

Find The Perimeter Of The Parallelogram Whose Vertices Are A (-1,1), B (-9,6), C(-12,11) And D(-4,6)

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Understanding how to find the perimeter of a parallelogram when given the coordinates of its vertices is a fundamental skill in coordinate geometry. In this article, we will go through a detailed, step-by-step process to determine the perimeter of the parallelogram with vertices A (-1, 1), B (-9, 6), C (-12, 11), and D (-4, 6). We will explore concepts such as distance formula, properties of parallelograms, and coordinate calculations, ensuring that you not only find the answer but also understand the underlying principles involved.

Understanding the Problem: Coordinates and Parallelogram Properties

Before diving into calculations, it's essential to understand what is being asked and the properties of the shapes involved.

Vertices of the Parallelogram

- Vertex A: (-1, 1)
- Vertex B: (-9, 6)
- Vertex C: (-12, 11)
- Vertex D: (-4, 6)

These points are given in a coordinate plane, and the goal is to find the total perimeter of the parallelogram formed by these points.

Properties of a Parallelogram

- Opposite sides are equal in length.
- Opposite sides are parallel.
- The diagonals bisect each other.

Knowing these properties helps verify the shape and guide which pairs of points form sides.

Verifying the Shape: Are the Given Points a Parallelogram?

Before calculating the perimeter, we should confirm that the points indeed form a parallelogram.

Method 1: Check if Opposite Sides are Parallel

- Calculate the slopes of the sides to see if opposite sides are parallel.
- Use the slope formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Calculating Slopes of Sides

- AB: between points A (-1, 1) and B (-9, 6)

$$m_{AB} = \frac{6 - 1}{-9 - (-1)} = \frac{5}{-8} = -\frac{5}{8}$$

- BC: between B (-9, 6) and C (-12, 11)

$$m_{BC} = \frac{11 - 6}{-12 - (-9)} = \frac{5}{-3} = -\frac{5}{3}$$

- CD: between C (-12, 11) and D (-4, 6)

$$m_{CD} = \frac{6 - 11}{-4 - (-12)} = \frac{-5}{8} = -\frac{5}{8}$$

- DA: between D (-4, 6) and A (-1, 1)

$$m_{DA} = \frac{1 - 6}{-1 - (-4)} = \frac{-5}{3} = -\frac{5}{3}$$

Analysis of Slopes

- Sides AB and CD have slopes of $-\frac{5}{8}$, indicating they are parallel.
- Sides BC and DA have slopes of $-\frac{5}{3}$, indicating they are parallel.

Since opposite sides are parallel, the shape is indeed a parallelogram.

Calculating the Lengths of Sides

To find the perimeter, we need the lengths of two adjacent sides. We will use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Calculating Side Lengths

1. Length of AB

$$AB = \sqrt{(-9 - (-1))^2 + (6 - 1)^2} = \sqrt{(-8)^2 + 5^2} = \sqrt{64 + 25} = \sqrt{89}$$

2. Length of BC

$$BC = \sqrt{(-12 - (-9))^2 + (11 - 6)^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$$

3. Length of CD

$$CD = \sqrt{(-4 - (-12))^2 + (6 - 11)^2} = \sqrt{8^2 + (-5)^2} = \sqrt{64 + 25} = \sqrt{89}$$

4. Length of DA

$$DA = \sqrt{(-1 - (-4))^2 + (1 - 6)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34}$$

Observation:

- AB and CD are equal in length ($\sqrt{89}$)
- BC and DA are equal in length ($\sqrt{34}$)

This confirms the shape's property of equal opposite sides, characteristic of a parallelogram.

Calculating the Perimeter

The perimeter (P) of a parallelogram is given by:

$$P = 2 \times (\text{length of one side}) + 2 \times (\text{length of adjacent side})$$

Using the side lengths calculated:

$$P = 2 \times AB + 2 \times BC = 2 \times \sqrt{89} + 2 \times \sqrt{34}$$

Simplify:

$$P = 2 (\sqrt{89} + \sqrt{34})$$

For a numerical approximation:

$$\begin{aligned} \sqrt{89} &\approx 9.43 \\ \sqrt{34} &\approx 5.83 \end{aligned}$$

Therefore,

$$P \approx 2 (9.43 + 5.83) = 2 \times 15.26 = 30.52$$

Final answer:

$$\boxed{\text{Perimeter} \approx 30.52 \text{ units}}$$

Summary and Key Takeaways

- The vertices provided form a parallelogram, verified by the slopes of opposite sides.
 - Calculated side lengths using the distance formula: $\sqrt{89}$ and $\sqrt{34}$.
 - The perimeter is twice the sum of the lengths of adjacent sides.
 - The approximate perimeter of the parallelogram is 30.52 units.
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Additional Tips for Coordinate Geometry Problems

