# Find The Number Of Combinations And Permutations Of Four Letters Each That Can Be Made From The Word

## Find The Number Of Combinations And Permutations Of Four Letters Each That Can Be Made From The Word

When working with words and selecting or arranging letters, it is essential to understand the concepts of permutations and combinations. These concepts help determine how many different arrangements or selections can be made from a set of items, in this case, letters of a word. The problem of finding the number of four-letter combinations or permutations from a specific word involves analyzing the letter composition of the word, considering whether the letters are distinct or repeated, and applying relevant mathematical formulas accordingly. This article explores the process step-by-step, providing comprehensive explanations and examples to clarify these concepts.

## Understanding Permutations and Combinations

### Permutations

Permutations refer to arrangements of items where the order matters. For example, arranging the letters A, B, C, D in different orders yields different permutations (ABCD, BACD, ACBD, etc.). The number of permutations depends on whether the items are distinct and whether repetition is allowed.

### Combinations

Combinations involve selecting items where the order does not matter. For example, choosing four letters out of the set {A, B, C, D} regardless of their order. The focus here is solely on the selection, not how they are arranged.

## Analyzing the Word and Its Letters

To determine the number of four-letter arrangements or selections, the first step is to analyze the given word's letter composition. Let's consider an example word to illustrate the process:
Example Word: "BALLOON"

This word contains the following letters:
- B (1 occurrence)
- A (1 occurrence)
- L (2 occurrences)
- O (2 occurrences)
- N (1 occurrence)

Total number of letters: 7

The presence of repeated letters (L and O) influences how many unique permutations and combinations are possible.

# Calculating Permutations of Four Letters

Permutations involve arranging four letters chosen from the word's letter set, considering the repetitions.

## Step 1: Determine Possible Cases Based on Repetition

Since the word contains repeated letters, different cases arise:
1. All four letters are distinct.
2. Two letters are the same, and the other two are distinct.
3. Two pairs of identical letters.
4. Three identical letters and one different.
5. Four identical letters (not possible here).

## Step 2: Calculate Permutations for Each Case

Case 1: All four letters are distinct

- Possible distinct letters: B, A, L, O, N
- Number of distinct letters in the word: 5

Number of ways to choose 4 distinct letters out of these 5:

$$
\binom{5}{4} = 5
$$

For each selection, the number of arrangements (permutations) of 4 distinct letters:

$$
4! = 24
$$

Total permutations in this case:

$$
5 \times 24 = 120
$$

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Case 2: Two identical letters and two other distinct letters

- Letters with duplicates: L (2), O (2)

Possible choices:
- L as the repeated letter: select L twice, plus 2 other distinct letters from the remaining (B, A, O, N)
- O as the repeated letter: select O twice, plus 2 other distinct letters from (B, A, L, N)

Number of ways:
- For L:

$$
\binom{4}{2} = 6
$$

choices of the remaining 2 distinct letters from {B, A, O, N}

- For O:

$$
\binom{4}{2} = 6
$$

\]
choices of remaining 2 distinct letters from {B, A, L, N}

Total number of 4-letter sets:
\[
(6 + 6) = 12
\]

Number of arrangements for each set with two identical letters:
\[
\frac{4!}{2!} = \frac{24}{2} = 12
\]

Total permutations:
\[
12 \times 12 = 144
\]

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Case 3: Two pairs of identical letters

- Pairs: (L, L) and (O, O)

Number of such sets:
\[
1 \text{ (L, L, O, O)}
\]

Number of arrangements:
\[
\frac{4!}{2! \times 2!} = \frac{24}{4} = 6
\]

---

Case 4: Three identical letters and one different

- Not possible here, since no letter appears 3 times.

Total permutations of four letters:

Adding all cases:
\[
120 + 144 + 6 = 270
\]

Thus, from the word "BALLOON," the total number of four-letter permutations is 270.

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## Calculating Combinations of Four Letters

In contrast to permutations, combinations focus on selecting four letters regardless of their order.

