0.150.

What is the probability that at least 9 components function

correctly?

Sum the probabilities of exactly 9 and 10 components working: $P = C(10,9)p^9(1-p) + p^{10}$.

If the components are 100% reliable, what is the probability that the machine functions?

The probability is 1, since all components are guaranteed to work independently.

How would you compute the probability that the machine fails if failure occurs when at least one component fails?

Calculate the complement of all components functioning: $1 - p^{10}$, where p is the probability each component works.

What is the expected number of components that function correctly in the machine?

The expected value is $10p$, where p is the probability each component functions independently.

If two components are known to be dependent, how does that affect calculating the probability of all components functioning?

Dependence complicates the calculation, requiring joint probability distributions; the simple product rule for independent events no longer applies.

Additional Resources

Find The Indicated Probability. A machine has 10 identical components which function independently.

Understanding how to calculate the probability of specific outcomes in systems composed of multiple independent components is a fundamental skill in probability theory and quality engineering. Whether you're designing reliable machinery, troubleshooting, or conducting risk assessments, mastering the process of finding the indicated probability for such systems enables more informed decision-making. In this article, we will explore the principles behind calculating these probabilities, walk through detailed examples, and provide practical tips to approach similar problems effectively.

Introduction to System Reliability and Component Independence

In many real-world applications, complex machines or systems are built from multiple components. Each component typically performs a specific function, and the overall system's performance depends on the functioning of these individual parts. When components are independent—meaning the functioning or failure of one component does not influence the others—the probability calculations

become more straightforward, but they still require careful analysis.

In our specific scenario, we are considering a machine with 10 identical components, each functioning independently. Our goal is often to determine the probability that the system as a whole meets certain criteria, such as:

- All components are working
- Exactly a certain number of components are working
- At least a certain number of components are functioning

Understanding these probabilities helps in assessing system reliability, planning maintenance schedules, and designing redundancy strategies.

Fundamental Concepts and Terminology

Before diving into calculations, let's clarify some key concepts and terminology:

- Component Reliability (p): The probability that a single component functions correctly during a specified period.
- Component Failure Probability (q): The probability that a component fails, which is $1 - p$.
- System Reliability: The probability that the entire system functions as intended.
- Independent Components: The functioning of one component does not affect the functioning of others.

Modeling the System: Binomial Distribution

When dealing with multiple independent and identical components, the binomial distribution becomes a powerful tool. It models the number of successes (e.g., functioning components) in a fixed number of independent Bernoulli trials (each component's operation), each with the same probability of success.

The probability mass function (PMF) of the binomial distribution is:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

where:

- n = total number of components (here, 10)
- k = number of components functioning successfully
- p = probability that an individual component functions
- $\binom{n}{k}$ = binomial coefficient, "n choose k"

Note: The binomial distribution assumes independence and identical probability across components, fitting our scenario.

Step-by-Step Approach to Find the Indicated Probability

Let's outline the general process:

1. Identify the event or condition: What specific probability are we asked to find? For example, "Probability that at least 8 components function," or "Probability that exactly 5 components function."
2. Determine the relevant parameters: Values of (n) , (p) , and the specific (k) values involved.
3. Use the binomial formula: Calculate the probability for each relevant (k) value.
4. Sum probabilities if needed: For "at least" or "at most" types of questions, sum over the appropriate range of (k) .

Practical Examples and Calculations

Let's explore some typical problems with solutions, assuming a component success probability $(p = 0.9)$.

Example 1: Probability that all components function

Question: What is the probability that all 10 components are functioning?

Solution:

Since each component functions independently with probability $(p = 0.9)$:

$$P(\text{all 10 work}) = p^{10} = (0.9)^{10}$$

Calculating:

$$(0.9)^{10} \approx 0.3487$$

Interpretation: There's approximately a 34.87% chance that the entire machine operates without any component failure.

Example 2: Probability that at least 8 components function

Question: What's the probability that 8 or more components are functioning?

Solution:

Calculate:

$$P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$$

Using the binomial PMF:

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\begin{aligned}
P(X=8) &= \binom{10}{8} (0.9)^8 (0.1)^2 \\
P(X=9) &= \binom{10}{9} (0.9)^9 (0.1)^1 \\
P(X=10) &= \binom{10}{10} (0.9)^{10} (0.1)^0
\end{aligned}

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Calculations:

$$- \binom{10}{8} = 45$$

$$P(X=8) = 45 \times (0.9)^8 \times (0.1)^2 \approx 45 \times 0.4305 \times 0.01 \approx 0.1937$$

$$- \binom{10}{9} = 10$$

$$P(X=9) = 10 \times (0.9)^9 \times 0.1 \approx 10 \times 0.3874 \times 0.1 \approx 0.3874$$

$$- \binom{10}{10} = 1$$

$$P(X=10) = 1 \times (0.9)^{10} \times 1 \approx 0.3487$$

Adding these:

$$P(X \geq 8) \approx 0.1937 + 0.3874 + 0.3487 = 0.9298$$

Interpretation: There's approximately a 92.98% chance that at least 8 components are functioning, indicating high system reliability.

Example 3: Probability that fewer than 5 components function

Question: What's the probability that fewer than 5 components are functioning?

Solution:

Calculate:

$$P(X < 5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

Compute each:

$$- \ P(X=k) = \binom{10}{k} (0.9)^k (0.1)^{10-k}$$

For brevity, calculations:

$$- \ P(X=0) = \binom{10}{0} (0.9)^0 (0.1)^{10} = 1 \times 1 \times 10^{-10} \approx 1 \times 10^{-10}$$

$$- \ P(X=1) = 10 (0.9)^1 (0.1)^9 \approx 10 \times 0.9 \times 10^{-9} \approx 9 \times 10^{-9}$$

$$- \ P(X=2) = 45 (0.9)^2 (0.1)^8 \approx 45 \times 0.81 \times 10^{-8} \approx 36.45 \times 10^{-8}$$

Similarly, these probabilities are very small, indicating that the chance of fewer than 5 components functioning is negligible with high p .

General Tips for Solving These Problems

- Identify the correct probability event: Are you calculating the probability of exactly k components functioning, at least k , or fewer than k ?
- Use binomial coefficients accurately: Remember that $\binom{n}{k}$ can be calculated via factorials or binomial coefficient tables.
- Leverage technology: Use scientific calculators, spreadsheet software, or statistical packages to compute binomial probabilities efficiently.
- Check your assumptions: Ensure that components are indeed independent and identical, as the binomial model relies on these assumptions.
- Interpret results in context: Think about what the probability implies for system reliability and maintenance planning.

Advanced Topics and Variations

Beyond basic calculations, more sophisticated questions may involve:

- Conditional probabilities: What is the probability the system functions given certain component failures?
- Reliability functions: Modeling how the probability of functioning changes over time.
- Redundancy strategies: Designing systems that maintain high reliability even with component failures.

Conclusion

Finding the indicated probability in systems with multiple independent components requires a solid understanding of binomial probability principles. By identifying the event of interest, applying the binomial formula, and summing over relevant outcomes, you can accurately assess system reliability under various scenarios. This approach not only enhances your analytical skills but also supports

effective engineering design, maintenance, and risk management.

Remember, the key is to clearly define what you are calculating, understand the assumptions involved, and utilize the binomial distribution as a reliable tool for these types of probability assessments. With practice, these calculations become intuitive, empowering you to handle complex reliability problems with confidence.

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