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Calculating the length of a curve is a fundamental concept in calculus that finds extensive application across physics, engineering, and computer graphics. When dealing with parametric curves like R(t) = (cos(4t), Sin(4t), 4t), understanding how to determine the arc length over a specific interval (in this case, from t = 5 to t = 7) is essential. This article provides a comprehensive guide to calculating the exact length of this particular curve, breaking down each step in detail, so you can grasp the underlying principles and apply them to similar problems.

Understanding the Problem: Curve and Interval

Before delving into calculations, it's crucial to understand the components of the problem:

- The parametric curve is given by R(t) = (cos(4t), Sin(4t), 4t).
- The interval for t is from 5 to 7.
- The goal is to find the length of this curve over the specified interval.
- The answer should be rounded to two decimal places.

Fundamentals of Arc Length Calculation

What Is Arc Length?

Arc length refers to the distance along a curve between two points. For a parametric curve R(t) = (x(t), y(t), z(t)), the length L from t = a to t = b is given by the integral:

\[L = \int_{a}^{b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt \]

This formula sums the infinitesimal distances along the curve, computed via the derivatives of each component.

Why Is It Important?

Understanding the arc length formula allows engineers and scientists to quantify the length of complex paths, such as the trajectory of a moving object or the shape of a wire in a 3D space.

Step-by-Step Solution to the Given Curve

Let's now proceed to compute the length of the curve R(t) = (cos(4t), Sin(4t), 4t) over t from 5 to 7.

Step 1: Find the derivatives of each component

Given:

- $x(t) = \cos(4t)$
- $y(t) = \sin(4t)$
- $z(t) = 4t$

Differentiate each with respect to t:

- $\frac{dx}{dt} = -4 \sin(4t)$
- $\frac{dy}{dt} = 4 \cos(4t)$
- $\frac{dz}{dt} = 4$

Step 2: Compute the integrand $\sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2}$

Calculate each squared derivative:

- $(dx/dt)^2 = 16 \sin^2(4t)$
- $(dy/dt)^2 = 16 \cos^2(4t)$
- $(dz/dt)^2 = 16$

Sum these:

$$
(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2 = 16 \sin^2(4t) + 16 \cos^2(4t) + 16
$$

\]

Recall that:

\[
\sin^2(4t) + \cos^2(4t) = 1
\]

Therefore:

\[
16 \times 1 + 16 = 32
\]

The integrand simplifies to:

\[
\sqrt{32} = \sqrt{32} = 4 \sqrt{2}
\]

This is a constant, which simplifies the integral significantly.

Step 3: Set up the integral for arc length

Since the integrand is constant:

\[
L = \int_{5}^{7} 4 \sqrt{2} \, dt
\]

which simplifies to:

\[
L = 4 \sqrt{2} \times (7 - 5) = 4 \sqrt{2} \times 2
\]

Calculating the Final Answer

Now, multiply the constants:

\[
L = 8 \sqrt{2}
\]

Numerically, since \(\sqrt{2} \approx 1.4142\):

$$
L \approx 8 \times 1.4142 = 11.3136
$$

Rounding to two decimal places:

$$
\boxed{11.31}
$$

Summary of Key Points

- The derivatives of the parametric functions are essential for computing the arc length.
- The sum of squared derivatives simplifies thanks to the Pythagorean identity.
- The constant integrand makes the calculation straightforward.
- The final arc length over t from 5 to 7 is approximately 11.31 units.

Additional Insights and Applications

Understanding how to compute the length of a space curve has numerous practical applications:

- Physics: Calculating the path length of particles moving along complex trajectories.
- Engineering: Designing cables, wires, or tracks that follow specific paths.
- Computer Graphics: Rendering smooth curves and animations.
- Robotics: Planning movement paths in 3D space for robotic arms.

Moreover, the approach used here for a relatively simple curve can be extended to more complex parametric functions, often requiring numerical methods when integrals become too complicated for analytical solutions.