## Step 1: Identify Possible Cases Based on Repetition

Similar to permutations, the cases are:
1. All four letters are distinct.
2. Two identical letters and two others.
3. Two pairs of identical letters.
4. Three identical letters and one different.

## Step 2: Count the Number of Combinations for Each Case

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Case 1: All four letters are distinct

Number of ways:
- Choose 4 distinct letters from {B, A, L, O, N}:

$$
\binom{5}{4} = 5
$$

---

Case 2: Two identical letters and two others

- For L (since it appears twice), choose:
- The pair L, L
- Plus 2 other distinct letters from {B, A, O, N}:

$$
\binom{4}{2} = 6
$$

- For O (similarly):
- The pair O, O
- Plus 2 other distinct letters from {B, A, L, N}:

$$
\binom{4}{2} = 6
$$

Total combinations:

$$
6 + 6 = 12
$$

---

Case 3: Two pairs of identical letters

- Only one possibility: {L, L, O, O}
- Number of combinations: 1

---

Case 4: Three identical letters and one different

- Not possible here, as no letter appears three times.

---

Total combinations:

Sum of all cases:

$$
5 + 12 + 1 = 18
$$

Therefore, there are 18 different four-letter combinations that can be formed from "BALLOON".

---

## Summary and Final Remarks

In summary, calculating the number of permutations and combinations of four letters from a word requires careful analysis of the letter composition, especially when repetitions exist. For the example word "BALLOON," the calculations reveal:

- Number of four-letter permutations: 270
- Number of four-letter combinations: 18

These calculations exemplify how combinatorial principles are applied in practical scenarios involving word arrangements. When approaching similar problems, the key steps involve:

- Analyzing the frequency of each letter
- Categorizing cases based on the presence of repeated letters
- Applying appropriate formulas for permutations and combinations
- Summing across all valid cases to obtain the total counts

Understanding these concepts is fundamental in fields like cryptography, word puzzles, and probability theory, where arrangements and selections are commonplace.

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## Additional Notes and Tips

- Always start by counting the frequency of each letter in the word.
- For permutations, remember to divide by factorials of repeated letters to avoid overcounting.
- For combinations, focus solely on the selection, not the order, and adjust calculations accordingly.
- Use binomial coefficients $\binom{n}{k}$ to determine the number of ways to select items.
- Be cautious about the cases where certain repetitions are impossible due to the word's letter composition.

By mastering these methods, you can efficiently solve a wide range of combinatorial problems involving words and letter arrangements.

# Frequently Asked Questions

## How many different 4-letter arrangements can be formed from the word 'COMBINE' if repetition is allowed?

If repetition is allowed and the word has 7 distinct letters, then the total arrangements are 7^4 = 2401.

## What is the number of 4-letter permutations that can be made from the word 'BANANA' considering the repeated letters?

Since 'BANANA' has repeated letters, the total permutations depend on the selection of distinct letters. For example, choosing 4 different letters out of the unique ones (B, A, N), and accounting for repetitions, the calculation involves permutations of multisets. The exact number varies based on selection.

## How many 4-letter combinations can be formed from 'DEED' if order does not matter?

Considering combinations without regard to order and accounting for repeated letters, the total is determined by selecting unique sets such as {D, E} with repetitions, resulting in a specific count based on combinations with repetitions.

## From the word 'FOUR', how many 4-letter permutations are possible?

Since all 4 letters are distinct, the total number of permutations is 4! = 24.

## If I select 4 letters from the word 'BALLOON', how many arrangements are possible considering repeated letters?

With repeated letters (L and O), the total arrangements depend on which letters are chosen. For example, selecting all distinct or some repeated letters, the calculation varies, typically involving permutations of multisets.

## Can you determine the number of 4-letter arrangements from 'DRIVE' if no letter repeats?

Yes, since all 5 letters are distinct, the number of arrangements of 4 letters is $P(5,4) = 5\times4\times3\times2 = 120$.

