Find The Point On The Curve Y= X^2 - 3x + 5 Where The Normal Is Parallel To The Tangent To The Curve

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Understanding the geometric properties of curves is fundamental in calculus, especially when dealing with tangents and normals. The problem of finding a point on a given curve where the normal line is parallel to the tangent line at another point or to a specific line is a classic application of derivatives and differential calculus. In this article, we will explore how to determine the point on the curve $y = x^2 - 3x + 5$ where the normal is parallel to the tangent to the curve, providing a comprehensive step-by-step solution, explanations, and related concepts.

Introduction to the Problem

The problem involves two key concepts:

- The tangent line to a curve at a point
- The normal line to a curve at a point

In particular, we are asked to find the point(s) on the curve $y = x^2 - 3x + 5$ where the normal line is parallel to the tangent line to the curve. Since the normal line is perpendicular to the tangent at the same point, typically, the normal cannot be parallel to the tangent at the same point, as they are perpendicular by definition. Therefore, the problem might be interpreted as:

- Find the point on the curve where the normal line at that point is parallel to the tangent line at another point on the curve, or
- Find the point on the curve where the normal line is parallel to the tangent line at that same point— which would imply the normal and tangent are parallel, which contradicts the perpendicularity unless the slope is zero.

To clarify, the most common interpretation is:

Find the point on the curve where the normal line is parallel to the tangent line at some other point on the curve, or equivalently, find the points where the normal line's slope equals the slope of the tangent line at some other point.

Alternatively, the problem might involve the line being parallel to a specific line or to the tangent at some point. For this reason, we will assume the problem asks:

Find the point(s) on the curve where the normal line is parallel to the tangent line at the same point.

But since the normal line at a point is perpendicular to the tangent line at that same point, they cannot be parallel unless the slope is zero, implying the tangent line is horizontal, and the normal is vertical, or vice versa.

Another common scenario is:

Find the points on the curve where the normal line is parallel to the tangent line at some other point, which requires setting the slopes accordingly.

In this article, we will explore the general case: Find the point(s) on the curve where the normal line is parallel to the tangent line at some point, i.e., their slopes are equal.

Understanding the Geometry of Tangents and Normals

The Tangent Line

Given a function $y = f(x)$, the slope of the tangent line at a point $x = x_0$ is given by the derivative:

$$
\text{Slope of tangent} = f'(x_0)
$$

For the curve $y = x^2 - 3x + 5$, the derivative is:

$$
f'(x) = 2x - 3
$$

At a point $x = x_0$, the tangent slope is:

$$
m_{tangent} = 2x_0 - 3
$$

The Normal Line

The normal line at a point $x = x_0$ is perpendicular to the tangent at that point. Its slope is the negative reciprocal of the tangent slope:

$$
m_{normal} = -\frac{1}{f'(x_0)} = -\frac{1}{2x_0 - 3}
$$

The equation of the normal line at $x = x_0$ is:

$$
y - y_0 = m_{normal}(x - x_0)
$$

where $y_0 = f(x_0) = x_0^2 - 3x_0 + 5$.

Formulating the Problem Mathematically

Suppose the problem asks:

Find the point(s) on the curve where the normal line is parallel to the tangent line at some other point on the curve.

This involves:

- The tangent slope at point $x = x_0$: $m_{tangent} = 2x_0 - 3$
- The normal slope at point $x = x_1$: $m_{normal} = -\frac{1}{2x_1 - 3}$

Set these equal to find when the normal at x_1 is parallel to the tangent at x_0:

$$
m_{normal}(x_1) = m_{tangent}(x_0)
$$

which gives:

$$
-\frac{1}{2x_1 - 3} = 2x_0 - 3
$$

Similarly, the point (x_1, y_1) on the curve is:

$$
y_1 = x_1^2 - 3x_1 + 5
$$

and the point (x_0, y_0):

$$
y_0 = x_0^2 - 3x_0 + 5
$$

Step-by-Step Solution

Step 1: Set Up the Equation for Parallel Slopes

From the above relation:

$$
-\frac{1}{2x_1 - 3} = 2x_0 - 3
$$

Cross-multiplied:

$$
-1 = (2x_0 - 3)(2x_1 - 3)
$$

which simplifies to:

$$
(2x_0 - 3)(2x_1 - 3) = -1
$$

This is our key equation relating x_0 and x_1.

Step 2: Express y_0 and y_1

Recall:

$$
y_0 = x_0^2 - 3x_0 + 5
$$
$$
y_1 = x_1^2 - 3x_1 + 5
$$

The problem reduces to finding x_0 and x_1 satisfying the above relation.

Step 3: Find x_0 and x_1 satisfying the relation

From:

$$
(2x_0 - 3)(2x_1 - 3) = -1
$$

Express x_1 in terms of x_0:

$$
2x_1 - 3 = - \frac{1}{2x_0 - 3}
$$

$$
2x_1 = 3 - \frac{1}{2x_0 - 3}
$$

$$
x_1 = \frac{3}{2} - \frac{1}{2(2x_0 - 3)}
$$

Now, to find specific points, pick values of x_0 that make the calculations manageable or solve for x_0 satisfying the overall condition.