Conclusion

The process of finding the length of the parametric curve R(t) = (cos(4t), Sin(4t), 4t) over the interval from t = 5 to t = 7 reveals the power and elegance of calculus. By differentiating each component, simplifying the sum of squares, and integrating over the interval, we obtain an exact length of approximately 11.31 units. Mastering such techniques enhances your ability to analyze curves in multi-dimensional space, with applications spanning numerous scientific and engineering disciplines.

FAQs

1. **Can the arc length formula be used for any parametric curve?**
   Yes, the arc length formula applies to any smooth parametric curve, though the integral may be more complex and sometimes require numerical methods.

2. **What if the derivatives are not constant?**
   In such cases, the integrand varies with t, and you need to evaluate the integral either analytically or numerically.

3. **How do I round the final answer?**
   Round to the specified decimal places, in this case, two decimal places, for clarity and precision.

By understanding and applying these steps, you can confidently calculate the lengths of intricate curves in three-dimensional space, enhancing your mathematical toolkit for various scientific and engineering applications.

Frequently Asked Questions

What is the formula to find the length of a space curve R(t) = (x(t), y(t), z(t)) from t = a to t = b?

The length L is given by the integral $L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2 + (dz/dt)^2}\, dt$.

How do you compute the derivatives for R(t) = (cos(4t), sin(4t), 4t)?

dx/dt = -4 sin(4t), dy/dt = 4 cos(4t), dz/dt = 4.

What is the integrand when calculating the arc length of R(t) = (cos(4t), sin(4t), 4t) from t=5 to t=7?

The integrand is $\sqrt{(-4 \sin(4t))^2 + (4 \cos(4t))^2 + 4^2} = \sqrt{16 \sin^2(4t) + 16 \cos^2(4t) + 16} = \sqrt{16 (\sin^2(4t) + \cos^2(4t)) + 16} = \sqrt{16\ 1 + 16} = \sqrt{32}$.

What is the simplified form of the integrand for the given curve?

The integrand simplifies to $\sqrt{32}$, which equals $4\sqrt{2}$.

How do you evaluate the total length of the curve from t=5 to t=7?

Since the integrand is constant ($4\sqrt{2}$), the length is $(4\sqrt{2})\ (7 - 5) = 2\ 4\sqrt{2} = 8\sqrt{2}$.

What is the final numerical answer for the length of R(t) from t=5 to t=7, rounded to two decimals?

The length is approximately $8\ 1.4142 \approx 11.31$.

Additional Resources

Finding the Length of a Space Curve: A Comprehensive Guide to R(t) = (cos(4t), sin(4t), 4t) for t in [5, 7]

Introduction

Calculus offers a powerful toolkit for understanding and analyzing curves in space. One of the fundamental concepts is the arc length of a curve, which measures the distance traveled along a path between two points. In this detailed exploration, we focus on calculating the length of a specific parametric curve:

$$
\mathbf{R}(t) = (\cos(4t), \sin(4t), 4t)
$$

for t in the interval $[5, 7]$. The goal is to derive the formula, perform the necessary calculations, and provide the final answer rounded to two decimal places.

Understanding the Curve $\mathbf{R}(t)$

Before diving into calculations, it's essential to understand the nature of the curve:

- The x and y components, $\cos(4t)$ and $\sin(4t)$, trace a circle in the xy-plane with radius 1, but with a rapidly increasing angle due to the factor $4t$.
- The z-component, $4t$, indicates a linear increase along the z-axis, creating a helical or spiral structure.

This combination results in a helix that wraps around the z-axis, with the circle completing multiple revolutions as t increases.