## How many 4-letter combinations can be formed from 'MATHS' when order is not important?

With all distinct letters, the number of combinations is $C(5,4) = 5$. These are the different sets of 4 letters chosen from the 5.

## What is the difference between permutations and combinations in the context of forming 4-letter words from 'PEAR'?

Permutations count arrangements where order matters (e.g., 'PEAR' vs. 'REAP'), totaling 4! = 24. Combinations count unique sets regardless of order, which is $C(4,4) = 1$ for all four letters.

### How do repeated letters in a word affect the calculation of permutations and combinations of 4-letter selections?

Repeated letters reduce the total number of unique permutations because swapping identical letters doesn't produce a new arrangement. When calculating permutations of multisets, factorials are divided by the factorials of repeated counts to account for indistinguishable arrangements.

## Additional Resources

Find The Number Of Combinations And Permutations Of Four Letters Each That Can Be Made From The Word

When exploring the fascinating world of combinatorics, one of the fundamental questions involves determining how many different arrangements or selections can be made from a given set of elements. In particular, the problem of finding the number of combinations and permutations of four letters each, derived from a specific word, is a classic example that encompasses many core principles of discrete mathematics. This comprehensive review aims to dissect this problem in depth, providing clarity on the concepts, formulas, methods, and applications involved.

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## Understanding the Foundations: Permutations and Combinations

Before diving into specific calculations related to the word, it's essential to understand the core definitions and differences between permutations and combinations, as they form the backbone of the problem.

### Permutations

- Definition: Permutations refer to the arrangements of a set of elements where order matters.
- Example: Arranging the letters A, B, C in different orders, such as ABC, BAC, CAB, etc.
- Formula for permutations of selecting r elements from n distinct elements:

$$
P(n, r) = \frac{n!}{(n - r)!}
$$

- Key points:
- When all elements are distinct, permutations are straightforward.
- If some elements are repeated, adjustments are necessary (covered later).

### Combinations

- Definition: Combinations refer to the selection of elements from a set where order does not matter.
- Example: Choosing 3 letters out of A, B, C, D, where ABC is the same as CAB or BAC.
- Formula for combinations of selecting r elements from n distinct elements:

$$
C(n, r) = \binom{n}{r} = \frac{n!}{r! (n - r)!}
$$

- Key points:

- Focused purely on selection, not arrangements.
- Useful when the order of selection is irrelevant.

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# The Word and Its Characteristics

To accurately determine the number of possible four-letter arrangements or selections, understanding the specific characteristics of the word in question is crucial.

## Analyzing the Word

Suppose the word is "BALLOON" as an example—this is a commonly used word with multiple repeated letters. Alternatively, the word could be "COMPUTER," which contains all distinct letters.

Case 1: Word with all distinct letters

- Example: "COMPUTER" (C, O, M, P, U, T, E, R)
- Total letters: 8
- All are unique.

Case 2: Word with repeated letters

- Example: "BALLOON" (B, A, L, L, O, O, N)
- Total letters: 7
- Repetition:
- L appears twice
- O appears twice
- Total distinct letters: B, A, L, O, N (5 distinct letters)
- Frequencies:
- B: 1
- A: 1
- L: 2
- O: 2
- N: 1

The approach will differ based on the presence or absence of repeated letters.

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# Counting Permutations and Combinations in Different Scenarios

The core of the problem lies in understanding how repetitions affect the total counts. We will explore both scenarios: all distinct letters and repeated letters.

## Scenario 1: All Distinct Letters

Total distinct letters: n

- For a word with n unique letters, the number of permutations of 4 letters:

$$
P(n, 4) = \frac{n!}{(n-4)!}
$$

- The number of combinations of 4 letters:

$$
C(n, 4) = \binom{n}{4}
$$

Example: With n=8 (COMPUTER)

- Permutations:

$$
P(8, 4) = \frac{8!}{(8-4)!} = \frac{8!}{4!} = \frac{40320}{24} = 1680
$$

- Combinations:

$$
C(8, 4) = \binom{8}{4} = \frac{8!}{4!4!} = 70
$$

Interpretation:

- There are 1,680 ways to arrange 4 different letters selected from "COMPUTER."
- There are 70 ways to select 4 different letters, disregarding order.

