

Find The Lengths Of The Sides Of The Triangle PQR. (a) $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$ (b) $P(1, -2,$

Find The Lengths Of The Sides Of The Triangle PQR. (a) $P(3, -2, -3)$, $Q(7, 0, 1)$, $R(1, 2, 1)$ (b) $P(1, -2,$ This comprehensive guide explores how to determine the lengths of the sides of a triangle in three-dimensional space, specifically focusing on triangle PQR with given coordinates. Understanding how to compute these lengths is fundamental in coordinate geometry, which has applications across numerous fields such as engineering, physics, computer graphics, and more. Whether you're a student preparing for exams or a professional working on spatial computations, mastering the distance formula in 3D space is essential.

In this article, we will delve into the step-by-step process of calculating the side lengths of triangle PQR, analyze the significance of these measurements, and provide detailed explanations to ensure clarity. The discussion is structured into sections covering the basics of 3D distance calculations, applying the distance formula to specific points, and exploring related concepts such as triangle properties and applications.

Understanding the Basics of Coordinate Geometry in 3D Space

Before we proceed to calculations, it's crucial to understand the foundational concepts involved in coordinate geometry, especially in three dimensions.

What Is Coordinate Geometry?

Coordinate geometry, also known as analytic geometry, is a branch of mathematics that uses algebraic equations to represent geometric shapes and analyze their properties. In 3D space, points are represented using triples of coordinates (x, y, z) .

Coordinates of Points in Space

Any point in three-dimensional space is specified by its coordinates:

- Point P: (x_1, y_1, z_1)
- Point Q: (x_2, y_2, z_2)
- Point R: (x_3, y_3, z_3)

The positions of these points determine the shape and size of the triangle formed.

Calculating the Lengths of Triangle Sides in 3D

The key to finding the lengths of the sides of a triangle in 3D space is the 3D distance formula, an extension of the Pythagorean theorem.

The 3D Distance Formula

Given two points, $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$, the distance between them, denoted as $d(P, Q)$, is:

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

This formula calculates the straight-line (Euclidean) distance in three-dimensional space.

Applying the Distance Formula to Triangle PQR

Let's now analyze the specific points of triangle PQR provided in the problem.

Part (a): Coordinates of P, Q, and R

- $P(3, -2, -3)$
- $Q(7, 0, 1)$
- $R(1, 2, 1)$

Our goal is to find the lengths of sides PQ, QR, and PR.

Calculating Side PQ

Using the distance formula:

$$PQ = \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2}$$

Plugging in the values:

$$PQ = \sqrt{(7 - 3)^2 + (0 - (-2))^2 + (1 - (-3))^2}$$

Simplify each term:

$$\begin{aligned} & - \sqrt{(7 - 3)^2} = 4 \\ & - \sqrt{(0 - (-2))^2} = 2 \\ & - \sqrt{(1 - (-3))^2} = 4 \end{aligned}$$

Now, compute:

$$PQ = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$$

Result: The length of side PQ is 6 units.

Calculating Side QR

Similarly,

$$QR = \sqrt{(x_R - x_Q)^2 + (y_R - y_Q)^2 + (z_R - z_Q)^2}$$

Plugging in:

$$QR = \sqrt{(1 - 7)^2 + (2 - 0)^2 + (1 - 1)^2}$$

Simplify:

$$\begin{aligned} & - \sqrt{(1 - 7)^2} = 6 \\ & - \sqrt{(2 - 0)^2} = 2 \\ & - \sqrt{(1 - 1)^2} = 0 \end{aligned}$$

Compute:

$$QR = \sqrt{(-6)^2 + 2^2 + 0^2} = \sqrt{36 + 4 + 0} = \sqrt{40} \approx 6.3246$$

Result: The length of side QR is approximately 6.32 units.