- Always verify the shape using properties such as parallel sides or diagonals bisecting each other.
 - Use the distance formula carefully, and double-check calculations.
 - When exact values are preferred, leave the answer in terms of square roots; for approximate numerical answers, use calculator approximations.
 - Visualizing the shape on a graph can help better understand the problem.
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Conclusion

Finding the perimeter of a parallelogram from its vertices involves understanding properties of the shape, verifying the shape with slopes, calculating side lengths with the distance formula, and then applying the perimeter formula. For the given vertices A (-1, 1), B (-9, 6), C (-12, 11), and D (-4, 6), the perimeter is approximately 30.52 units. Mastering these steps enhances your ability to solve similar geometry problems efficiently and accurately.

Frequently Asked Questions

How do you find the perimeter of a parallelogram given its vertices?

To find the perimeter, calculate the lengths of two adjacent sides using the distance formula, then multiply one of those lengths by 2 since opposite sides are equal.

What is the distance formula used to find the length between two points?

The distance between two points (x_1, y_1) and (x_2, y_2) is given by $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Given vertices A(-1,1), B(-9,6), C(-12,11), and D(-4,6), how do we confirm they form a parallelogram?

Check if the midpoints of diagonals are the same; if they coincide, the vertices form a parallelogram.

How do we verify that points A, B, C, D form a parallelogram using midpoints?

Calculate the midpoints of diagonals AC and BD; if both midpoints are the same, the quadrilateral is a parallelogram.

What are the lengths of sides AB and AD in the given parallelogram?

Using the distance formula, $AB \approx 7.81$ units and $AD \approx 6.40$ units.

How do we compute the perimeter once we find the lengths of the sides?

Add the lengths of two adjacent sides and multiply by 2, since opposite sides are equal: $\text{Perimeter} = 2 \times (AB + AD)$.

What is the approximate perimeter of the parallelogram with given vertices?

The perimeter is approximately 28.42 units.

Why is it important to verify the shape before calculating the perimeter?

Verifying ensures the vertices do form a parallelogram, so the calculated side lengths and perimeter are accurate for that shape.

Additional Resources

Find The Perimeter Of The Parallelogram Whose Vertices Are A (-1,1), B (-9,6), C(-12,11), And D(-4,6)

Understanding how to find the perimeter of a parallelogram given its vertices is a fundamental skill in

coordinate geometry, often encountered in high school mathematics and beyond. This problem involves analyzing the vertices of a parallelogram—points A, B, C, and D—and calculating the total length around the shape. By carefully applying the distance formula and understanding the properties of parallelograms, we can determine the perimeter with precision. Let's delve into the process step by step, exploring the geometric concepts and calculations involved.

Introduction to the Problem

In this scenario, we are given four points:

- A (-1, 1)
- B (-9, 6)
- C (-12, 11)
- D (-4, 6)

These points are vertices of a parallelogram, a four-sided polygon with opposite sides parallel and equal in length. Our goal is to find the perimeter of this shape.

Why Is This Important?

Calculating the perimeter of a parallelogram in the coordinate plane has applications in fields such as architecture, engineering, computer graphics, and mathematics education. It enhances spatial reasoning and understanding of geometric transformations.

Step 1: Confirming the Shape is a Parallelogram

Before calculating the perimeter, it's essential to verify that the given points indeed form a parallelogram. One way to do this is to check if the vectors representing opposite sides are equal or if the diagonals bisect each other.

Method 1: Check Opposite Sides

Calculate the vectors for:

- AB and DC
- BC and AD

If these pairs of vectors are equal, the shape is a parallelogram.

Method 2: Check Diagonals

Calculate midpoints of diagonals AC and BD, and see if they coincide. If midpoints are the same, the diagonals bisect each other, confirming a parallelogram.

Let's proceed with the midpoint method, as it's straightforward.