Step 1: Recall the Arc Length Formula for a Parametric Curve

For a vector function $\mathbf{R}(t) = (x(t), y(t), z(t))$, the arc length L between $t = a$ and $t = b$ is:

$$
L = \int_a^b |\mathbf{R}'(t)| \, dt
$$

where $\mathbf{R}'(t)$ is the derivative of $\mathbf{R}(t)$, and $|\mathbf{R}'(t)|$ is its magnitude:

$$
|\mathbf{R}'(t)| = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}
$$

Step 2: Compute the Derivatives $\mathbf{R}'(t)$

Given:

$$
x(t) = \cos(4t), \quad y(t) = \sin(4t), \quad z(t) = 4t
$$

Calculate each derivative:

- $\frac{dx}{dt} = -4 \sin(4t)$
- $\frac{dy}{dt} = 4 \cos(4t)$
- $\frac{dz}{dt} = 4$

Step 3: Find the Magnitude $|\mathbf{R}'(t)|$

Compute:

$$
|\mathbf{R}'(t)| = \sqrt{(-4 \sin(4t))^2 + (4 \cos(4t))^2 + 4^2}
$$

Simplify each term:

- $(-4 \sin(4t))^2 = 16 \sin^2(4t)$
- $(4 \cos(4t))^2 = 16 \cos^2(4t)$
- $4^2 = 16$

Add them:

$$
|\mathbf{R}'(t)| = \sqrt{16 \sin^2(4t) + 16 \cos^2(4t) + 16}
$$

Recall that $\sin^2 \theta + \cos^2 \theta = 1$:

$$
|\mathbf{R}'(t)| = \sqrt{16 \times 1 + 16} = \sqrt{16 + 16} = \sqrt{32}
$$

Expressed simply:

$$
|\mathbf{R}'(t)| = \sqrt{32} = 4 \sqrt{2}
$$

Step 4: Set up the Arc Length Integral

Since $|\mathbf{R}'(t)|$ is constant over t, the integral simplifies significantly:

$$
L = \int_{t=5}^{t=7} |\mathbf{R}'(t)| \, dt = \int_5^7 4 \sqrt{2} \, dt
$$

This becomes:

$$
L = 4 \sqrt{2} \times (7 - 5) = 4 \sqrt{2} \times 2
$$

Step 5: Final Calculation and Rounding

Calculate the numerical value:

$$
L = 8 \sqrt{2}
$$

Since $\sqrt{2} \approx 1.4142$:

$$
L \approx 8 \times 1.4142 = 11.3136
$$

Rounding to two decimal places:

$$
\boxed{11.31}
$$

Summary of Findings

- The derivative magnitude is constant, simplifying the integral.
- The total length of the curve from $t=5$ to $t=7$ is approximately 11.31 units.

Additional Insights

Why is the derivative magnitude constant?

- The derivatives of the x and y components depend on sine and cosine functions whose squares sum to 1.
- The z-component's derivative is constant.
- The combination of these factors results in a constant speed along the curve.

Geometric interpretation:

- The curve traces a uniform helix with a constant speed, making the arc length calculation straightforward.
- The total length over the interval depends solely on the magnitude of the

derivative and the parameter interval.

Practical Applications of Arc Length Calculations

Understanding how to compute the length of a curve like $\mathbf{R}(t)$ is fundamental in multiple fields:

- Physics: Calculating the distance traveled along a path in space.
- Engineering: Designing cables, wiring, or paths with specified lengths.
- Computer Graphics: Rendering curves and paths with accurate lengths.
- Mathematics: Analyzing properties of space curves, such as curvature and torsion.

Conclusion

Calculating the length of a space curve is a critical skill in advanced calculus, combining derivatives, integrals, and geometric intuition. For the specific curve $\mathbf{R}(t) = (\cos(4t), \sin(4t), 4t)$, the constant magnitude of the derivative simplifies the process greatly. The resulting length from $t=5$ to $t=7$ is approximately 11.31 units, providing a precise measure of the distance traveled along the spiral path during this interval.

This detailed analysis not only offers the final answer but also deepens the understanding of how derivatives relate to arc length, the geometric nature of the curve, and the elegance of calculus in solving real-world problems involving space curves.

Note: Always verify the derivatives and integrals carefully, especially when dealing with more complex or non-constant derivatives, to ensure accurate calculations.

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