Calculating Side PR

$$PR = \sqrt{(x_R - x_P)^2 + (y_R - y_P)^2 + (z_R - z_P)^2}$$

Plug in:

$$PR = \sqrt{(1 - 3)^2 + (2 - (-2))^2 + (1 - (-3))^2}$$

Simplify:

$$\begin{aligned} & - \sqrt{(1 - 3)^2} = -2 \\ & - \sqrt{(2 - (-2))^2} = 4 \\ & - \sqrt{(1 - (-3))^2} = 4 \end{aligned}$$

Calculate:

$$PR = \sqrt{(-2)^2 + 4^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$$

Result: The length of side PR is 6 units.

Summary of Part (a): Triangle PQR Side Lengths

Side	Length in Units	Approximate Value
PQ	6	6
QR	$\sqrt{40}$	6.32
PR	6	6

This calculation shows that triangle PQR has two sides of equal length (PQ and PR), indicating that it could be an isosceles triangle.

Part (b): Coordinates of P in the second scenario

The problem mentions an "optimized for SEO" approach, but in the context of coordinate geometry, Part (b) likely involves similar calculations with different points or conditions. Suppose the points are:

- $P(1, -2, z)$, where z is to be determined or optimized.

Without explicit data, the general approach involves:

1. Identifying the known coordinates.
2. Applying the distance formula to find side lengths.
3. Analyzing the properties or optimizing the triangle's configuration based on specific criteria (e.g., minimizing or maximizing side lengths, area, or angles).

Additional Concepts Related to Triangle Side Lengths in 3D Space

Understanding how to compute side lengths is just the beginning. Several other concepts are essential for a comprehensive grasp of triangles in three dimensions.

1. Triangle Inequality Theorem in 3D

This fundamental theorem states that, for any triangle, the sum of the lengths of any two sides must be greater than the length of the remaining side. In 3D space, this theorem applies similarly:

- $(PQ + QR > PR)$
- $(PQ + PR > QR)$
- $(QR + PR > PQ)$

Verifying these inequalities confirms the three points form a valid triangle.

2. Properties of Triangles in 3D

- Equilateral Triangle: All sides are equal.
- Isosceles Triangle: Two sides are equal.
- Scalene Triangle: All sides are different.
- Right Triangle: One angle is 90° , which can be checked using the dot product of vectors.

3. Using Vectors to Determine Triangle Properties

Vectors are powerful tools in 3D geometry:

- Side vectors: $(\vec{PQ} = \vec{Q} - \vec{P})$, etc.
- Dot product: Used to determine if an angle is right.

For example, to check if triangle PQR is right-angled at Q:

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\[
\text{If } \vec{PQ} \cdot \vec{QR} = 0, \text{ then angle at Q is } 90^\circ
\]
```

Practical Applications of Finding Triangle Side Lengths in 3D

Understanding how to compute side lengths in 3D space has numerous real-world applications:

- Engineering: Designing structures with precise measurements.
- Computer Graphics: Rendering 3D models accurately.
- Navigation: Calculating distances between points in space (e.g., GPS, aerospace).
- Robotics: Determining the positions and movements of robotic arms.
- Physics: Calculating distances between particles or celestial bodies.

Summary and Key Takeaways

- The 3D distance formula extends the Pythagorean theorem to three dimensions.
- To find the lengths of sides in triangle PQR, substitute the coordinates

into the formula.

- Calculations

Frequently Asked Questions

How do you find the length of side PQ in triangle PQR with points P(3, -2, -3) and Q(7, 0, 1)?

Use the distance formula for 3D points: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. For PQ: $\sqrt{(7 - 3)^2 + (0 + 2)^2 + (1 + 3)^2} = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6$.

What is the length of side PR in triangle PQR with points P(3, -2, -3) and R(1, 2, 1)?

Calculate using the distance formula: $\sqrt{(1 - 3)^2 + (2 + 2)^2 + (1 + 3)^2} = \sqrt{(-2)^2 + 4^2 + 4^2} = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$.

How do you determine the length of side QR in triangle PQR with points Q(7, 0, 1) and R(1, 2, 1)?

Apply the distance formula: $\sqrt{(1 - 7)^2 + (2 - 0)^2 + (1 - 1)^2} = \sqrt{(-6)^2 + 2^2 + 0^2} = \sqrt{36 + 4 + 0} = \sqrt{40} \approx 6.32$.

What are the side lengths of triangle PQR with points P(3, -2, -3), Q(7, 0, 1), R(1, 2, 1)?

Side PQ = 6, PR = 6, QR $\approx$ 6.32.

How do you find the length of a side in 3D space given two points?

Use the 3D distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Can you explain the step-by-step process to find the length of side PQ in triangle PQR?

Yes. First, identify the coordinates of P and Q. Then, subtract the corresponding coordinates: (7 - 3), (0 + 2), (1 + 3). Square each difference, sum them up, and take the square root: $\sqrt{4^2 + 2^2 + 4^2} = \sqrt{36} = 6$.

What is the significance of calculating side lengths

in a triangle in 3D space?

Calculating side lengths helps in determining the triangle's type, area, perimeter, and in solving geometric and spatial problems involving 3D figures.

If given only two points in 3D, how can you find the distance between them?

Apply the 3D distance formula: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$, substituting the coordinates of the two points.

What is the approximate length of side QR in triangle PQR with points Q(7, 0, 1) and R(1, 2, 1)?

Using the distance formula, $QR \approx \sqrt{40} \approx 6.32$ units.

What additional information is needed to find the length of a side if only partial data is given?

You need the coordinates of the endpoints of the side in 3D space to apply the distance formula and find the length.

Additional Resources

Triangle Side Lengths Calculation: A Comprehensive Guide

Understanding the lengths of the sides of a triangle given its vertices in a three-dimensional space is a fundamental skill in geometry, with applications spanning from engineering and architecture to computer graphics and physics. In this article, we delve deeply into the process of finding these side lengths, exemplified through two detailed problems involving points in 3D space.

Introduction to Distance Calculation in 3D Space

Before diving into specific problems, it's essential to grasp the underlying principle used to determine the length of a side in three-dimensional coordinates. The distance between two points $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ in 3D space is given by the 3D distance formula:

$\sqrt{$

$$d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

This formula extends the familiar Pythagorean theorem into three dimensions, accounting for differences along each axis.

Part A: Finding the Lengths of Sides of Triangle PQR with Coordinates P(3, -2, -3), Q(7, 0, 1), R(1, 2, 1)

Step 1: Listing the Coordinates

Let's first clearly identify the given points:

- \(\ P(3, -2, -3)\ \)
- \(\ Q(7, 0, 1)\ \)
- \(\ R(1, 2, 1)\ \)

Step 2: Calculating Side PQ

Applying the distance formula:

$$\begin{aligned} & \sqrt{(x_Q - x_P)^2 + (y_Q - y_P)^2 + (z_Q - z_P)^2} \\ & \sqrt{(7 - 3)^2 + (0 - (-2))^2 + (1 - (-3))^2} \\ & \sqrt{(4)^2 + (2)^2 + (4)^2} \\ & \sqrt{16 + 4 + 16} = \sqrt{36} = 6 \end{aligned}$$

Step 3: Calculating Side QR

Similarly:

$$\begin{aligned} & \sqrt{(x_R - x_Q)^2 + (y_R - y_Q)^2 + (z_R - z_Q)^2} \\ & \sqrt{(1 - 7)^2 + (2 - 0)^2 + (1 - 1)^2} \end{aligned}$$

```
\[
QR = \sqrt{(-6)^2 + (2)^2 + (0)^2}
\]
\[
QR = \sqrt{36 + 4 + 0} = \sqrt{40} \approx 6.32
\]
```

Step 4: Calculating Side PR

Finally:

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\[
PR = \sqrt{(x_R - x_P)^2 + (y_R - y_P)^2 + (z_R - z_P)^2}
\]
\[
PR = \sqrt{(1 - 3)^2 + (2 - (-2))^2 + (1 - (-3))^2}
\]
\[
PR = \sqrt{(-2)^2 + (4)^2 + (4)^2}
\]
\[
PR = \sqrt{4 + 16 + 16} = \sqrt{36} = 6
\]
```

Summary of Side Lengths in Triangle PQR:

Side	Length
PQ	6
QR	approximately 6.32
PR	6

This precise calculation reveals that triangle PQR is nearly equilateral, with two sides of length 6 and one slightly longer side approximately 6.32 units.

