Find The Limit, If It Exists. (if An Answer Does Not Exist, Enter Dne.) Lim (x, Y)(0, 0) X2 Y2 X2 Y2

Find The Limit, If It Exists. (if An Answer Does Not Exist, Enter Dne.) Lim (x, Y)(0, 0) X2 Y2 X2 Y2 is a fundamental problem in multivariable calculus that involves determining whether the limit of a function of two variables exists as the point (x, y) approaches (0, 0). This type of problem is essential for understanding the behavior of functions near specific points, especially when the functions involve multiple variables and complicated expressions. In this article, we will explore how to evaluate such limits systematically, understand the conditions under which they exist, and discuss strategies to identify when a limit does not exist (Dne).

Understanding Limits in Multiple Variables

What Is a Multivariable Limit?

A multivariable limit examines the behavior of a function $f(x, y)$ as the point (x, y) approaches a specific point (a, b). Unlike single-variable limits, which involve approaching a point along a single path, multivariable limits require considering all possible paths approaching the point. The limit exists if and only if all paths lead to the same value.

Importance of Path-Dependence

In multivariable calculus, a key concept is that the limit must be path-independent. If the limit varies depending on the path taken toward (a, b), then the limit does not exist. Therefore, testing multiple paths is a crucial step in analyzing such limits.

Evaluating the Limit: Step-by-Step Approach

1. Understand the Function and Its Domain

Given the expression:

$$
\lim_{(x, y) \to (0, 0)} \frac{X^2 Y^2}{X^2 + Y^2}
$$

the first step is to clearly identify the function and its domain. Here, the numerator is $X^2 Y^2$, and the denominator is $X^2 + Y^2$. Note that at $(0, 0)$, the denominator is zero, so the function is undefined there, and we need to analyze the limit carefully.

2. Simplify the Expression if Possible

The given function is:

$$
f(x, y) = \frac{x^2 y^2}{x^2 + y^2}
$$

since the variables are standard, and the expression is already simplified.

3. Test Along Different Paths

To determine whether the limit exists, evaluate the function along different paths approaching $(0, 0)$:

- Path 1: $y = 0$
- Path 2: $y = x$
- Path 3: $y = kx$, where k is a constant

Path 1: $y = 0$

$$
f(x, 0) = \frac{x^2 \cdot 0^2}{x^2 + 0} = 0
$$

as $x \to 0$, $f(x, 0) \to 0$.

Path 2: $y = x$

$$
f(x, x) = \frac{x^2 \cdot x^2}{x^2 + x^2} = \frac{x^4}{2x^2} = \frac{x^2}{2}
$$

as $x \to 0$, $f(x, x) \to 0$.

Path 3: $y = kx$

$$
f(x, kx) = \frac{x^2 (kx)^2}{x^2 + (kx)^2} = \frac{x^2 \cdot k^2 x^2}{x^2 + k^2 x^2} = \frac{k^2 x^4}{(1 + k^2) x^2} = \frac{k^2 x^2}{1 + k^2}
$$

As $x \to 0$, $f(x, kx) \to 0$ regardless of k.

All these paths suggest the limit approaches 0, but to be certain, consider more general paths.

Confirming the Limit: Polar Coordinates

Using Polar Coordinates

Expressing the function in polar coordinates is often the most straightforward way to analyze limits involving x and y:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$
where $r \to 0$.

The function becomes:
$$
f(r, \theta) = \frac{(r \cos \theta)^2 (r \sin \theta)^2}{(r \cos \theta)^2 + (r \sin \theta)^2}
$$
Simplify numerator and denominator:
$$
f(r, \theta) = \frac{r^4 \cos^2 \theta \sin^2 \theta}{r^2 (\cos^2 \theta + \sin^2 \theta)} = \frac{r^4 \cos^2 \theta \sin^2 \theta}{r^2}
$$
since $\cos^2 \theta + \sin^2 \theta = 1$:

$$
f(r, \theta) = r^2 \cos^2 \theta \sin^2 \theta
$$

As $r \to 0$, regardless of θ, $f(r, \theta) \to 0$. Thus, the limit is 0.

Conclusion: Does the Limit Exist?

Since along all paths—including the polar coordinate approach—the function approaches 0, we conclude:

$$
\boxed{
\text{Limit} = 0
}
$$

Therefore, the limit exists and equals 0.

Summary and Key Takeaways

- When evaluating multivariable limits, always consider multiple paths to check for path dependence.

- Using polar coordinates simplifies the process, especially when the expression involves \(x^2

$+ y^2$).

- If all paths lead to the same value, the limit exists; otherwise, it does not (Dne).
- In this case, the limit is 0 as $(x, y) \to (0, 0)$.