Step 4: Find the corresponding points and verify

Alternatively, if the question is to find points where the normal to the curve is parallel to the tangent at the same point, then the slopes are equal:

$$
m_{normal} = m_{tangent}
$$

which gives:

$$
-\frac{1}{2x - 3} = 2x - 3
$$

Multiply both sides by $2x - 3$:

$$
-1 = (2x - 3)^2
$$

$$
(2x - 3)^2 = -1
$$

which has no real solutions because the square of a real number cannot be negative.

Thus, the only meaningful solutions arise from the case where the slopes at different points are equal, which is the previous scenario.

Complete Solution: Finding the Specific Point(s)

Let's proceed with solving:

$$
(2x_0 - 3)(2x_1 - 3) = -1
$$

Suppose we pick x_0 and compute x_1.

Example:

Let $x_0 = 1$:

$$
2(1) - 3 = -1
$$

Then:

$$
(-1)(2x_1 - 3) = -1
$$

$$
2x_1 - 3 = \frac{-1}{-1} = 1
$$

\[

$$
2x_1 = 4 \Rightarrow x_1 = 2
$$

Check whether these points satisfy the curve:

$$
y_0 = 1^2 - 3(1) + 5 = 1 - 3 + 5 = 3
$$

$$
y_1 = 2^2 - 3(2) + 5 = 4 - 6 + 5 = 3
$$

So, points:

- $(x_0, y_0) = (1,$

Frequently Asked Questions

How do you find the point on the curve $y = x^2 - 3x + 5$ where the normal is parallel to the tangent?

First, find the derivative $y' = 2x - 3$ to get the slope of the tangent. Set the normal's slope equal to the tangent's slope (since they are parallel) and solve for x to find the point.

What is the slope of the tangent to the curve $y = x^2 - 3x + 5$ at any point?

The slope of the tangent at any point x is given by the derivative $y' = 2x - 3$.

How is the slope of the normal related to the slope of the tangent for the curve $y = x^2 - 3x + 5$?

The slope of the normal is the negative reciprocal of the slope of the tangent. If the tangent slope is m, the normal slope is $-1/m$.

What condition must be satisfied for the normal to be parallel to the tangent on the curve?

Since a normal and tangent are perpendicular, for them to be parallel, their slopes must be equal. Therefore, the slope of the normal must equal the slope of the tangent, which is only possible if the slope is zero, indicating a special case or a mistake in interpretation.

Is it possible for the normal to be parallel to the tangent at the same point on the curve?

No, at a single point, the tangent and normal are perpendicular by definition; they cannot be parallel at the same point unless the slope is zero, which implies a horizontal tangent and a vertical normal, but they are not parallel in that case. The problem likely refers to finding a point where the normal line is parallel to the tangent line at some other point.

How do you set up the equation to find the point where the normal is parallel to the tangent?

Set the slope of the tangent $y' = 2x - 3$ equal to the slope of the normal, which is the same as the tangent's slope at that point, and solve for x to find the specific point(s).

What is the final step after finding the x-coordinate where the normal is parallel to the tangent?

Substitute the x-value back into the original curve $y = x^2 - 3x + 5$ to find the corresponding y-coordinate, thus determining the point(s) on the curve.

Additional Resources

Mathematics Analysis

Introduction: Deciphering the Geometry of Curves and Normals

In the world of calculus and analytical geometry, understanding the relationship between tangents and normals to a curve is fundamental. Whether you're a student striving to master the concepts or an educator seeking to illustrate the elegance of mathematical relationships, the problem of finding a point on a curve where the normal is parallel to the tangent is both intriguing and instructive.

Today, we delve into the specific quadratic curve given by $Y = X^2 - 3X + 5$, exploring how to locate the point(s) where the normal to the curve is parallel to the tangent. This problem is not just a theoretical exercise; it embodies core principles of differential calculus, including derivatives, slopes, and geometric interpretations.

Understanding the Curve and Its Derivative

Analyzing the Function

The curve under consideration is:

$$ y = x^2 - 3x + 5 $$

This is a quadratic function, opening upwards, with its vertex and axis of symmetry critical for understanding its geometric properties. The shape and position of the parabola influence the slopes of tangents and normals at various points.

Key features:

- Vertex: The vertex of the parabola can be calculated using the formula $x = -\frac{b}{2a}$. Here, $a=1$, $b=-3$, so:

$$ x_{vertex} = -\frac{-3}{2 \times 1} = \frac{3}{2} = 1.5 $$

- Vertex coordinate: To find y at $x=1.5$:

$$ y = (1.5)^2 - 3(1.5) + 5 = 2.25 - 4.5 + 5 = 2.75 $$

Thus, the vertex is at (1.5, 2.75).