Part B: Finding the Lengths of Sides of Triangle P with Coordinates P(1, -2,)

Step 1: Clarifying the Coordinates

The problem provides:

- $P(1, -2,)$

The double asterisks indicate missing data, which is common in real-world scenarios where data might be incomplete. To proceed, we need to assume or clarify the missing coordinate, which might be a typographical error or an intentional placeholder.

Assumption: For the purpose of this analysis, let's assume the missing coordinate is $z = 4$. This is a reasonable assumption to illustrate the process.

Thus, the point P becomes:

- $P(1, -2, 4)$

If you have the actual third coordinate, replace the assumed value accordingly.

Step 2: Understanding the Problem Context

Since only one point is provided here, and the problem asks for side lengths, it suggests that perhaps the second point Q and third point R are missing or the problem intends to demonstrate a method for similar calculations with incomplete data.

In the case of incomplete data, the procedure remains the same:

- Obtain complete coordinate data for all relevant points.
- Use the 3D distance formula to calculate each side.

Step 3: Hypothetical Extension

Suppose you have another point $Q(3, 0, 2)$ and $R(5, -1, 3)$, then:

- The length of PQ
- The length of PR
- The length of QR

for illustration, would be calculated similarly:

$$\begin{aligned} & \left[\right. \\ PQ &= \sqrt{(3 - 1)^2 + (0 - (-2))^2 + (2 - 4)^2} \\ & \left. \right] \\ & \left[\right. \\ PQ &= \sqrt{(2)^2 + (2)^2 + (-2)^2} = \sqrt{4 + 4 + 4} = \sqrt{12} \approx 3.46 \\ & \left. \right] \end{aligned}$$

and so on for other sides.

Additional Considerations and Applications

Practical Relevance of Side Lengths in 3D Geometry

- **Structural Engineering:** Precise measurements of side lengths are critical for designing stable frameworks and ensuring structural integrity.
- **Computer Graphics:** Calculating distances in 3D space enables rendering realistic models and animations.
- **Physics:** Distance calculations underpin motion analysis, collision detection, and spatial reasoning.
- **Navigation and Robotics:** Determining exact positions and distances allows for precise movement and object interaction in three-dimensional environments.

Calculating Other Triangle Properties

Once side lengths are known, you can further explore:

- **Perimeter:** Sum of all side lengths.

- Area: Using Heron's formula adapted for 3D or vector cross products.
- Type of Triangle: Equilateral, isosceles, or scalene based on side lengths.

Conclusion: Mastering 3D Distance Calculations for Triangles

Calculating the lengths of the sides of a triangle in three-dimensional space hinges on a straightforward yet powerful formula—the distance formula. By carefully applying this formula to each pair of vertices, you can accurately determine side lengths, which then serve as a foundation for further geometric analysis.

In the example involving points $P(3, -2, -3)$, $Q(7, 0, 1)$, and $R(1, 2, 1)$, we illustrated the process step-by-step, emphasizing precision and clarity. The process remains similar across different problems, with adjustments made for incomplete data or varying coordinate values.

Understanding these fundamental calculations enhances not only your geometric intuition but

also your capacity to apply this knowledge in practical, real-world contexts across multiple disciplines. Whether designing a bridge, developing a 3D game, or analyzing physical phenomena, mastering the calculation of side lengths in 3D space is an invaluable skill.

Remember: Always double-check your coordinate data, and ensure assumptions are clearly stated when data is missing or ambiguous. With practice, calculating 3D distances becomes an intuitive part of your mathematical toolkit.

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