Additional Tips for Finding Limits in Multivariable Calculus

1. Check for Indeterminate Forms

Many limits involve expressions like $0/0$ or ∞/∞. Simplify or use alternative methods such as substitution or coordinate transformation.

2. Use Symmetry

If the function is symmetric, testing symmetric paths can quickly reveal the limit's behavior.

3. Apply the Squeeze Theorem

If the function can be bounded between two functions that approach the same limit, then the limit of the original function exists and equals that common limit.

4. Recognize When the Limit Does Not Exist

If different paths yield different limits, then the overall limit does not exist (Dne).

Final Remarks

Evaluating limits like $\lim_{(x, y) \to (0, 0)} \frac{x^2 y^2}{x^2 + y^2}$ demands careful analysis and multiple approaches. Using path testing and polar coordinates provides clarity and confidence in the result. In this example, the limit exists and is equal to 0, illustrating a common technique in multivariable calculus to handle such problems efficiently.

Understanding the principles behind these limit evaluations is crucial for progressing in calculus topics such as continuity, differentiability, and the behavior of functions near critical points. Whether you're a student studying for exams or a professional analyzing multivariable functions, mastering the art of limit evaluation is an invaluable skill.

Frequently Asked Questions

What is the limit of the function lim (x, y) → (0, 0) of (x^2 y^2) / (x^2 + y^2)?

Dne (Does not exist)

Does the limit lim (x, y) → (0, 0) of (x^2 y^2) / (x^2 + y^2) exist?

No, the limit does not exist.

How can I determine whether the limit lim (x, y) → (0, 0) of (x^2 y^2) / (x^2 + y^2) exists?

By approaching (0, 0) along different paths and checking if the limit is the same; if not, the limit does not exist.

What happens if I approach (0, 0) along y = 0 or x = 0 for the given function?

Along y=0, the function simplifies to 0; along x=0, it also simplifies to 0. However, these paths alone are insufficient to conclude the limit exists, so further analysis is needed.

Can the limit lim (x, y) → (0, 0) of (x^2 y^2) / (x^2 + y^2) be found using polar coordinates?

Yes, converting to polar coordinates reveals the limit approaches 0 along some paths, but since the limit varies along different paths, it does not exist.

What is the final conclusion about the limit lim (x, y) → (0, 0) of (x^2 y^2) / (x^2 + y^2)?

Dne (The limit does not exist).

Additional Resources

Understanding and Finding the Limit of a Multivariable Function as \((x, y) \to (0, 0)\)

When studying multivariable calculus, one of the fundamental concepts is understanding limits of functions as their variables approach a specific point. Unlike single-variable limits, multivariable limits can be more complex due to the multiple paths along which the point can be approached. This detailed review explores the process of evaluating the limit:

\[
\lim_{(x, y) \to (0, 0)} \frac{X^2 Y^2}{X^2 + Y^2}

\]

and determining whether this limit exists.

Introduction to the Problem

The function under consideration is:

$$
f(x, y) = \frac{x^2 y^2}{x^2 + y^2}
$$

with the goal of evaluating:

$$
\lim_{(x, y) \to (0, 0)} f(x, y)
$$

Key points:

- The function involves both x and y approaching zero.
- The numerator is $x^2 y^2$, which tends to zero as either x or y approaches zero.
- The denominator is $x^2 + y^2$, which also tends to zero as $(x, y) \to (0, 0)$.
- The main challenge is determining whether the ratio approaches a single finite value regardless of the path taken toward the origin.

Why Are Multivariable Limits More Complex Than Single-Variable Limits?

In single-variable calculus, the limit as $x \to a$ is straightforward: if the function approaches the same value from both sides, the limit exists. However, in two variables:

- The point $(0, 0)$ can be approached along infinitely many paths.
- The limit exists only if all paths approaching $(0, 0)$ lead to the same value.
- Different paths can sometimes produce different limit values, indicating the limit does not exist.

This complexity necessitates examining multiple paths to test the existence of the limit.

Approach to the Limit: Path Analysis

To analyze whether the limit exists, consider approaching $(0, 0)$ along various paths and observe the behavior of $f(x, y)$.

Path 1: Approach Along the x-axis ($y = 0$)

- Substituting $y = 0$:

$$
f(x, 0) = \frac{x^2 \cdot 0^2}{x^2 + 0^2} = \frac{0}{x^2} = 0
$$

- As $x \to 0$, $f(x, 0) \to 0$.

Result: Along the x-axis, the limit approaches 0.

Path 2: Approach Along the y-axis ($x = 0$)

- Substituting $x = 0$:

$$
f(0, y) = \frac{0^2 \cdot y^2}{0^2 + y^2} = \frac{0}{y^2} = 0
$$

- As $y \to 0$, $f(0, y) \to 0$.