Step 2: Calculating Midpoints of Diagonals

Midpoint of AC:

$$M_{\{AC\}} = \left(\frac{x_A + x_C}{2}, \frac{y_A + y_C}{2} \right)$$

$$M_{\{AC\}} = \left(\frac{-1 + (-12)}{2}, \frac{1 + 11}{2} \right) = \left(\frac{-13}{2}, \frac{12}{2} \right) = (-6.5, 6)$$

Midpoint of BD:

$$M_{\{BD\}} = \left(\frac{x_B + x_D}{2}, \frac{y_B + y_D}{2} \right)$$

$$M_{\{BD\}} = \left(\frac{-9 + (-4)}{2}, \frac{6 + 6}{2} \right) = \left(\frac{-13}{2}, 6 \right) = (-6.5, 6)$$

Since both midpoints are the same, the diagonals bisect each other, confirming that ABCD is a parallelogram.

Step 3: Calculating Side Lengths

To find the perimeter, we need the lengths of all sides. Since opposite sides are equal in a parallelogram, calculating the length of one pair of adjacent sides suffices, then doubling for the total perimeter.

Let's calculate the distances:

- AB and BC (adjacent sides)

- Alternatively, we can choose any two adjacent sides, as the opposite sides will have the same length.

Distance Formula:

Given two points (x_1, y_1) and (x_2, y_2) , the distance d between them is:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Calculating AB:

$$A(-1, 1), \text{quad } B(-9, 6)$$

$$AB = \sqrt{(-9 - (-1))^2 + (6 - 1)^2} = \sqrt{(-8)^2 + 5^2} = \sqrt{64 + 25} = \sqrt{89}$$

Calculating BC:

$$B(-9, 6), \text{quad } C(-12, 11)$$

$$BC = \sqrt{(-12 - (-9))^2 + (11 - 6)^2} = \sqrt{(-3)^2 + 5^2} = \sqrt{9 + 25} = \sqrt{34}$$

Calculating CD:

$$C(-12, 11), \text{quad } D(-4, 6)$$

$$CD = \sqrt{(-4 - (-12))^2 + (6 - 11)^2} = \sqrt{8^2 + (-5)^2} = \sqrt{64 + 25} = \sqrt{89}$$

Calculating DA:

$$\begin{aligned} & \backslash[\\ & D(-4, 6), \quad A(-1, 1) \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & DA = \sqrt{(-1 - (-4))^2 + (1 - 6)^2} = \sqrt{3^2 + (-5)^2} = \sqrt{9 + 25} = \sqrt{34} \\ & \backslash] \end{aligned}$$

Step 4: Confirming Opposite Sides are Equal

From the calculations:

$$\begin{aligned} - AB &= \sqrt{89} \\ - CD &= \sqrt{89} \\ - BC &= \sqrt{34} \\ - DA &= \sqrt{34} \end{aligned}$$

This confirms that the opposite sides are equal, consistent with the properties of a parallelogram.

Step 5: Calculating the Perimeter

Now, the perimeter (P) is:

$$\begin{aligned} & \backslash[\\ & P = 2 \times (\text{length of AB} + \text{length of BC}) \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & P = 2 \times (\sqrt{89} + \sqrt{34}) \\ & \backslash] \end{aligned}$$

Expressed as a numeric approximation:

$$\begin{aligned} & \backslash[\\ & \sqrt{89} \approx 9.43 \\ & \backslash[\\ & \sqrt{34} \approx 5.83 \end{aligned}$$

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$$P \approx 2 \times (9.43 + 5.83) = 2 \times 15.26 = 30.52$$

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Therefore, the perimeter of the parallelogram is approximately 30.52 units.

Additional Insights and Tips

Using Vectors for Distance Calculation

In coordinate geometry, vectors simplify the process of finding side lengths and verifying shapes:

- The vector $\vec{AB} = \vec{B} - \vec{A}$
- The magnitude of the vector gives the length of the side.

Recognizing Parallelogram Properties

- Opposite sides are parallel and equal in length.
- Diagonals bisect each other.
- Opposite angles are equal.

Practical Applications

Knowing how to find the perimeter from vertices allows for:

- Calculating fencing needed around a plot
- Determining material lengths in construction
- Analyzing shapes in computer graphics

Summary

To find the perimeter of a parallelogram given its vertices:

1. Verify the shape (diagonals bisect or opposite sides are equal).
2. Calculate side lengths using the distance formula.
3. Identify pairs of equal sides.
4. Sum the lengths of adjacent sides and multiply by 2 to get the total perimeter.

In this specific case, the vertices A (-1,1), B (-9,6), C (-12,11), and D (-4,6) form a parallelogram with side lengths approximately $\sqrt{89}$ and $\sqrt{34}$. The perimeter is approximately 30.52 units.

Mastering these steps enhances your geometric reasoning and problem-solving skills, providing a solid foundation for tackling more complex coordinate geometry problems in academic and practical contexts.

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