---

## Scenario 2: Repeated Letters Present

When the word contains repeated letters, the calculations become more nuanced, as identical letters reduce the total number of distinct arrangements.

Key considerations:

- How many repeated letters are involved?
- How many times each repeated letter appears?
- Which combinations involve repetitions, and which do not?

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# Calculating Permutations with Repeated Letters

When selecting 4 letters from a set that contains repeated characters, the total number of permutations depends on the composition of the selected letters.

## Possible Cases for Selection

Assuming the set of distinct letters is $D = \{L_1, L_2, ..., L_k\}$, with some letters repeated, the possible cases for choosing 4 letters are:

1. All four letters are distinct.
2. One letter is repeated twice, and two other distinct letters are selected.
3. Two letters are each repeated twice.
4. One letter is repeated three times, and one other distinct letter is selected.
5. One letter is repeated four times. (If such exists; unlikely with the given word unless a letter appears four times)

Note: For the word "BALLOON," for example, the maximum repetition is 2, so cases involving three or four repetitions are impossible.

---

## Calculating Each Case

Let's analyze each case with detailed formulas:

Case 1: All four letters are distinct

- Number of ways to choose 4 distinct letters:

$$
\text{Number of combinations} = \binom{k}{4}
$$

- Number of permutations for each combination:

$$
P(4, 4) = 4! = 24
$$

- Total permutations:

$$
\text{Total} = \binom{k}{4} \times 4! = \binom{k}{4} \times 24
$$

Example: For "BALLOON," with 5 distinct letters, $k=5$:

$$
\binom{5}{4} = 5
$$
$$
\text{Total permutations} = 5 \times 24 = 120
$$

---

Case 2: One letter repeated twice, plus two other distinct letters

- Number of ways to choose the repeated letter:

$$
\text{Number of choices} = \text{Number of letters with at least 2 repetitions}
$$

In "BALLOON," L and O are repeated.

- For each such letter:

- Choose 2 instances of the repeated letter (fixed, since the repeats are identical).

- Select 2 other distinct letters from remaining letters (excluding the repeated one):

$$
\binom{k-1}{2}
$$

- Number of arrangements of the selected 4 letters (with one letter repeated twice):

$$
\frac{4!}{2!} = 12
$$

- Total arrangements across all such choices:

$$
\text{Total} = \text{Number of repeated letters} \times \binom{k-1}{2} \times 12
$$

Example: For "BALLOON," repeated letters are L and O:

- Repeated letters: L, O (2 options)

- For L:

$$
\binom{4}{2} = 6
$$

- For O:

$$
\binom{4}{2} = 6
$$

- Total arrangements:

$$
(2) \times (6 + 6) \times 12 = 2 \times 12 \times 12 = 288
$$

(Note: The actual calculation may vary based on the specific set; the above is for illustration.)

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Case 3: Two letters each repeated twice

- For example, selecting L and O, each repeated twice:

- Number of such combinations:

$$
\binom{\text{Number of letters with at least 2 repetitions}}{2}
$$

- Number of arrangements for each such combination:

\[
\frac{4!}{2!2!} = 6
\]

- Total permutations:

\[
\text{Number of options} \times 6
\]

In "BALLOON," with L and O both repeated twice, only one such combination exists:

\[
\text{Total} = 1 \times 6 = 6
\]

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### Calculating Combinations in the Presence of Repititions

When considering combinations (where order does not matter), the calculations involve selecting sets of letters with possible repetitions.

Approach:

- For repeated letters, use multisets.
- The number of multisets of size 4

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