Result: Along the y-axis, the limit also approaches 0.

Path 3: Approach Along the line $y = m x$

- Substituting $y = m x$:

$$
f(x, m x) = \frac{x^2 (m x)^2}{x^2 + (m x)^2} = \frac{x^2 \cdot m^2 x^2}{x^2 + m^2 x^2} = \frac{m^2 x^4}{x^2 (1 + m^2)} = \frac{m^2 x^4}{x^2 (1 + m^2)}
$$

- Simplify:

$$
f(x, m x) = \frac{m^2 x^4}{x^2 (1 + m^2)} = \frac{m^2 x^2}{1 + m^2}
$$

- As $x \to 0$:

$$
f(x, m x) \to 0
$$

Result: Along any straight line $y = m x$, the function approaches 0.

Examining Other Paths to Confirm Limit Behavior

Since the previous paths all lead to 0, initial evidence suggests the limit might be 0. However, to confirm the limit exists, we need to examine more complex paths that could potentially yield different results.

Path 4: Approach Along $y = k x^n$ for various n

- For example, take $y = x^2$:

$$
f(x, x^2) = \frac{x^2 (x^2)^2}{x^2 + (x^2)^2} = \frac{x^2 x^4}{x^2 + x^4} = \frac{x^6}{x^2 (1 + x^2)} = \frac{x^4}{1 + x^2}
$$

- As $x \to 0$:

$$
f(x, x^2) \to 0
$$

Similarly, other polynomial paths tend to zero, reinforcing the idea that the limit might be zero.

Path 5: Approach Along $y = x$

- Substituting $y = x$:

$$
f(x, x) = \frac{x^2 x^2}{x^2 + x^2} = \frac{x^4}{2 x^2} = \frac{x^2}{2}
$$

- As $x \to 0$:

$$
f(x, x) \to 0
$$

Again, the limit is zero.

Path 6: Approach Along $y = \sqrt{x}$

- Substituting $y = \sqrt{x}$:

$$
f(x, \sqrt{x}) = \frac{x^2 (\sqrt{x})^2}{x^2 + (\sqrt{x})^2} = \frac{x^2 \cdot x}{x^2 + x} = \frac{x^3}{x^2 + x}
$$

- Simplify denominator:

$$
= \frac{x^3}{x(x + 1)} = \frac{x^2}{x + 1}
$$

- As $x \to 0$:

$$
f(x, \sqrt{x}) \to 0
$$

Observation: No path investigated yields a limit different from zero.

Using Polar Coordinates to Confirm the Limit

While path analysis strongly suggests the limit is zero, the most rigorous approach in multivariable calculus involves converting to polar coordinates.

Transforming the Function to Polar Coordinates

Recall that:

$$
x = r \cos \theta, \quad y = r \sin \theta
$$

and:

$$
x^2 + y^2 = r^2
$$

The function becomes:

$$
f(r, \theta) = \frac{(r \cos \theta)^2 (r \sin \theta)^2}{r^2} = \frac{r^2 \cos^2 \theta \cdot r^2 \sin^2 \theta}{r^2}
$$

Simplify numerator:

$$
= \frac{r^4 \cos^2 \theta \sin^2 \theta}{r^2} = r^2 \cos^2 \theta \sin^2 \theta
$$

Limit in Polar Coordinates

- As $(x, y) \to (0, 0)$, $r \to 0$.

- The limit becomes:

$$
\lim_{r \to 0} r^2 \cos^2 \theta \sin^2 \theta
$$

- For any fixed θ:

$$
\lim_{r \to 0} r^2 \times (\text{bounded term}) = 0
$$

since $\cos^2 \theta$ and $\sin^2 \theta$ are bounded between 0 and 1.

- Because this limit is independent of θ, the overall limit is:

$$
\boxed{0}
$$

which exists and is finite.

Conclusion: Does the Limit Exist?

Based on the path analysis and the polar coordinate transformation, we see:

- All paths approaching \((0, 0)\) lead to the value 0.
- The polar coordinate approach confirms the limit is 0 regardless of the path taken.

Therefore, the limit:

\[
\boxed{\text{Find The Limit, If It Exists. (if an answer does not exist, enter DNE.)} = 0}
\]

Summary and Final Remarks

- The initial suspicion based on multiple path analysis is that the limit is zero.

Find The Limit If It Exists If An Answer Does Not Exist Enter Dne Lim X Y 0 0 X2 Y2 X2 Y2

Find The Limit If It Exists If An Answer Does Not Exist Enter Dne Lim X Y 0 0 X2 Y2 X2 Y2

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