Calculating the Slope of the Tangent

The derivative $\frac{dy}{dx}$ gives the slope of the tangent at any point x:

$$ \frac{dy}{dx} = 2x - 3 $$

This derivative is pivotal because:

- It provides the slope of the tangent line at any point (x, y).
- The normal to the curve at that point is perpendicular to the tangent, with slope:

$$ m_{normal} = -\frac{1}{m_{tangent}} $$

where $m_{tangent} = 2x - 3$.

Establishing the Condition: When Is the Normal Parallel to the Tangent?

The core question is: Find the point on the curve where the normal is parallel to the tangent.

This appears paradoxical at first glance because:

- The tangent at a point has slope $m_{tangent}$.
- The normal at the same point has slope $m_{normal} = -1/m_{tangent}$.

For the normal to be parallel to the tangent, they must have equal slopes:

$$m_{normal} = m_{tangent}$$

Substituting:

$$-\frac{1}{m_{tangent}} = m_{tangent}$$

This leads to the key algebraic relation:

$$m_{tangent}^2 = -1$$

Solving the Critical Equation

Let's analyze what the equation:

$$m_{tangent}^2 = -1$$

means in real numbers:

- Since $m_{tangent}$ is real, $m_{tangent}^2 \geq 0$.
- The equation $m_{tangent}^2 = -1$ has no real solutions because a square cannot be negative.

Implication: There are no points on the curve where the normal is parallel to the tangent in the usual real-valued Euclidean plane.

Re-examining the Problem: Is There a Misinterpretation?

Given the above, the direct interpretation suggests no such points exist. But to ensure clarity, let's explore alternative interpretations.

Alternate interpretation:

- Could the problem be asking for the point where the normal is parallel to the tangent at some other point? This would involve two different points on the curve.

- Or perhaps, the problem is asking where the normal line is parallel to the tangent line at another point on the curve, i.e., the normal at one point is parallel to the tangent at a different point.

Exploring the Second Interpretation: Normal at one point parallel to tangent at another point

This is a typical approach in such problems, and it often yields solutions.

Restated problem:

Find points P and Q on the curve such that:

- The normal at P is parallel to the tangent at Q.

Step-by-Step Solution Approach

1. Parameterize points:

- Let P be at $x = x_1$, with point (x_1, y_1).
- Let Q be at $x = x_2$, with point (x_2, y_2).

2. Calculate slopes:

- $m_{tangent\, at\, Q} = 2x_2 -3$.
- $m_{normal\, at\, P} = - \frac{1}{2x_1 - 3}$.

3. Set the slopes equal for parallelism:

$$ m_{normal\, at\, P} = m_{tangent\, at\, Q} $$

$$ -\frac{1}{2x_1 - 3} = 2x_2 - 3 $$

4. Express x_2 in terms of x_1:

$$ 2x_2 - 3 = -\frac{1}{2x_1 - 3} $$

$$ 2x_2 = -\frac{1}{2x_1 - 3} + 3 $$

$$ x_2 = \frac{1}{2} \left(-\frac{1}{2x_1 - 3} + 3 \right) $$

Determining the Points on the Curve

- The points P and Q lie on the parabola, so:

$$ y_1 = x_1^2 - 3x_1 + 5 $$

$$ y_2 = x_2^2 - 3x_2 + 5 $$

- For the points to correspond correctly, the normal at P must pass through P, and the tangent at Q must pass through Q.

Line equations:

- Normal at P:

Slope: $m_{normal\, at\, P} = -\frac{1}{2x_1 - 3}$

Equation:

$$ y - y_1 = m_{normal\, at\, P} (x - x_1) $$

- Tangent at Q:

Slope: $2x_2 - 3$

Equation:

$$ y - y_2 = (2x_2 - 3)(x - x_2) $$

For the normal at P to be parallel to the tangent at Q:

- The lines must have the same slope:

$$ m_{normal\, at\, P} = m_{tangent\, at\, Q} $$

which we already have.

Finalizing the Solution: Numerical Approach

Given the algebraic complexity, an exact symbolic solution might be cumbersome. Instead, a numerical approach can be adopted:

- Pick specific values of \(x_1 \) and compute \(x_2 \) accordingly.
- Verify whether the corresponding points satisfy the curve equations.
- Check the slopes and confirm the parallelism condition.

Sample calculation:

Suppose \(x_1 = 0 \):

- \(m_{normal\, at\, P} = -\frac{1}{2 \times 0 - 3} = -\frac{1}{-3} = \frac{1}{3} \)

- \(x_2 = \frac{1}{2} \left(-\frac{1}{-3} + 3 \right) = \frac{1}{2} \left(\frac{1}{3} + 3 \right) = \frac{1}{2} \left(\frac{1 + 9}{3} \right) = \frac{1}{2} \times \frac{10}{3} = \frac{5}{3} \approx 1.6667 \)

- \(y_2 = (1.6667)^2 - 3(1.6667) + 5

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